UNIT I
TRANSMISSION LINE THEORY

General theory of transmission lines - Transmission lines - General Solution, the infinite line, wavelength, velocity of propagation, Waveform distortion - Distortion less line, loading and different methods of loading, Line not terminated in Z_0, Reflection Coefficient, Calculation of current, voltage, power delivered and efficiency of transmission line – Input and transfer impedance - Open and short circuited lines- Reflection factor and reflection loss.

1. INTRODUCTION

Transmission lines are used to transmit electric energy and signals from one point to another, specifically from a source to a load. Examples include the connection between a transmitter and an antenna, connections between computers in a network, or connections between a hydroelectric generating plant and a substation several hundred miles away. Other familiar examples include the interconnects between components of a stereo system and the connection between a cable service provider and your television set. Examples that are less familiar include the connections between devices on a circuit board that are designed to operate at high frequencies.

What all of these examples have in common is that the devices to be connected are separated by distances on the order of a wavelength or much larger, whereas in basic circuit analysis methods, connections between elements are assumed to have negligible length. The latter condition enabled us, for example, to take for granted that the voltage across a resistor on one side of a circuit was exactly in phase with the voltage source on the other side, or, more generally, that the time measured at the source location is precisely the same time as measured at all other points in the circuit. When distances are sufficiently large between source and receiver, time delay effects become appreciable, leading to delay-induced phase differences. In short, deal with wave phenomena on transmission lines in the same manner that deal with point-to-point energy propagation in free space or in dielectrics.

The transmission line is a structure which can transport electrical energy from one point to another. At low frequencies, a transmission line consists of two linear conductors separated by a distance. When an electrical source is applied between the two conductors, the line gets energized and the electrical energy flows along the length of the conductors.

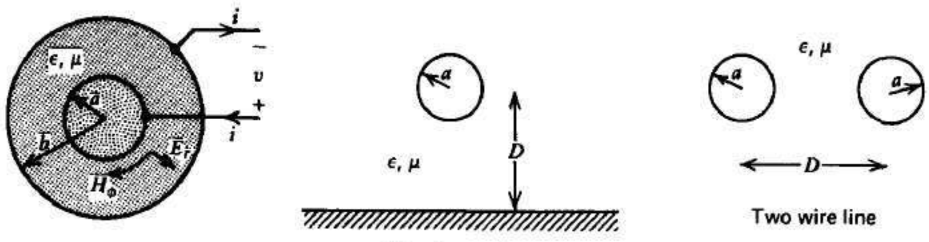


Fig 1.1. Example transmission lines

A two-conductor transmission line may appear in any of the forms shown in the figure. **Co-axial cable** Consists of a solid conducting rod surrounded by the two conductors. This line has good isolation of the electrical energy and therefore has low Electromagnetic Interference (EMI). **Parallel wire transmission line** Consists of two parallel conducting rods. In this case the electrical energy is distributed between and around the rods. Theoretically the electric and magnetic fields extend over infinite distance though their strength reduces as the distance from the line. Obviously this line has higher EMI. **Micro strip line** Consists of a dielectric substrate having ground plane on one side and a thin metallic strip on the other side. The majority of the fields are confined in the dielectric substrate between the strip and the ground plane. Some fringing field exist above the substrate which decay rapidly as a function of height. This line is usually found in printed circuit boards at high frequencies.

1.1. GENERAL THEORY OF TRANSMISSION LINES

1.1.1. Types Of Transmission Lines

The various types of the transmission lines are,

1. Open wire line:

These lines are the parallel conductors open to air hence called open wire line. The conductors are separated by air as the dielectric and mounted on the posts or the towers. The telephone lines and the electrical power transmission lines are the best example of the open wire lines.

Advantages and disadvantages of open wire line

The open wire line easy to construct. It is comparatively cheaper. Since insulation between the conductor is air, the dielectric loss is very small. This line is balanced to the earth.

The main disadvantage of this line is that there is significant energy loss due to radiations. So it is unsuitable at higher frequencies. The open wire lines are requirement of telephone posts and towers hence high initial cost, affected by atmospheric conditions like wind, air ice etc., maintenance is difficult and possibility of shorting due to flying objects and birds. But less capacitance compared to underground cable is the advantages of open wire line.

2. Cables:

These are underground lines. The telephone cables consists of hundred of conductors which are individually insulated with paper. These are twisted in pairs and combined together and placed inside a protective lead or plastic sheath. While underground electrical transmission cables consists of two or three large conductors which are insulated with oil impregnated paper or other solid dielectric and placed inside protective lead sheath. Both these types are still considered as parallel conductors separated by a dielectric.

3. Coaxial line:

As name suggests there are two conductors which are co axially placed. One conductor is hollow and other is placed co axially inside the first conductor. The dielectric may be solid or gaseous. These lines are used for high voltage levels.

Advantages and disadvantages of coaxial cable

The main advantages of the coaxial cable is that the electromagnetic fields cannot leak into the free space; hence radiation losses are totally absent. Outer conductor provides very effective shielding to the external electromagnetic fields. The coaxial cable transmission line is costlier. The losses in the dielectric increase as the frequency of the signal increase. Hence above 1 GHz this line cannot be used.

4. Waveguides:

These types of transmission lines are used to transmit to transmit the electrical waves at microwave frequencies. Constructionally these are the hollow conducting tubes having uniform cross section. The energy is transmitted from inner walls of the tube by the phenomenon of total internal reflection.

Different types of transmission lines are shown in fig 2.2.

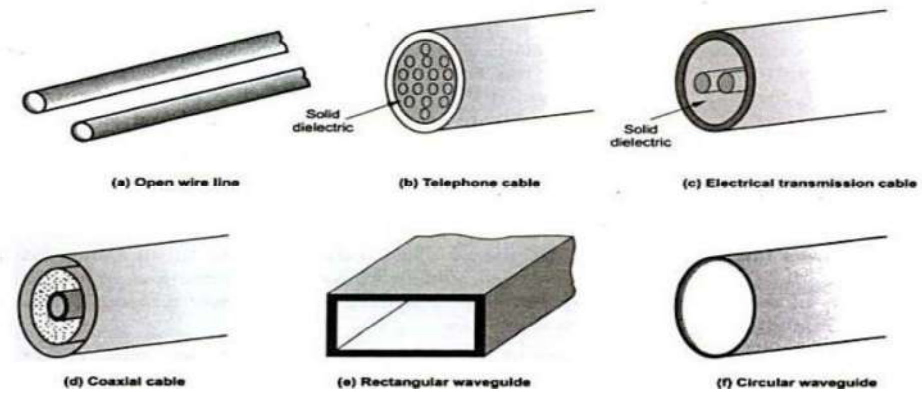


Fig 1.2. Types of transmission lines

1.1.2. Characterization In Terms Of Primary And Secondary Constants

The transmission lines has following constants,

R = Resistance per unit length, measured in ohm (Ω)

G = Conductance per unit length, measured in mho

L = Inductance per unit length, measured in Henry (H)

C = Capacitance per unit length, measured in Farad (F)

All these assumptions to be independent of frequency and are called primary constants of the transmission line. All these constants are measured considering both the wire of transmission line.

Apart from R, G, L, C few other constants related to the transmission line are characteristic impedance Z_0 the propagation constant γ, attenuation constant α, phase constant β. All these constants fixed at one particular frequency but change their values as the frequency changes. Hence these constants are called secondary constants.

The relationship between primary and secondary constants of the transmission line is, The total series impedance per unit length is denoted as Z while the total parallel impedance per unit length is denoted as Y.

$$Z_1 = R + j\omega L$$
$$Y = G + j\omega C$$

Hence the equivalent T network can be shown in fig 2.3.

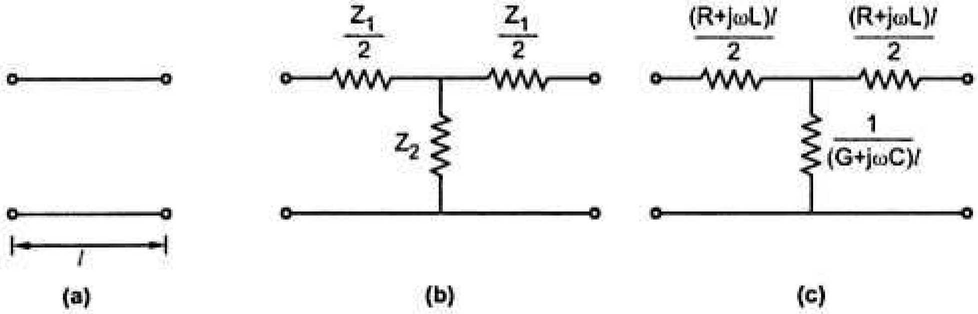


Fig 1.3. Short transmission line representation

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

If γ is representing in the polar form as α+jβ the value of α and β can be directly obtained. While expressing γ from rectangular form to polar form, β must be in radians.

The analysis of the transmission of the electric waves along a line can be done by considering a uniform and symmetrical transmission line. Before starting analysis of the symmetrical transmission line, the electric properties of the symmetrical network.

Any symmetrical network has two important electrical properties namely,

1. Characteristics impedance (Z_0)
2. Propagation constant (γ)

Characteristics impedance (Z_0)

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

The significances of characteristic impedance

a) When the line is terminated in its characteristic impedance, there is no reflection

b) A line of finite length terminated in characteristic impedance acts as an infinite line.

c) When a uniform transmission line is terminated in its characteristic impedance. Its Input impedance will be equal to the characteristic impedance.

d) A network terminated in characteristic impedance at the input as well as at the output terminals is said to be correctly terminated network.

Propagation Constant

Definition

The propagation constant, symbol γ, for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x, such that $\frac{I_1}{I_n} = e^{\gamma n}$. Since the propagation constant is a complex quantity can write

$$\gamma = \alpha + j\beta$$

where α, the real part, is called the attenuation constant

β, the imaginary part, is called the phase constant

For a copper transmission line, the propagation constant can be calculated from the primary line coefficients by means of the relationship $\gamma = \sqrt{ZY}$ where,

$Z = R + j\omega L$ the series impedance of the line per metre,

$Y = G + j\omega C$, the shunt admittance of the line per metre.

Attenuation constant

In telecommunications, the term attenuation constant, also called attenuation parameter or coefficient, is the attenuation of an EM wave propagating through a medium per unit distance from the source. It is the real part of the propagation constant and is measured in nepers per metre. A neper is approximately 8.7dB. Attenuation constant can be defined by the amplitude ratio

$$\left|\frac{I_1}{I_n}\right| = e^{\alpha x}$$

Phase constant

In electromagnetic theory, the phase constant, also called phase change constant, parameter or coefficient is the imaginary component of the propagation constant for a plane wave. It represents the change in phase per meter along the path travelled by the wave at any instant and is equal to the angular wave number of the wave. It is represented by the symbol β and is measured in units of radians per meter. From the definition of angular wave number

$$\beta = \frac{2\pi}{\lambda}$$

1dB= 0.11 Np

1.2. TRANSMISSION LINES - GENERAL SOLUTION

The general solution of a transmission line included the expression for current and voltage at any point along a line of any length having uniformly distributed constants.

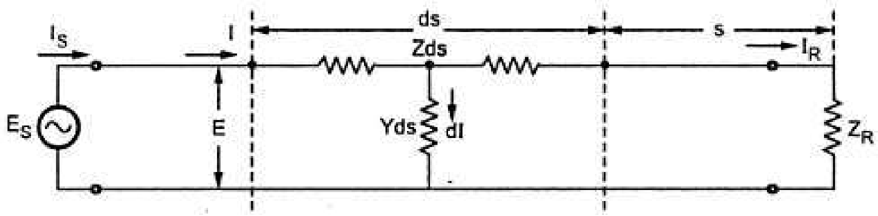


Fig 1.4 T section of a long line

The Various notations used,

R - Series resistance [Ω/ unit length]

L - Series inductance [H / unit length]

C - Capacitance between conductors [F / unit length]

G – Shunt leakage conductance between conductors [mho / unit length]

WL – Series reactance [Ω / Unit length]

Z = R + jwL – Series impedance [Ω/ unit length]

Wc – Shunt susceptance [mho / unit length]

Y = G + jwC – Shunt admittance [mho/ unit length]

S – Distance to the point of observation

I – Current in the line at any point

E – Voltage between conductors at any point

L – Length of the line

Voltage drop in the length ds is,

$$dE = Iz\,ds$$

$$\frac{dE}{dS} = Iz \ldots\ldots\ldots\ldots\ldots..(1)$$

Leakage current flowing through shunt admittance

$$dI = Ey\,ds$$

$$\frac{dI}{dS} = Ey \ldots\ldots\ldots.(2)$$

Differentiate (1) & (2) with respect to S

$$\frac{d^2E}{dS^2} = z\frac{dI}{dS} \qquad \frac{d^2E}{dS^2} = z\frac{dI}{dS}$$

$$\frac{d^2E}{dS^2} = ZEy \ldots\ldots..(3) \qquad \frac{d^2I}{dS^2} = YIz \ldots\ldots..(4)$$

These two equation are Second Order diff. equation

$$\left(\frac{d^2E}{dS^2} - zy\right) E = 0$$

$$(m^2 - zy)E = 0$$

$$\therefore m = \pm\sqrt{zy}$$

The general solution of the equation is,

$$E = A\,e^{\sqrt{zy}S} + Be^{-\sqrt{zy}S} \ldots\ldots\ldots..(5)$$

$$I = C\,e^{\sqrt{zy}S} + De^{-\sqrt{zy}S} \ldots\ldots\ldots..\ (6)$$

A, B, C & D are arbitrary constants. Now necessary to obtain the values of A, B, C & D. As distance measured from the receiving end. S = 0 indicate the receiving end. E = E_R & I = I_R

$$(5) \Rightarrow \quad E_R = A + B$$

$$I_R = C + D \quad \ldots\ldots\ldots\ldots (A)$$

Diff. eqn (5) & (6) W.r.to S,

$$\frac{dE}{dS} = A\sqrt{zy}\,e^{\sqrt{zy}}s - B\sqrt{zy}\,e^{-\sqrt{zy}}s$$

$$\frac{dI}{dS} = C\sqrt{zy}\,e^{\sqrt{zy}}s - D\sqrt{zy}\,e^{-\sqrt{zy}}s$$

Sub. (1) & (2)

$$Iz = A\sqrt{zy}\ e^{\sqrt{zy}}s - B^{\sqrt{zy}}\ e^{-\sqrt{zy}}s$$

$$Ey = C\sqrt{zy}\ e^{\sqrt{zy}} - D^{\sqrt{zy}}\ e^{-\sqrt{zy}}s$$

$$I = \frac{A}{z}\sqrt{zy}\ e^{\sqrt{zy}}s - \frac{B^{\sqrt{zy}}}{z}\quad e^{-\sqrt{zy}}$$

$$E = \frac{C}{y}\sqrt{zy}\ e^{\sqrt{zy}}s - \frac{D^{\sqrt{zy}}}{y}\quad e^{-\sqrt{zy}}$$

$$I = A\sqrt{\frac{y}{z}}\ e^{\sqrt{zy}}s - B^{\sqrt{\frac{y}{z}}}\ e^{\sqrt{zy}}s$$

$$E = C\sqrt{\frac{z}{y}}\ e^{\sqrt{zy}} - D^{\sqrt{\frac{z}{y}}}\ e^{\sqrt{zy}}s$$

Use S = 0 $E = E_R$, $I = I_R$

$$IR = A\sqrt{\frac{y}{z}} - B\sqrt{y/z}$$

$$ER = C\sqrt{\frac{z}{y}} - D\sqrt{z/y}$$

We know that,

$$Z_R = \frac{E_R}{I_R} \quad \& \quad Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}}$$

Adding (A) & (B) Simplifying

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{z}{y}} = \frac{E_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]$$

$$B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{z}{y}} = \frac{E_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{y}{z}} = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]$$

$$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{y}{z}} = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]$$

Hence the general Solution of diff. eqn. is

$$A \Rightarrow E = \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{zy}\,s} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{zy}\,s}\right] \ldots\ldots \qquad I$$

$$I = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right) e^{\sqrt{zy}\,s} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{zy}\,s}\right] \ldots\ldots\ldots \qquad II$$

$$Taking\ LCM\ as\ Z_R\ and\ taking\ \frac{Z_R + Z_0}{Z_R}$$

$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[e^{\sqrt{zy}\,s} + \frac{Z_R - Z_0)}{Z_R + Z_0} e^{-\sqrt{zy}\,s}\right] \ldots\ldots\ldots\ldots\ldots.. \quad A$$

$$Taking\ LCM\ as\ Z_0\ and\ taking\ \frac{Z_R + Z_0}{Z_0}$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\sqrt{zy}\,s} - \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\sqrt{zy}\,s}\right] \ldots\ldots\ldots\ldots\ldots\ldots \quad B$$

This is the general solution for transmission line. It can be arranged as,

(A) ⇒ $$E = E_R\left[\frac{e^{\sqrt{zy}\,s}+e^{-\sqrt{zy}\,s}}{2}\right] + I_R Z_0\left[\frac{e^{\sqrt{zy}\,s}-e^{-\sqrt{zy}\,s}}{2}\right]$$

(B) ⇒ $$I = I_R\left[\frac{e^{\sqrt{zy}\,s}+e^{-\sqrt{zy}\,s}}{2}\right] + \frac{E_R}{Z_0}\left[\frac{e^{\sqrt{zy}\,s}-e^{-\sqrt{zy}\,s}}{2}\right]$$

$$E = E_R \operatorname{Cos h}\sqrt{zy}\,s + I_R Z_0 \operatorname{Sin h}\sqrt{zy}\,s$$

$$I = I_R \operatorname{Cos h}\sqrt{zy}\,s + \frac{E_R}{Z_0}\operatorname{Sin h}\sqrt{zy}\,s$$

1.2.1. Physical Signification Of The Equations The Infinite Line

We know,

$$E = E_S \operatorname{Cos h}(\gamma s) + I_S Z_0 \operatorname{Sin h}(\gamma s)$$

$$I = I_S \operatorname{Cos h}(\gamma s) + \frac{E_S}{Z_0}\operatorname{Sin h}(\gamma s)$$

The sending end current can be obtained by substituting S = l measured from the receiving end $I = I_S$

$$E_S = E_R \cos h\,\gamma l + I_R Z_0 \operatorname{Sin h}\,\gamma l$$

$$I_S = I_R \cos h\,\gamma l + \frac{E_R}{Z_0}\operatorname{Sin h}\,\gamma l$$

$$\therefore \quad I_S = I_R \cos h\,\gamma l + \frac{Z_R}{Z_0} I_R \operatorname{Sin h}\,\gamma l$$

$$I_S = I_R \cos h\,\gamma l + \frac{Z_R}{Z_0}\operatorname{Sin h}\,\gamma l \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Input impedance of the line is terminated in its characteristic impedance Z_0 then Z_R then $Z_R = Z_0$.

$$I_S = I_R[\cos h\,\gamma l + \operatorname{Sin h}\,\gamma l] \qquad e^{\theta} = \cos h\,\theta + \sin h\,\theta$$

$$\frac{I_S}{I_R} = \cos h\,\gamma l + \sin h\,\gamma l = e^{\gamma l} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

When a line terminated in Z_0 then $Z_R = Z_0$

$\therefore \quad Z_S = Z_0$

1.2.3. Infinite Line

Now consider the infinite line $l \to \infty$

$$Z_S = \frac{Z_0(Z_R + Z_0 \tan h\,\gamma l)}{Z_0 + Z_R \tan h\,\gamma l}$$

$\tan h\,\gamma l \to 1$ as $l \to \infty$ $Z_s = Z_0$ (or)

From (A) & (B)

$$\frac{E}{I} = \frac{E_R/Z_R}{I_R/Z_0}\left[\frac{e^{\sqrt{zy}\,s} + \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\sqrt{zy}\,s}}{e^{\sqrt{zy}\,s} - \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\sqrt{zy}\,s}}\right]$$

$$= \frac{Z_0}{Z}\left[\frac{e^{\sqrt{zy}\,s} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\sqrt{zy}\,s}}{e^{\sqrt{zy}\,s} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\sqrt{zy}\,s}}\right]$$

$$\frac{E_s}{I_S} = z_0\left[\frac{e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{\gamma l}}{e^{\gamma l} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{\gamma l}}\right]$$

Now for infinite line $l \rightarrow \infty$ $\therefore\ Z_S = Z_0$

The portion of the infinite line (or) finite line laminated in its characteristic impedance, $Z_R = Z_0$

(A) $\Rightarrow E = E_R e^S$ & $I = I_R e^S$

At the sending end

$$E_R = E_S e^{\gamma S} \quad \& \quad I_R = I_s e^{\gamma S}$$

Since $\gamma = \propto + j\beta$

$$E = E_S e^{S - \propto s} e^{-j\beta s}$$

$$I = I_S e^{S - \propto s} e^{-j\beta s}$$

If we want the effective leading neglect phase angle

$$E = E_S e^{S - \propto s}$$

$$I = I_S e^{S - \propto s}$$

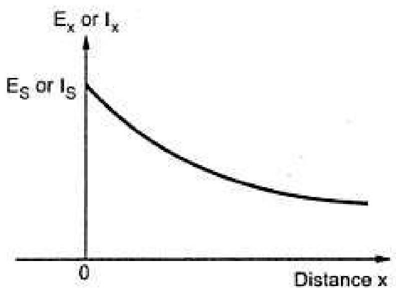


Fig 1.5. General Solution curve

1.3. WAVELENGTH

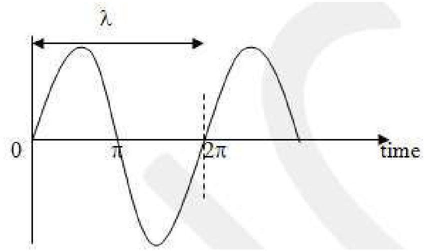
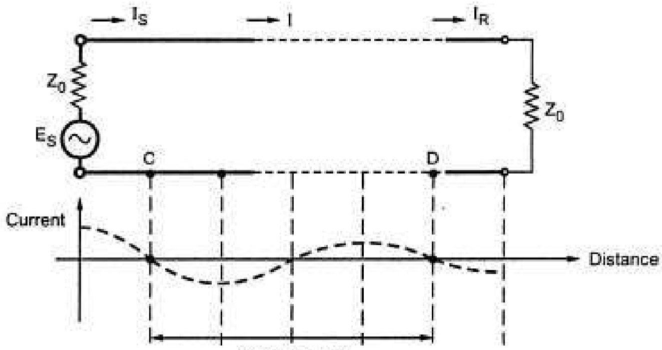


Fig 1.6 Wavelength

Wave length is defined as the distance that a wave travels along the line in order that the total phase shift is 2π rad. Denoted by λ and its unit is meter. At distance S,



The current amplitude and phase decreases down the transmission line from the sending end. The voltage also varies similarly.

The distance between two points along the line at which currents or voltages differ in phase by 2π radians is called wavelength. It is denoted by λ.

It can also be defined as the distance between any point and the next point along the line at which the current or voltage is in the same phase.

The distance between points C and D along the line shown in the Fig. is wavelength λ.

The phase constant of the line β is defined as radians per unit length of line. So if β is defined as rad/km it indicates that there is a phase change of β radians for a distance of 1 km of the line. Hence for a phase shift of 2π radians, the distance will be $2\pi/\beta$ km. This distance corresponding to phase shift of 2π radians is wavelength λ.

$$\lambda = \frac{2\pi}{\beta}$$

In one wavelength, one electrical cycle is completed. If the frequency is f Hz i.e. cycles/sec then for one cycle the time required is called time period given by,

$$T = \frac{1}{f} \text{ sec/cycle}$$

The wave travels distance of λ in one cycle, for which the time required is 1/f sec. Hence the velocity of propagation v can be written as,

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{\left(\frac{1}{f}\right)} = f\lambda$$

$$\therefore \quad v = \frac{2\pi f}{\beta} = \frac{\omega}{\beta}$$

It is measured in km/sec if β is in rad/km and in m/sec if β is in rad/m and so on. As it is related to phase constant of the line, the velocity is called **phase velocity.**

When β is a function of ω then velocity is produced by introduction of a group of frequencies travelling through the system. Such a velocity is called **group velocity** and can be obtained as,

$$v_g = \frac{d\omega}{d\beta}$$

This velocity plays an important role in wave guides and not in transmission line.

1.4. VELOCITY OF PROPAGATION

Velocity :

$$V = \frac{distance}{time} = \frac{x}{t}$$

$$x = vt \quad C = ?\lambda \quad C = \mu\pi$$

$$V = f\lambda$$

$$V = \lambda f = \frac{2\pi}{\beta} f$$

$$V = \frac{w}{\beta}$$

This is the Velocity of propagation along the line it measured in miles per sec. If us in β read per mile. Velocity of propagation is defined as the velocity with which a signal of single frequency propagates along the line as a particular frequency f denoted V_P. $V_p = \frac{W}{\beta}$

$w.k.t,$ $\quad z = R + jwL$

$$y = G + jwC$$

Then the propagation constant $\gamma = \propto + j\beta = \sqrt{zy}$

$$\propto + j\beta = \sqrt{(R + jwL)(G + jwC)}$$

$$\sqrt{RG + jRWC + jWLG - W^2LC}$$

Squaring both sides, $\quad \propto^2 \div \beta^2 + j^2 \propto \beta = RG\, jW\,(RC + LG) - W^2LC$

Equating the real Part on both side, $\propto^2 - \beta^2 = RG - W^eLC$

$$\propto^2 = \beta^2 + RG - W^eLC \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Equating the imaginary part on both side, $\quad 2 \propto \beta = W(RC + LG$

Squaring $\quad 4 \propto^2 \beta^2 = W^2 (Rc + LG)^2 \quad \ldots\ldots\ldots\ldots.(2)$

Sub eqn. (1) in (2) $\quad 4(\beta^2 + RG - W^2LC)\beta^2 = W^2 (RC + LG)^2$

$$(4\beta^2 + 4RG - 4W^2LC)\beta^2 = W^2 (RC + LG)^2$$

$$(4\beta^4 + 4\beta^2RG - 4\beta^2W^2LC) = W^2 (RC + LG)^2$$

÷ ing throught by 4,

$$\beta^4 + \beta^2 (RG - W^2LC) - \frac{W^2}{4}(RC + LG)^2 = 0$$

$$\beta^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\beta^2 = \frac{-(RG - W^2LC) \pm \sqrt{(RG - W^2LC)^2 + W^2(RC + LG)^2}}{2}$$

$$= \frac{W^2LC - RG \pm \sqrt{(RG - W^2LC)^2 + W^2(RC + LG)^2}}{2}$$

$$\frac{\sqrt{W^2LC - RG \pm \sqrt{(RG - W^2LC)^2 + W^2(RC + LG)^2}}}{2}$$

Sub β in (1)

$$\propto^2 = \frac{W^2LC - RG \pm \sqrt{(RG - W^2LC)^2 + W^2(RC + LG)^2}}{2}(RG + W^2LC)$$

$$\frac{W^2LC - RG \pm \sqrt{(RG - W^2LC)^2 + W^2(RC + LG)^2 + 2RG + 2W^2LC}}{2}$$

$$\propto = \sqrt{1/_2 [RG - W^2LC \pm \sqrt{(RG - W^2LC)^2 + W^2(LG + RC)^2}}$$

In Perfect line, R = 0, G = 0 ∴ $\beta = W\sqrt{LC}$

Velocity of propagation

$$VP = \frac{w}{\beta}$$

$$VP = \frac{1}{\sqrt{LC}} m/s$$

R and G for Minimum Attenuation

When R or G is variable, then by differentiating α and equating it to zero, no minima can be found out mathematically. So theoretically no values of R and G can be obtained for minimum attenuation.

But practically it can be seen that when R = 0 there are no losses along the line while when G = 0 there is no leakage thus in all when R and G are zero, the attenuation is zero. This can be observed from the expression for α at R = 0 and G = 0.

1.5. WAVEFORM DISTORTION

When the received signal is not the exact replica of the transmitted signal, then the signal is said to be distorted. Type of distortions are,

i. Distortion due to characteristic impedance
ii. Frequency distortion
iii. Phase (or) delay distortion.

i. Distortion due to Z_0 Varying with Frequency

The characteristic impedance Z_0 of the line varies with the frequency while the line is terminated in an impedance which does not vary with frequency in similar fashion as that of Z_0. This causes the distortion. The power is absorbed at certain frequencies while it gets reflected for certain frequencies. So there exists the **selective power absorption**, due to this type of distortion.

It is known that,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R}{G}\frac{\left(1+j\omega\frac{L}{R}\right)}{\left(1+j\omega\frac{C}{G}\right)}}$$

If for the line, the condition LG = CR is satisfied then $\frac{L}{R}=\frac{C}{G}$ and hence

$$\left(1+j\omega\frac{L}{R}\right) = \left(1+j\omega\frac{C}{G}\right)$$

$$Z_0 = \sqrt{\frac{R}{G}}\angle 0^\circ = \sqrt{\frac{L}{C}}\angle 0^\circ\ \Omega$$

For such a line Z_0 does not vary with frequency ω and it is **purely resistive** in nature.

Such a line can be easily and correctly terminated in an impedance which matches with Z_0 at all the frequencies. For such a line, $Z_R = \sqrt{\frac{R}{G}}$ or $\sqrt{\frac{L}{C}}$.

ii*) Frequency distortion*

All the frequencies transmitted on a line will not be attenuated equally. When a Complex Voltage containing many frequencies is transmitted over the line will not have all frequencies transmitted will equal attenuation and the received waveform will not be identical with the input waveform at the sending end. This variation is known as frequency distortion. It is the type of distortion in which the time required to transmit the various frequency component of the signal undergoes different attenuation. The attenuation constant

$$\propto = \sqrt{1/2\,[\,RG - W^2LC + \sqrt{(RG - W^2LC)^2 + W^2(LG+RC)^2}}$$

$\propto$ is the function of frequency $w = 2\pi f$. If $\propto$ is not a function of frequency, frequency distortion does not exists on transmission lines.

iii. Phase (or) Delay distortion:

The Phase constant β is,

$$\beta = \sqrt{1/2\,[\,W^2LC - RG + \sqrt{(RG - W^2LC)^2 + W^2(LG+RC)^2}}.$$

It is also the function of frequency. The velocity propagation Vp = ω/β. It is also the function of frequency. All frequencies applied to transmission line will not have the same time of transmission, some frequencies being delayed more than the others. For an applied Voice Voltage wave, the received waveform will not be identical with the input waveform at the sending end, since some components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion. Frequency distortion is reduced by using equalizers at the line terminals. These circuits are networks whose frequency and phase characteristics are adjusted to be inverse those of the lines resulting in an overall freq. response over the desired frequency range band. Delay distortion is very serious in picture transmission. This is overcome by the internal inductance, resistance and capacitance are small. Also the leakages are neglected because of air dielectric. Hence the velocity of propagation is raised and made more nearly equal for all frequencies.

1.6. DISTORTION LESS LINE

A line in which there is no phase or frequency distortion and also it is correctly terminated, is called a **distortionless line.**

To derive the condition for distortionless line consider,

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\therefore \quad \gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\therefore \quad \gamma^2 = \left(RG-\omega^2 LC\right)+j\omega C(RC+LG) \quad \dots (1)$$

It is known that for minimum attenuation $L=\frac{CR}{G}$ i.e. LG = CR. Substituting this condition in equation (1) we get,

$$\gamma^2 = RG-\omega^2 LC+j\,2\omega RC$$

$$\text{But} \quad RC = LG = \sqrt{RCLG}$$

$$\therefore \quad \gamma^2 = RG-\omega^2 LC+j2\omega\sqrt{RCLG}$$

$$\therefore \quad \gamma^2 = \left(\sqrt{RG}+j\omega\sqrt{LC}\right)^2$$

$$\therefore \quad \gamma = \sqrt{RG}+j\omega\sqrt{LC} \quad \dots (2)$$

$$\text{But} \quad \gamma = \alpha+j\beta$$

$$\therefore \quad \boxed{\alpha = \sqrt{RG}} \quad \dots (3)$$

$$\text{and} \quad \boxed{\beta = \omega\sqrt{LC}} \quad \dots (4)$$

It can be seen from the equation (3) that α does not vary with frequency which elliminates the frequency distortion.

Now $\beta = \omega\sqrt{LC}$ ... for the condition LG = CR

Now $v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$ km/sec ... (5)

Thus for the condition LG = CR, the velocity becomes independent of frequency. This eliminates the phase distortion.

It is already proved that for RC = LG, the Z_0 becomes resistive and line can be correctly terminated to eliminate distortion due to Z_0 varying with frequency. Thus all the distortions are eliminated for a condition,

$$RC = LG \quad \text{i.e.} \quad \frac{R}{G} = \frac{L}{C} \qquad \text{... (6)}$$

This is the required condition for a **distortionless line.** For such a line, received signal is exact replica of the signal at the sending end, though it is delayed by constant propagation time and its amplitude reduces.

Another important observation is that the condition for a distortionless line is identical to the condition for a minimum attenuation with L or C varied.

Thus if the primary line constants do not mutually naturally satisfy the condition of equation (6), then this condition will have to be satisfied artificially by increasing L or decreasing C. When this is done artificially, the line is said to be **loaded line** and the process of artificially achieving the condition is called **loading of a line.**

However if the frequency of operation is very high, then though the condition of distortionless line is not satisfied then $j\omega L >> R$ and $j\omega C >> G$ due to high ω. Hence R and G can be neglected. Thus Z_0 becomes equal to $\sqrt{L/C}$ which is automatically real and resitive and line can be perfectly terminated. Hence for very high frequency operation like radio frequencies though distortionless condition is not satisfied, the line is automatically distortionless and hence loading is not essential. Practically for the line, R/G is higher than L/C and usually L is increased artificially to match the distortionless condition.

1.7. LOADING AND DIFFERENT METHODS OF LOADING

It is seen earlier that if the primary constants of a line, mutually satisfy the relationship RC = LG then the distortionless transmission results.

For a practical line, R/G is always more than L/C and hence the signal is distorted. Thus the preventive remedy is to make the condition $\frac{R}{G} = \frac{L}{C}$ satisfy artificially.

To satisfy the condition, it is necessary to reduce R/G or increase L/C. Let us consider all the possibilities. To reduce R/G, it is necessary to decrease R or increase

G. The resistance R can be decreased by increasing the area of cross-section i.e. diameter of the conductors. This increases the size and cost of the line. Hence this possibility is uneconomical.

To increase G, it is necessary to use poor insulators. To get poor insulator is easy and economical but from the receiving end point of view, increase in G is very much uneconomical. When G is increased, the leakage of the signal will increase, though it becomes distortionless. So quality improves but quantity decreases. Thus increase in G is quality at the cost of quantity. The signal at receiving end must activate the receivers. But if leakage is more, then received signal becomes so weak that amplifiers are required at the intermediate stages. This makes the design complicated. Hence advice of increasing G to reduce ratio R/G is 'penny wise pound foolish' advice. It is the worst advice and hence this possibility is ruled out in practice.

Now to increase L/C, it is necessary to increase L or decrease C. If C is to be reduced, then the seperation between the lines will be more. Thus the brackets which were carrying previously more wires will now carry very less number of wires due to increased separation. Hence more number of brackets are required. Taller towers and posts are required and also number of towers and posts per unit length of the line will be increased. Thus for the same strength of the line, the line will become very much costlier due to decrease in C. Hence this possibility is also ruled out.

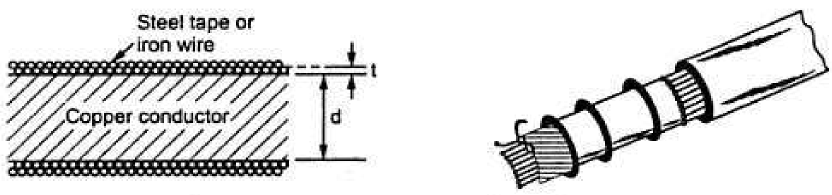
Thus the only alternative left is to increase L. This is opted in practice. The process of increasing the inductance L of a line artificially is called **loading of a line.** And such a line is called **loaded line.**

There are two methods of loading a line which are,

1. **Continuous loading** which is also called **Krarup loading** or **Heavyside loading.**
2. **Lump loading** which is also called **Pupin loading** or **Coil loading.**

1. Continuous Loading

In this method of loading, to increase the inductance, on each conductor the tapes of magnetic material having high permeability such as permalloy or μ-metal are wound. This is shown in the Fig. 1.7 (a) while the loaded cable is shown in the Fig. 1.7 (b).



(a) Continuous loading (b) Continuously loaded cable

The increase in the inductance for a continuously loaded line is,

$$L \approx \frac{\mu}{\frac{d}{nt}+1} \text{ mH}$$

where μ = Permeability of surrounding material

d = Diameter of copper conductor

t = Thickness per layer of tape or iron wire

n = Number of layers

Propagation Constant of Continuously Loaded Cable

For the continuously loaded cable it can be assumed that its $G = 0$ and L is increased such that $\omega L >> R$.

Now $\quad Z = R + j\omega L \quad$ and $\quad Y = j\omega C \quad$ as $G = 0$

Thus

$$\gamma = \sqrt{YZ} = \sqrt{(R+j\omega L)(j\omega C)}$$

$$= \sqrt{\sqrt{R^2+\omega^2 L^2} \angle \frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L} \quad \omega C \angle \frac{\pi}{2}}$$

The angle of Z which is $\tan^{-1}\frac{\omega L}{R}$ is written as $\frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}$

$$\gamma = \sqrt{\omega L \sqrt{1+\frac{R^2}{\omega^2 L^2}} \quad \omega C \angle \pi - \tan^{-1}\frac{R}{\omega L}}$$

Neglecting $R^2/\omega^2 L^2$ as $\omega L >> R$,

$$\gamma = \omega\sqrt{LC} \quad \angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}$$

Angle becomes half out of the square root sign.

Let

$$\theta = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}$$

$$\therefore \quad \cos\theta = \cos\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = \sin\left(\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

But for a small angle, $\quad \sin\theta = \tan\theta = \theta \quad$ i.e. $\tan^{-1}\theta = \theta$

$$\therefore \quad \tan^{-1}\frac{R}{\omega L} = \frac{R}{\omega L} \text{ and } \sin\left(\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = \frac{R}{2\omega L}$$

$$\therefore \quad \cos\theta = \frac{R}{2\omega L}$$

and
$$\sin\theta = \sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) \approx \sin\frac{\pi}{2} = 1 \quad \text{... as } \omega L >> R$$

$$\therefore \quad \gamma = \omega\sqrt{LC}\ \angle\theta = \omega\sqrt{LC}\ [\cos\theta + j\sin\theta]$$

As $\cos\theta + j\sin\theta = \sqrt{\cos^2\theta + \sin^2\theta}\ \angle\tan^{-1}[\tan\theta] = 1\angle\theta$

$$\therefore \quad \gamma = \omega\sqrt{LC}\cos\theta + j\,\omega\sqrt{LC}\sin\theta$$

$$\therefore \quad \boxed{\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}} \text{ and } \boxed{\beta = \omega\sqrt{LC}}$$

and
$$\boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

Thus the attenuation factor α is not the function of frequency while velocity v is also independent of frequency. And hence continuously loaded cable is distortionless.

Advantages

The advantages of the continuous loading are,

1. The attenuation to the signal is independent of the frequency and it is same to all the frequencies.
2. The attenuation can be reduced by increasing L, provided that R is not increased greatly.
3. The increase in the inductance upto 100 mH per unit length of line is possible.

Disadvantages

The disadvantages of the continuous loading are,

1. The method is very costly.
2. Existing lines can not be modified by this method. Hence total replacement of the existing cables by the new cables wound with magnetic tapes is required. This is again costly and uneconomical.
3. Extreme precision care must be taken while manufacturing continuously loaded cable, otherwise it becomes irregular.

4. The size is increased. Thus the capacitance increases. Hence the benefit obtained by increase in L is partly nullified.
5. All along the conductor, there will be huge mass of iron. Thus for a.c. signals there will be large eddy current and hysteresis losses. The eddy current losses increase directly with square of frequency while the hysteresis losses increase directly with the frequency. Hence overall this puts the upper limit to increase inductance.
6. The maximum value by which inductance can be increased is fixed as 100 mH per unit length of line.

Thus this method of loading is not used for the landlines but are preferred for the submarine cables. For underwater circuits lumped loading is difficult to use. It is not necessary to load the submarine cable continuously while the sections of loaded cable separated by the sections of unloaded cable can be used. This reduces the cost, still enjoying the advantages of continuous loading. This is called **patch loading.**

2 Lumped Loading

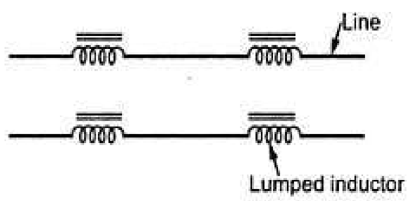


Fig. 1.8 Lumped loading

In this type of loading, the inductors are introduced in lumps at the uniform distances, in the line. Such inductors are called **lumped inductors.** The inductors are introduced in both the limbs to keep the line as balanced circuit. The lumped inductors are in the form of coils called loading coils. The method is shown in the Fig. 1.8

The lumped loading is preferred for the open wire lines and cables for the transmission improvement. The loading coil design is very much important in this method. The core of the coils is usually toroidal in shape and made of permalloy. This type of core produces the coil of high inductance, having small dimensions, very low eddy current losses and negligible external field which restricts the interference with neighbouring circuits.

The loading coil is wound of the largest guage of wire consistent with small size. Each winding is divided into equal parts, so that exactly half the inductance can be inserted into each leg of the circuit. These are built into steel pots which are made in several standard sizes to accomodate one or more coils. The pots protect the coils from

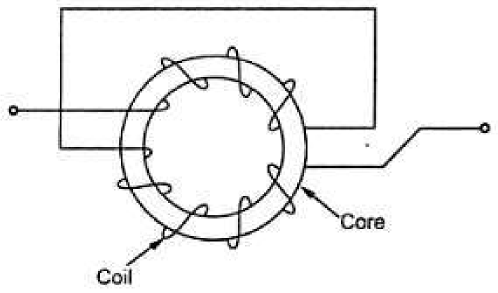


Fig. 1.9 Winding of single loading coil

external magnetic fields, weather and mechanical damage. The Fig. 1.9 shows the construction of loading coils. While installing the coils, the care must be taken so that the circuit balance is maintained. No winding is reversed. If winding is reversed, it will neutralize the inductance of other winding reducing the overall inductance.

In case of lumped loading the line behaves properly provided spacing is uniform and loading is balanced, upto a certain frequency called **cut-off frequency** of the line. Upto this frequency, the added inductance behaves as if it is distributed uniformly along the line. But above this cut-off frequency the attenuation constant increases rapidly. The line acts as low pass filter. The graph of α against the frequency called the **attenuation frequency characteristics** of the line is shown in the Fig. 1.10. It can be seen that for continuous loading, the attenuation is independent of frequency while for lumped loading it increases rapidly after the cut-off frequency.

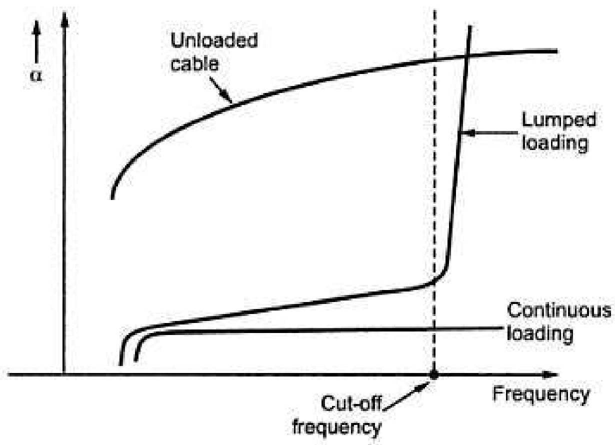


Fig. 1.10 Attenuation frequency characteristics

If the loading section distance is d then keeping inductance L_S of the loading coil constant, the cut-off frequency is found to be proportional to the $1/\sqrt{d}$. Hence to get the higher cut-off frequency, small lumped inductances must be used at smaller distances.

Campbell's Equation

This equation gives the analysis of the performance of a loaded line.

Let Z_0 = Characteristic impedance of the line before loading

γ = Propagation constant of the line before loading

Z_L = Impedance of the loading coil

N = Coil spacing i.e. distance in km between loading coils

γ' = Propagation constant of the line after loading

The analysis can be done by considering a symmetrical section from one loading to the next loading coil. The T equivalent section of the line is used for the analysis. The Fig. 1.11 (a) shows the unloaded line and corresponding T equivalent while the Fig. 1.11 (b) shows the loaded line and corresponding T equivalent section.

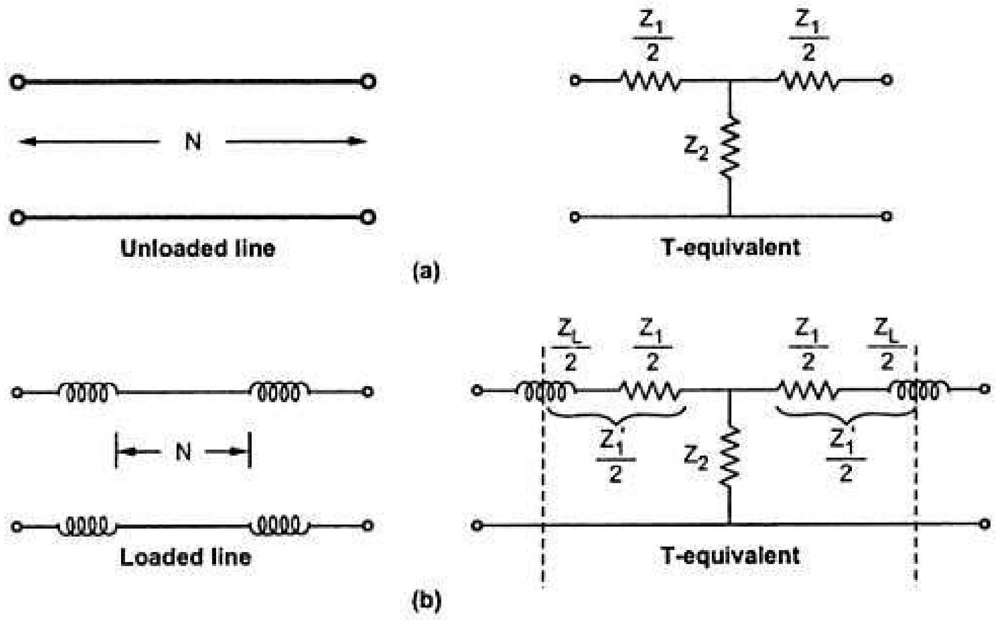


Fig. 1.11

Consider equivalent T section before loading the line.

for a line of length N,

$$\sinh(N\gamma) = \frac{Z_0}{Z_2} \quad \text{... (1)}$$

and $$\cosh(N\gamma) = 1+\frac{Z_1}{2\,Z_2} \quad \text{... (2)}$$

When the loading section is added, the equivalent series arm of the loaded section becomes,

$$\frac{Z_1'}{2} = \frac{Z_L}{2} + \frac{Z_1}{2} \quad \text{... (3)}$$

The shunt arm of the equivalent T section remains unchanged as $Z_2 = Z_0 / \sinh(N\gamma)$.

Now from (2),

$$\frac{Z_1}{2} = Z_2\left[\cosh(N\gamma) - 1\right] = \frac{Z_0}{\sinh(N\gamma)}\left[\cosh(N\gamma) - 1\right] \quad \text{... (4)}$$

$$\frac{Z_1'}{2} = \frac{Z_L}{2} + \frac{Z_0}{\sinh(N\gamma)}\left[\cosh(N\gamma) - 1\right] \quad \text{... (5)}$$

Now γ' is the new propagation constant after loading,

$$\therefore \quad \cosh(N\gamma') = 1 + \frac{Z_1'}{2Z_2} = 1 + \frac{(Z_1'/2)}{Z_2}$$

$$= 1 + \frac{\left\{\frac{Z_L}{2} + \frac{Z_0}{\sinh(N\gamma)}\left[\cosh(N\gamma) - 1\right]\right\}}{\left(\frac{Z_0}{\sinh(N\gamma)}\right)}$$

$$= 1 + \frac{\sinh(N\gamma)}{2Z_0}\left\{Z_L + \frac{2Z_0}{\sinh(N\gamma)}\left[\cosh(N\gamma) - 1\right]\right\}$$

$$= 1 + \frac{Z_L \sinh(N\gamma)}{2Z_0} + \cosh(N\gamma) - 1$$

$$\therefore \quad \cosh(N\gamma') = \cosh(N\gamma) + \frac{Z_L}{2Z_0}\sinh(N\gamma) \quad \text{... (6)}$$

This expression is known as the Campbell's equation for a loaded line. It gives the expression for the propagation constant of loaded line, interms of the propagation constant of a loaded line.

For a cable, Z_2 i.e. shunt arm is essentially capacitive. The cable capacitance and lumped inductance appear similar to the circuit of low pass filter. The cut-off frequency of the low pass filter is given by,

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

where L is the total inductance per unit length which is addition of inductance per unit length of line and that of the loading coil added to the line.

The attenuation reduces below f_c, due to the loading effect. But due to the filter action, it increases rapidly after f_c is crossed. Thus f_c puts an upper limit for the successful transmission over the lines.

In practice R and L are to some extent are the functions of frequency hence truely distortionless line is not possible. But loaded cables perform much better than the unloaded cables.

Advantages

The advantages of lumped loading are,

1. There is no practical limit to the value by which the inductance can be increased.
2. The cost involved is small.
3. With this method, the existing lines can be tackled and modified.
4. Hysteresis and eddy current losses are small.

Disadvantages

The only disadvantage of this method is its action like low pass filter. The attenuation increases considerably after the cut-off frequency. The cut-off frequency must be at the top of voice frequency. Hence fractional loading is used. Whatever distortion results due to fractional loading is corrected using equalizers. The care must be taken while installing the lumped inductors so as to maintain the exact balancing of the circuit.

1.8. LINE NOT TERMINATED IN Z_0

If a line is not terminated in Z_0 [$Z_R \neq Z_0$] then part of the wave is reflected back from the receiving end. Such a reflection is maximum. When a line is open circuit $Z_R = \infty$ (or) short circuit $Z_R = 0$. From the general solution of the line, we can write.[1]

$$E = \frac{E_R(Z_0 + Z_R)}{2Z_R}\left[e^{\sqrt{ZY}S} + \frac{Z_R - Z_0}{Z_R + Z_0}\, e^{-\sqrt{ZY}S}\right]$$

$$I = \frac{I_R(Z_R + Z_0)}{2Z_0}\left[e^{\sqrt{ZY}S} - \frac{Z_R - Z_0}{Z_R + Z_0}\, e^{-\sqrt{ZY}S}\right]$$

When Z_R is not equal to Z_0, each equation consists of two terms, one varies exponentially with +s and other with –s. When a wave travels from sending and (S max) to the receiving end of the line, its amplitude decreases as (S min) it approaches the receiving end. At S max amplitude is increases. At S_{min} it gets decreased. This wave of voltage (or) Current is known as the incident wave. It can be identified as the component varying with $e^{\gamma S}$. When the waves travels from receiving end towards the sending end its amplitudes goes on decreasing. ie., at S min amplitude increases and at S max amplitude decreases, This wave is known as reflected wave can be identified as the component varying with $e^{\gamma S}$. The total instantaneous voltage at any point on the line is the vector sum of the voltage of the incident and reflected waves. In the case of an infinite line (S = ∞) (or) for $Z_R = Z_0, \frac{Z_R - Z_0}{Z_R + Z_0} = k$ is Zero. In open circuit, the identical value of the reflected wave is equal to incident Voltage at the load.

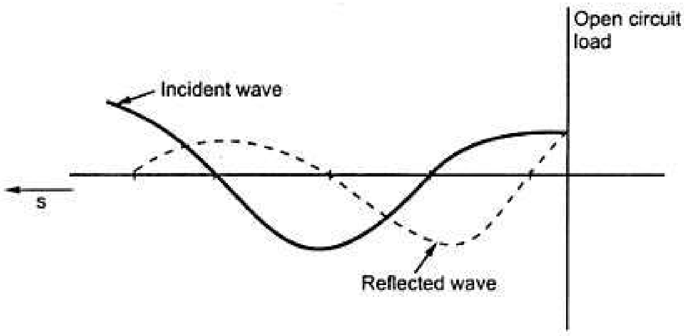


Fig 1.7: Instantaneous Voltage for open circuit line

The only difference between the curve for voltage current for open circuited line is the revered phase of the reflected current wave. The wave of instantaneous current are, open circuit load.

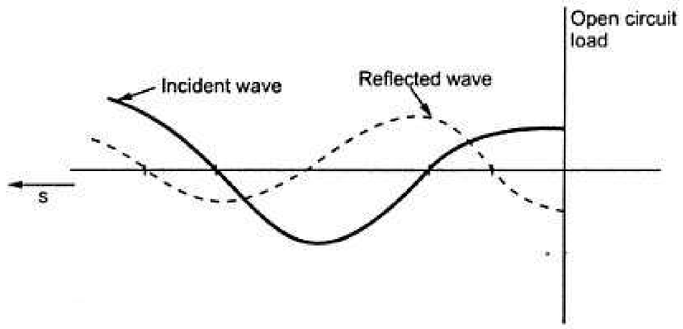


Fig 1.8: Instantaneous current for open circuit line

Thus addition of instantaneous current at the open circuited receiving end is always Zero. The term $\frac{Z_R - Z_0}{Z_R + Z_0}$ decides the relative phase angles between incident and reflected waves. In the infinite line with (S = ∞), the waves travels smoothly along the line and the energy is absorbed in the load Z_o without any reflected wave. Such a finite line terminated in Z_0 without having any reflection is called a smooth line. For an ideal lossless line with R=G=0 the ratio of voltage to current is a constant given by $Z_0 = \frac{E}{I}$ and the distribution of energy between the electric and magnetic fields are fixed. The energy conveyed in the electric field is,

$$W_m = \frac{CE^2}{2} \text{Joules/m}^2$$

The energy conveyed in the magnetic field is, $W_m = \frac{LI^2}{2} \text{Joules/m}^2$

If $R = G = 0, Z_0 = \sqrt{L/C}$

$$E = IZ0$$

$$\therefore\ Z_0^2 = \frac{L}{C}\ hence\ C = \frac{L}{Z^2}$$

$$\therefore\ w_e = \frac{1}{2}\frac{L}{Z_0^2}E^2 = \frac{1}{2}\frac{1}{Z_0^2}(IZ_0)^2$$

$$= \frac{1}{2}LI^2$$

$$we = wm$$

Thus for a ideal line terminated in Z_0 at all the points along the line, electric field energy is equal to the magnetic field energy.

1.8. REFLECTION COEFFICIENT

Reflection in a transmission line is either due to impedance irregularity or when the line is not correctly terminated. The ratio of amplitudes of the reflected and incident voltage wave at the receiving end of the line is frequently called reflection coefficient.

$$K = \frac{\text{reflected voltage at load}}{\text{incident voltage at load}}$$

The equation for voltage at any point on a transmission line is,

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\sqrt{ZY}S} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}S}\right] \quad \ldots\ldots\ldots\ldots\ldots(1)$$

S – distance to the point of observation, measured from the receiving end of the line.
Z – Series impedance Y – Shunt admittance
E_R – Voltage across Z_R I_R - Current flowing through Z_R

$$k = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}[e^{\gamma S} + Ke^{-\gamma S}]$$

$$I = \frac{I_R(Z_R + Z_0)}{2Z_0}[e^{\gamma S} + Ke^{-\gamma S}]$$

The sign of k is dependent on the angle and magnitudes of Z_R and Z_0. For a line terminated in Z_0 ($Z_R = Z_0$) the reflection coefficient is Zero.

1.9. CALCULATION OF CURRENT, VOLTAGE

The absolute value of the ratio of input current to output current of a given symmetrical network was defined as an exponential function. When a symmetrical network terminated by Z_0 the exponential can be extended to include the Phasor current ratios. $\frac{I_1}{I_2} = e^{\gamma}$ Where γ is the complex number, defined as $\gamma = \alpha + j\beta$ If $\frac{I_1}{I_2} = A\angle\beta$ then $A = \frac{I_1}{I_2} = e^{\gamma}$; $\angle\beta = e^{j\beta}$

Since the input and output impedance are equal. Similarly $\frac{V_1}{V_2} = e^{\gamma}$. The term γ has been given the propagation constant. The exponent α is known as attenuation constant. The unit of α are Nepers. The exponent β is the Phase constant as it is determines the phase angle between input and output quantities. The unit of β is radians. If the number of sections all having a common Z_0 value are cascaded the ratio of current is, $\frac{I_1}{I_2} x \frac{I_2}{I_3} x \frac{I_3}{I_4} x \ldots\ldots\ldots\ldots = \frac{I_1}{I_n}$

Which is, $e^{\gamma_1} x e^{\gamma_2} x e^{\gamma_3} x \ldots\ldots\ldots\ldots = e^{\gamma_n}$

Taking log on both sides, $\gamma_1 x \gamma_2 x \gamma_3 x \ldots\ldots\ldots\ldots\ldots. = \gamma_n$

Thus the overall propagation constant is equal to the sum of individual propagation constant.

1.12. INPUT AND TRANSFER IMPEDANCE

It is derived in earlier section that the input impedance of a transmission line is given by,

$$Z_{in} = Z_S = \frac{Z_0\left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)\right]}{\left[Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l)\right]} \quad \ldots (1)$$

$$Z_S = Z_0\left[\frac{1+\frac{Z_0}{Z_R}\tanh(\gamma l)}{\frac{Z_0}{Z_R}+\tanh(\gamma l)}\right] \quad \ldots (2)$$

Let $Z_T = \frac{E_S}{I_R}$ = **Transfer impedance** of a line

Now $I_R = I_S \cosh(\gamma l) - \frac{E_S}{Z_0}\sinh(\gamma l)$

$$\therefore \quad 1 = \frac{I_S}{I_R}\cosh(\gamma l) - \frac{Z_T}{Z_0}\sinh(\gamma l) \quad \ldots (3)$$

While $E_R = E_S \cosh(\gamma l) - I_S Z_0 \sinh(\gamma l)$

$$\therefore \quad I_S = \frac{E_S \cosh(\gamma l) - E_R}{Z_0 \sinh(\gamma l)} \quad \ldots (4)$$

Substituting in (3),

$$1 = \frac{\cosh(\gamma l)}{I_R}\left[\frac{E_S \cosh(\gamma l) - E_R}{Z_0 \sinh(\gamma l)}\right] - \frac{Z_T}{Z_0}\sinh(\gamma l)$$

$$1 = \left(\frac{E_S}{I_R}\right)\frac{\cosh^2(\gamma l)}{Z_0 \sinh(\gamma l)} - \left(\frac{E_R}{I_R}\right)\frac{\cosh(\gamma l)}{Z_0 \sinh(\gamma l)} - \frac{Z_T \sinh(\gamma l)}{Z_0}$$

$$\ldots \frac{E_R}{I_R} = Z_R$$

$$1 = \frac{Z_T \cosh^2(\gamma l) - Z_R \cosh(\gamma l) - Z_T \sinh^2(\gamma l)}{Z_0 \sinh(\gamma l)}$$

$$Z_0 \sinh(\gamma l) = Z_T[\cosh^2(\gamma l) - \sinh^2(\gamma l)] - Z_R \cosh(\gamma l)$$

$$Z_0 \sinh(\gamma l) = Z_T - Z_R \cosh(\gamma l)$$

$$Z_T = Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)$$

In terms of exponential coefficients this can be expressed as,

$$Z_T = Z_R \left[\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right] + Z_0 \left[\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right]$$

This is the required transfer impedance of a transmission line, terminated in an impedance Z_R.

1.13. OPEN AND SHORT CIRCUITED LINES

When the line is open circuited, then IR = 0. Thus the magnetic field energy is Zero. The magnetic field energy carried cannot be dissipated but it get added, to the electric field energy causing increased voltage to appear. This increased voltage is the reflected current wave. When the line is short circuited, The $E_R = 0$. Thus the electric field energy is Zero. The electric field energy carried cannot be dissipated but gets added to the magnetic field set up a reflected voltage wave. As limiting cases it is convenient to consider lines terminated in open circuit or short circuit. $Z_R = \infty$ and $Z_R = 0 \rightarrow$ for Open and short circuited line. The input impedance of a line of length l is,

$$Zin = Z0 \frac{Z_R \, Cosh\gamma l + Z_0 Sin\, h\gamma l}{Z_0 \, Cosh\gamma l + Z_R \, Sin\, h\gamma l}$$

For the short circuit case, $Z_R = 0$ so that,

$$Z_{SC} = Z_0 \frac{Sin\, h\gamma l}{Cos\, h\gamma l} = Z_0 \tan h\, \gamma l \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

The input impedance equation can be rearranged as,

$$Zin = Z0 \left[\frac{Cosh\gamma l + \frac{Z_0}{Z_R} Sin\, h\gamma l}{\frac{Z_0}{Z_R} \, Cos\, h\gamma l + Sin\, h\gamma l}\right]$$

The input impedance of a open circuit line of length l with $Z_R = \infty$ is,

$$Z_{0C} = Z_0 \frac{\text{Cos } h\gamma l}{\text{Sin } h\gamma l} = Z_0 \text{ Cot } h\, \gamma l \quad \dots\dots\dots\dots\dots\dots (2)$$

By multiplying eqn. (1) & (2) $\quad Z_0 = \sqrt{Z_{OC} Z_{SC}}$

÷ (1) & (2) $\quad \tan h\gamma l = \sqrt{\frac{Z_{SC}}{Z_{OC}}}$

1.14. REFLECTION FACTOR AND REFLECTION LOSS.

It is seen that when the line is terminated in Z_0 then there is no reflection. But under mismatch condition, the ratio of voltage to current gets disturbed. The part of the energy is rejected and reflected by the load. Thus energy delivered to the load under mismatch condition is always less than the energy which would be delivered to the load under matched impedance condition. This is because of a loss called reflection loss. The reflection loss is determined from the ratio of current which actually flows under mismatch condition in the load to that which would flow if the impedances are matched at the terminals of load.

The **reflection loss** is defined as the number of nepers or decibels by which the current in the load under image matched conditions would exceed the current actually flowing in the load.

So if I'_2 is the load current under image matching condition and I_2 is the actual load current under image mismatch condition then the reflection loss in nepers is given by,

$$\text{Reflection loss} = ln\left[\frac{|I'_2|}{|I_2|}\right] \text{ nepers} \quad \text{... (1(a))}$$

$$\text{Reflection loss} = 20 \log\left[\frac{|I'_2|}{|I_2|}\right] \text{ dB}$$

Thus reflection loss is actually **mismatching loss.**

The reflection loss can be measured in terms of the power absorbed by the load and received

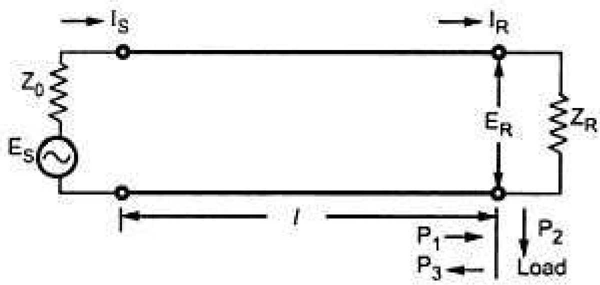


Fig. 1. 10.

P_1 = Power at the receiving end due to incident wave

P_2 = Power absorbed by the load

P_3 = Power reflected back down the line

$$P_1 = P_2 + P_3$$

$$P \propto I^2 \quad \text{i.e.} \quad I \propto P^{\frac{1}{2}}$$

$$\therefore \quad \text{Reflection loss} = 20 \log \left[\frac{|I'_2|}{|I_2|}\right]$$

$$= 20 \log \left[\frac{P_1^{\frac{1}{2}}}{P_2^{\frac{1}{2}}}\right] = 20 \log \left[\frac{P_1}{P_2}\right]^{\frac{1}{2}}$$

$$\therefore \quad \text{Reflection loss} = 10 \log \frac{P_1}{P_2} \text{ dB} = 20 \log \frac{1}{|k|}$$

The reflection coefficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

If E_R and I_R are the values of voltage and current at the receiving end due to incident wave then the values of voltage and current at the receiving end due to the reflected wave are $K E_R$ and $K I_R$.

P_1 = Received power = $E_R I_R$

P_2 = Power absorbed by load = $P_1 - P_3$

P_3 = Reflected power = $(|K| E_R)(|K| I_R) = |K|^2 P_1$

$P_2 = P_1 - |K|^2 P_1 = (1 - |K|^2) P_1$

Z_R and Z_0 are complex hence K is also complex but only magnitude of K is to be considered. The power calculated is the apparent power in volt amp and not the true power in watts.

Hence the reflection loss can be obtained as,

$$\text{Reflection loss} = 10 \log \frac{P_1}{P_2} = 10 \log \left\{ \frac{P_1}{P_1 [1 - |K|^2]} \right\}$$

$$= 10 \log \left\{ \frac{1}{1-|K|^2} \right\}$$

Now

$$\frac{1}{1-K^2} = \frac{1}{1 - \left[\frac{Z_R - Z_0}{Z_R + Z_0}\right]^2} = \frac{(Z_R + Z_0)^2}{(Z_R + Z_0)^2 - (Z_R - Z_0)^2}$$

$$= \frac{(Z_R + Z_0)^2}{Z_R^2 + Z_0^2 + 2 Z_R Z_0 - Z_R^2 - Z_0^2 + 2 Z_R Z_0}$$

$$= \frac{(Z_R + Z_0)^2}{4 Z_R Z_0}$$

$$\therefore \quad \frac{1}{1-|K|^2} = \frac{|Z_R + Z_0|^2}{|4 Z_R Z_0|}$$

$$\therefore \quad \text{Reflection loss} = 10 \log \left\{ \frac{|Z_R + Z_0|^2}{|4 Z_R Z_0|} \right\} = 10 \log \left[\frac{|Z_R + Z_0|}{\left|2\sqrt{Z_R Z_0}\right|} \right]^2$$

$$\therefore \quad \text{Reflection loss} = 20 \log \frac{|Z_R + Z_0|}{\left|2\sqrt{Z_R Z_0}\right|} \text{ dB} = 20 \log \frac{1}{|K|}$$

The ratio which indicates the change in current in the load due to reflection at the mismatched junction is called **reflection factor.** It is denoted by K and defined by,

$$K = \text{reflection factor} = \left| \frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0} \right| \qquad \dots (7)$$

The reflection loss is inversely proportional to the reflection factor.

Return Loss

The return loss is defined as,

$$\text{Return loss} = 10 \log \frac{P_1}{P_3} \text{ dB} \quad \dots (8)$$

It indicates the ratio of the power at the receiving end due to incident wave to the power reflected by the load.

$$\text{Return loss} = 10 \log \frac{P_1}{|K|^2 P_1} = 10 \log \frac{1}{|K|^2}$$

$$= 10 \log \left[\frac{1}{|K|}\right]^2 = 20 \log \left[\frac{1}{|K|}\right]$$

$$\therefore \quad \text{Return loss} = 20 \log \left|\frac{Z_R + Z_0}{Z_R - Z_0}\right| \text{ dB} \quad \dots (9)$$

This is also called Singing point.

1.15. INSERTION LOSS

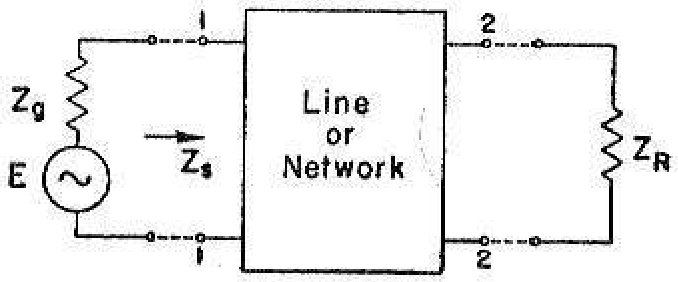


Fig: 1.9. Illustrating Insertion Loss

The insertion loss of a line or linear network is defined as the number of nepers or decibels by which the current in the load is changed by the insertion. If the load current increases it is called insertion gain.

The sending end current

$$I_s = \frac{I_R(Z_R+Z_0)}{2Z_0} e^{\gamma l} - \frac{I_R(Z_R+Z_0)}{2Z_0} e^{-\gamma l}$$

$$\frac{I_R(Z_R + Z_0)}{2Z_0}\left[e^{\gamma l} - \frac{(Z_R + Z_0)}{Z_R + Z_0} e^{-\gamma l}\right]$$

$$= \frac{I_R(Z_R + Z_0)}{2Z_0} e^{\gamma l} - ke^{-\gamma l} \quad \dots\dots\dots\dots\dots (1)$$

(or)

$$Is = \frac{E}{Z_g Z_s}$$

W.K.T,

$$Z_S = Z_0 \left[\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} + Ke^{-\gamma l}}\right]$$

$$\therefore \quad I_s = \frac{e}{Z_g + Z_0 \left[\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} + Ke^{-\gamma l}}\right]}$$

$$IS = \frac{E(e^{\gamma l} - Ke^{-\gamma l})}{Z_g(e^{\gamma l} + Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

equate (1) & (2)

$$\frac{E(e^{\gamma l} + Ke^{-\gamma l})}{Z_g(e^{\gamma l} + Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})} = \frac{I_R(Z_R + Z_0)}{2Z_0}(e^{\gamma l} + Ke^{-\gamma l})$$

$$I_R = \frac{2EZ_0}{Z_R + Z_0 + [Z_g(e^{\gamma l} + Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})]}$$

$$= \frac{2EZ_0}{(Z_R + Z_0) + [Z_g(e^{\gamma l} - Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})]}$$

$$= \frac{2EZ_0}{(Z_R + Z_0) + (Z_g + Z_0)e^{\gamma l} + (Z_0 - Z_g)\ (Z_R - Z_0)e^{\gamma l}}$$

Without the insertion loss of line, the current I_R is, $\quad I_R^1 = \frac{E}{Z_g + Z_R}$

$$\frac{I_R^1}{I_R} = \frac{E/(Z_g + Z_R)}{(2Z_R + Z_0) + (Z_g + Z_0)e^{\gamma l} + (Z_0 - Z_g)\ (Z_R - Z_0)e^{-\gamma l}}$$

$$= \frac{(Z_R + Z_0) + (Z_g + Z_0)e^{\propto l}\, e^{j\beta l} + (Z_0 - Z_g)\ (Z_R - Z_0)e^{-\propto l}\, e^{-j\beta l}}{2Z_0(Z_g + Z_R)}$$

If $\propto$ is large, the second term on the numerator is neglected

$$\frac{I_R^1}{I_R} = \frac{(Z_R + Z_0) + (Z_g + Z_0)e^{\propto l}}{2Z_0\ (Z_g + Z_R)}$$

X & ÷ the above eqn. by $\sqrt[2]{(Z_g + Z_R)}$

$$\frac{I_R^1}{I_R} = \frac{\sqrt[2]{(Z_g\, Z_R)}\ \ (Z_R + Z_0)(Z_g + Z_0)e^{\propto l}\ e^{-j\beta l}}{\sqrt[4]{(Z_g\, Z_R)}\, Z_0\ (Z_g - Z_R)}$$

The insertion loss is to be calculated as a function of current magnitudes only.

$$\left|\frac{I_R^1}{I_R}\right| = \frac{|Z_g + Z_0|}{\sqrt[2]{(Z_g Z_0)}} \cdot \frac{|Z_R + Z_0|}{\sqrt[2]{(Z_R Z_0)}} \frac{\sqrt[2]{(Z_g Z_R)}}{|Z_g + Z_R|} e^{\propto l}$$

Reflection factor K_s is, $\quad K_S = \frac{\sqrt[2]{(Z_g Z_0)}}{|Z_g + Z_0|} \qquad K_R = \frac{\sqrt[2]{Z_R Z_0)}}{|Z_R + Z_0|}$

Reflection factor for the terminates or junction between line and load. $\quad \frac{\sqrt[2]{(Z_g Z_0)}}{|Z_g + Z_0|} = KSR$

$$\therefore \quad \frac{I_R^1}{I_R} = \frac{K_{SR}\, e^{\propto l}}{K_S + K_R}$$

Insertion loss in nepers, $\quad = \ln\left(\frac{1}{K_S} + \ln\frac{1}{K_R} - \ln\frac{1}{K_{SR}} + \propto l\right.$

Insertion loss in dB $\quad = 20\left(\log\frac{1}{K_S} + \log\frac{1}{K_R} - \log\frac{1}{K_{SR}} + 0.4343\ \propto l\right.$

PROBLEMS

Example 1.1: A generator of IV, 1KH$_Z$ supplies power to a 100 Km long line terminated in Z_0 and having following constants .R = 10.4Ω/km, L = 0.00367 H/km, G = 0.8 × 10^{-6} mho/km C = 0.00835 10^{-6} F/Km Calculate Z_0, attenuation constant α, β, γand V_P, received current, voltage and power.

Solution: given:

$$f = 1 \times 10^3\ Hz,\ R = 10.4\frac{\Omega}{Km}\quad l = 100\ Km\quad L = 0.00367\frac{H}{Km}$$

$$G = 0.8\times 10^{-6}\ mho/km\qquad C = 0.00835\ x\ 10^{-6}\ F/Km$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + J\omega C}}\qquad \omega = 2\pi \times 10^3\ rad/Sec$$

$$= \sqrt{\frac{10.4 + j\ 23.059}{0.8 \times 10^{-6} + j\ 5.246 \times 10^{-5}}}$$

$$= 694.35\angle - 11.7030\quad \Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= 0.007928 + J\ 0.03553$$

$$= \propto + j\beta$$

$$\propto = 0.007928\ Nepers/m$$

$$\beta = 0.0553\ rad/km$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.03553} = 176.841\ km$$

$$\vartheta p = \frac{\omega}{\beta} = 1.95 \times 10^4\ km/sec$$

$$Z_0 = \frac{E_s}{I_s} = \frac{1\angle 0}{694.35\angle - 11.7030}$$

$$IR = IS\ e^{\gamma l}\qquad l = 100\ km$$

$$IR = IS\ e^{\gamma l}\angle \beta l$$

$$= 1.4402 \times 10^{-3} \times e^{-0.007928 \times 100} \angle - 0.03553 \times 100$$

$$= 6.518 \times 10^{-4} \angle - 8.144\ \ rad$$

$$\frac{E_R}{I_R} = Z_0$$

$$E_R = I_R Z_0 = 6.518 \times 10^4 \angle - 203.580 \times 694.32 \angle - 11.703$$

$$= 0.4525 \angle - 215.2830\ V$$

$$PR = E_R I_R\ Cos\ (E_R \wedge I_R)\qquad \theta = E_R \wedge I_R$$

$$= 6.518 \times 10^{-4} \times 0.4525\ Cos\ (11.7030)\qquad = 215.28 - 208.58$$

$$P_R = 288 \times 10^{-6}\ watts$$

Example 1.2: A generator of 1 volts 1000 cycles, supplies power to a 100 mile open wire line terminated in 200Ω resistance. The line parameters are, R = 10.4Ω /miles, L = 0.00367 H/miles G = 0.8 × 10^{-6} mho/miles C = 0.00835 $\mu F/miles$. Calculate the power input and Transmission efficiency.

Z = 25.2 66

gn: $Z_R = 200$ Y = 52.6 × 10^{-6} ∠ 90^0

= 0.0363 ∠ 78^0

Already calculated $Z_0 = 692\ \angle -12^0$ = 0.00755 + j 0.0355

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{200 - 692\angle -12^0}{200 - 692\angle -12^0}$$

$$= 0.558\ \angle\ 172.8^0$$

$$Z_S = Z_0 \left(\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}}\right)$$

$$e^{\gamma l} = e^{\propto l} + Ke^{j\beta l} = e^{0.755}\ \angle\ 203.8 = 2.12\ \angle 203.8$$

$$e^{-\gamma l} = e^{-\propto l} + Ke^{j\beta l} = e^{-0.755}\ \angle\ -203.8 = 0.472\angle\ -203.8$$

$$Zs = 692\angle -120\left[\frac{2.12\angle\ 203.8 + 0.558 + 172.8 \times 0.472\angle -203.8}{2.12\angle\ 203.8 + 0.558 - 172.8 \times 0.472\angle -203.8}\right]$$

$$= 692\angle -120\left[\frac{1.975\angle 210}{2.285\ \angle 198.5}\right]$$

$$= 692\angle\ -12^0 \times 0.864\ \angle 11.5$$

$$= 597\ \angle -0.5^0$$

The input current is,

$$Is = \frac{E_s}{Z_S} = \frac{1.0}{597\ \angle -0.5} = 0.00167\ \angle 0.50\ Amps$$

$$I_s = \frac{I_R(Z_R + Z_0)}{2Z_0}\left(e^{\gamma l} + Ke^{-\gamma l}\right)$$

$$0.00167\angle\ 0.50 = \frac{IR\ (888\ \angle -9.5)}{1384\angle\ -120}\ (2.285\ \angle 198.5)$$

$$IR = \frac{2.31\angle -11.5}{2030\angle 1890}$$

$$= 0.0018\ \angle\ -200.50\ \text{amp}$$

$E_R = I_R Z_R$ = 0.00113 ∠-200.5 × 200 = 0.226 ∠ -200.5 V

$P_R = I_R^2 R = 0.00113^2 \times 200$ = 0.000255 watts

$$Ps = EsIs\cos\theta = 1.0\ 0.00167\ \cos\ 0.50 = 0.00167\ watt$$

$$\eta = \frac{0.000255}{0.00167} \times 100\ \%$$

$$\eta = 15.2\%$$

Example 1.3: A Cable has an attenuation of 3.5 dB/km and a phase constant of 0.28 rad 1 km. If 3 V are applied to the sending end, what will be the voltage at point 10 km down the line when the line is terminated in its characteristic impedance?

$\boldsymbol{given}: E_s = 3\ V\ , \propto = 3.5\ {dB}/{km}, \beta = 0.28 \frac{rad}{km}, S = 10\ km$

$$1\ Nepers = 8.686\ dB$$

$$3.5\ dB/km = \frac{3.5}{8.686}\ Nepers\ /\ km = 0.4029\ Neper/km$$

$$E_s = E_{Se^{-\gamma s}} = E_s e^{-\propto s} \angle - \beta s$$

$$S = 10\ km$$

$$E_s = 3e^{-0.4029 \times 10} \angle - 028 \times 10\ \ rad$$

$$= 0.05337 \angle\ -2.8\ (\pi 880)$$

$$= 0.05337\ \angle - 160.440\ V$$

Example 1.4: Calculate the characteristic impedance, the attenuation constant and phase constant of transmission line if the following measurements have been made on the line. Z_{oc} = 550∠ 60^0 Ω & Z_{oc} = 500 ∠-14^0 Ω

given: For a transmission line, $Z_0 = \sqrt{Z_{0c} Z_{sc}} = \sqrt{500 \times 550 \angle - 60 \angle - 14^0}$

$$= 524.404 \angle - 370\ \Omega\ \ (or)\ 418.807 - j315.594\ \Omega$$

$$tan\ h\gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{500\ \angle - 14^0}\ \ = 0.9534\ \angle\ \frac{-14 + 60}{2}$$

$$= 0.9534 \angle 230\ \ = 0.877\ +\ j\ 0.3725$$

$$tan\ h\gamma = \frac{e^{2\gamma}\ - 1}{e^{2\gamma}\ + 1} = 0.877 + j\ o.3725$$

$$e^{2\gamma}\ - 1 = (0.877 + j\ 0.3725)(e^{2\gamma}\ + 1)$$

$$= 0.877\ e^{2\gamma}\ + j\ 0.3725\ e^{2\gamma}\ + 0.877 + j\ 0.3725$$

$$e^{2\gamma}\ [1 - 0.877 - j\ 0.3725] = 1 + 0.877 + j\ 0.3725$$

$$e^{2\gamma}\ (0.123 - j\ 0.3725) = 1.877 + j\ 0.3725$$

$$e^{2\gamma} = \frac{1.877 + j\ 0.3725}{0.123 - j\ 0.3725} = \frac{1.9136 \angle\ 11.220}{0.3922\ \ \angle - 71.720}$$

$$e^{2\gamma}\ = 4.8791 \angle 82.940$$

$$2\gamma = ln\ [4.8791 \angle\ 82.940]$$

$$\gamma = \frac{1}{2}\ ln\ [4.8791 \angle 82.940]$$

$$= ln\ (\propto\ + j\beta)$$

$$\propto = \frac{1}{2} ln\ 4.8791\ Nepers/km$$

$$\beta = \left(\frac{82.94}{2}\right) = 41.47\ /\ km\ = 0.7237\ rad\ /\ km$$

Example 1.5: For an open wire overhead line $\beta = 0.04$ rad / km. Find the wavelength and velocity at a frequency of 1600 Hz. Hence calculate the time taken by the wave to travel 90 km.

Solution:

$$\beta = 0.04\, rad\ /\ km \qquad f = 1600\, Hz$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.04} = 157.079\, km$$

$$V = \frac{\omega}{\beta} = \frac{2\pi \times 1600}{0.04} = 2.5132 \times 10^5 \quad Km\ /\ Sec$$

$$So\ time\ required\ to\ travel\ 90\ km\ distance\ is, \qquad t = \frac{90\ Km}{2.5132 \times 10^5\ Km/Sec}$$

$$= 3.581 \times 10^4\ Sec$$

Example 1.6: The constant of transmission line per Km are R = 6Ω L = 2 mH, C = 0.005μF and G = 0.25 μmho. Calculate at the frequency 1000 Hz i), The transmission impedance for which no reflection will be set up in the line ii) the attenuation in dB suffered by signal at 1000 Hz, while travelling a distance of 100 km when the line is properly terminated and iii) The phase velocity with which the signal would travel.

Solution:

$Given$: R $= 6\Omega/Km,\ L = 2.2 \times 10^{-3}\ H/Km,\ C = 0.005 \times 10^{-6}\ F/m. \quad G = 0.025 \times 10^{-6} mho/Km, f = 1000\, Hz\ \ \omega = 2\pi\ f = 6280$

$$i)\ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}}$$

$$Z = R + j\omega L$$

$$= 6 + j6280 \times 2.2 \times 10 - 3$$

$$= 15.04 \angle 66.50$$

$$Y = G + j\omega C$$

$$= 0.25 \times 10 - 6 + j\ 6280 \times 0.005 \times 10 - 6$$

$$= 31.42\ \ 10 - 6\ \angle 89.50$$

Error! Bookmark not defined. $= \sqrt{\frac{15.04\ \angle 66.5}{31.42 \times 10^{-6}\ \angle 89.5}}$

$$= 0.692 \times 10^3\ \ 71.5$$

$$= 692 \angle 71.5\ \Omega$$

$$\gamma = \sqrt{ZY}$$

$$= 21.73 \times 10^{-3} \angle 780$$

$$\gamma = 0.0046 + j\ 0.0215$$

$$\propto = 0.0046 \, nepers \, / \, Km$$

$$\beta = 0.0215 \, rad \, / \, Km$$

$$attenuation \; while \; travelling \; 100 \, Km = 100 \times \propto$$

$$in \, dB = 0.0046 \times 10 \times 8.66$$

$$= 3.99 \, dB$$

$$Phase \; velocity \; \vartheta\rho = \frac{\omega}{\beta}$$

$$= \frac{6280}{0.0215} = 2.9 \times 10^5 \, Km/Sec$$

$$= 2.9 \times 10^8 \, m/s$$

Example 1.7: The primary constant of a line per loop km are R = 196 Ω/Km,C = 0.09 μ F/Km , L = 0.71 μ mH and leakage conductance is neglible. Calculate Z_0 γ at $\frac{5000}{2\pi}$ H_z

Solution:

$$R = 196 \frac{\Omega}{Km}, C = 0.09 \times 10^{-6} \frac{F}{Km} L = 0.07 \times 10^{-3} \frac{H}{Km}. \; G = 0 \;\; f = \frac{5000}{2\pi} \; Hz$$

$$\omega = 5000 \, rad.$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= 10.4 \; \angle \; -39.75 \; \Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{199.2 \times 0.45 \times 10^{-3}}$$

$$= 0.290 \angle \, 50.25$$

Example 1.8: A transmission line has the following primary constants measured per Km,R = 10.15Ω , L = 3.93 mH, C = 0.0097 μ F/Km, G = 0.29 μ mho. Determine Z_0 and propagation constant at a frequency of 796 Hz. Also calculate the ratio of current at a point which is 100 Km down the line to the current at the sending end if the line is terminated in its characteristic impedance.

Solution:

$$\omega = 2\pi f = 2\pi \times 796 = 5 \times 10^3 \, rad/sec$$

$$R + j\omega L = 10.15 + j5 \times 10^3 \times 3.93 \times 10^{-3} = 10.15 + j \, 1965 \, \Omega$$

$$= 22.1166 \angle 62.68°\Omega$$

$$G + j\omega \, C = 0.28 \times 10^{-6} + j5 \times 10^3 \times 0.00797 \times 10^{-6}$$

$$= 0.29 \times 10^{-6} + j3.985 \times 10^{-5} = 3.985 \times 10^{-5} \angle 89.58°$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{(221166 \angle 62.68°)}{3.985 \times 10^{-5} \angle 89.58°}}$$

$$= 744.97 \angle -13.45° \Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{221166 \angle 62.68° 3.985 \times 10^{-5}}$$

$$= 0.02968 \, O \, 76.13° = 0.00711 + j\, 0.0288$$

$$\propto = 0.00711 \; nepers/km$$

$$and \qquad \beta = 0.0288 \; rad/km$$

As the line is properly terminated we can use,

$$I_x = I_S \; e^{-\gamma x} = I_S \; e^{-\propto x} \; \angle -\beta x$$

$$\therefore \qquad \frac{I_x}{I_S} = e^{-\propto x} \; \angle -\beta x$$

$$Now \qquad x = 100 \; km$$

$$\frac{I_x}{I_S} = e^{-0.00711 \times 100} \angle -0.0288 \times 100 \; rad$$

$$= 0.49115 \angle -2.88 \; rad = 0.49115 \angle -165.024°$$

$$2.88 \; rad = 2.88 \times 57.3°$$

Example 1.9: A transmission line 2 miles long operate at 10 kHz and the parameters R = 30 Ω/mile, C = 80 nF/mile, L = 2.2 m H/mile and G = $20nV/mile$. Find the characteristic impedance, propagation constant, attenuation and phase shift per miles.

Solution: For a transmission line of length l = 2 miles,

R = 30 Ω/mile L = 2.2 m H/mile

G = $20nV/mile$ C = 80 nF/mile and f = 10 kHz

The characteristic impedance, Z_0 is given by,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{30 + j(2\pi \times 10 \times 10^3) \times 2.2 \times 10^{-3}}{20 \times 10^{-9} + j(2\pi \times 10 \times 10^3)(80 \times 10^{-9})}}$$

$$= \sqrt{\frac{30 + j138.23}{20 \times 10^{-9} + j5.0265 \times 10^{-3}}}$$

$$= \frac{\sqrt{141.4479 \angle 77.75°}}{5.0265 \times 10^{-3} O 89.99°}$$

$$= 167.7511 \angle -6.12 \, \Omega$$

The propagation constant γ is given by, $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ =

$$\sqrt{30 + j2 \times \pi \times 10 \times 10^3 \times 2.2 \times 10^3)(20 \times 10^3 + j2 \times \pi \times 10^3} \times 80 \times 10$$

$$= \sqrt{(141.4479 \angle 77.75° (5.0265 \times 10^3 \angle 89.90°)}$$

$$\therefore \quad = 0.8432 \angle 83.87° \; /km$$

Representing γ *in rectangular* form, we get

$$\gamma = \propto + j\beta = 0.09 + j0.8383$$

Hence attenuation constant = $\propto$= 0.09 nepers/miles

Phase constant β = 0.8383 rad/miles

Example 1.10: A voltage of 45 V is applied to a 10 km long field quad cable. The receiving end voltage is 7.868 V and it lags behind by 110.2^0. Calculate the attenuation and phase constants of the cable, if it is properly terminated.

Solution:

$$x = 10\,km, Es = 45\,V, Ex = 7.868\,V, \quad \beta = 110.2°.$$

$$Now E_x = E_S\, e^{-\gamma x} = E_S\, e^{-\alpha x} \angle -\beta x$$

$$\beta x = 110.2°$$

$$\beta = \frac{110.2}{10} = 11.02°\, per\ Km = 0.1923\, rad/Km$$

and

$$E_x = E_S\, e^{-\propto x}$$

$$\therefore \quad 7.868 = 45e^{-\propto x 10}$$

$$\therefore \quad e^{-10\propto} = 0.1748$$

$$\therefore \quad -10 \propto = ln(0.1748) = 1.743$$

$$\therefore \quad \propto = 0.1743\, Nepers\ per\ Km$$

Example 1.11: A unloaded underground cable has the following constants R = 40 Ω/km, G = 0.5 μ mho/km, L = 1 μH/km, C = 0.08 μ F/km. Find the approximate value values of Z_0, $\propto$ & β at 400 H_Z

Solution:

$$Case\ (i)\quad f = 400\, Hz \quad w = 2\pi f = 2.5132 \times 103\, rad/Sec$$

$$Z_0 = \sqrt{\frac{R}{\omega C}} \angle -450\,\Sigma = \sqrt{\frac{40}{2.513 \times 10^3 \times 0.08 \times 10^{-6}}} \angle -450$$

$$= 446.03 \angle -450\,\Sigma$$

$$\propto = \sqrt{\frac{\omega RC}{2}} = 0.6341\, neper/km = \beta$$

Example 1.12: An unloaded underground cable has the following constant R = 40 Ω/Km, G = 0.05μ mho/Km, L = 1 μ H/Km, C = 0.08 μ F/Km. Find the approximate value of Z_o, $\propto, \beta$ at 400 H_z.

Solution:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

f = 400. for unloaded cable L & G is negligible,

$$Z_0 = \sqrt{\frac{R}{j\omega C}}$$

$$= \sqrt{\frac{40}{\sqrt{\omega}\ 0.08 \times 10^{-6}}}$$

$$= \sqrt{\frac{40}{(8 \times 10^{-8}\ 90)\ (2\pi \times 400}}$$

$$= \sqrt{\frac{0.0159}{8 \times 10^{-8}}}$$

Z = 446.03

$$\propto = \beta\ \frac{\omega RC}{2}$$

$$= \sqrt{\frac{2\pi \times 400 \times 40 \times 10^{-6} \times 0.08}{2}}$$

$\propto$ = 0.06341 Nepers / Km

β = 0.06341 rad / Km.

Example 1.13. The following characteristics have been measured on a lossy transmission line at 100 *MHz*: Z_0=50Ω, α=0.01dB/m=1.15×$10^{-3}$$Np/m$, β=0.8π(rad/m). Determine R, L, G, and C for the line.

Solution: $$50 = \sqrt{\frac{R + j2\pi 10^8 L}{G + j2\pi 10^8 C}}$$,

$$1.15 \times 10^{-3} + j0.8\pi = \sqrt{(R + j\omega L)(G + j\omega C)} = 50 \cdot (G + j2\pi 10^8 C)$$

$$\Rightarrow C = \frac{0.8\pi}{2\pi \times 10^8 \times 50} = 80(pF/m)$$

$$G = \frac{1.15}{50} \times 10^{-3} = 2.3 \times 10^{-5}(S/m),$$

$$R = 2500G = 0.0575(\Omega/m)$$

$$L = 2500C = 0.2(\mu F/m)$$

Example 1.15 : A transmission line has $Z_0 = 700\angle -13.4^0\ \Omega$ is inserted between generator of 200Ω and a load of 400Ω. The attenuation and phase constants of line are: $\propto = 0.00712$ nepers/Km and β 0.0.288 rad/km. Calculate the insertion loss if line length is 200 km

Solution:

$$Z_0 = 700 < -13.4^0\Omega,\ Z_g = 200 < 0°\Omega,\ Z_R = 400 < 0°\Omega$$

$$K_s = \sqrt[2]{\frac{Z_g Z_0}{|Z_g Z_0|}} = \sqrt[2]{\frac{200 \times 700}{|200 + j0 + 700\angle -13.4°|}} = \frac{748.3314}{|200 + j0 + 680.94 - j162.2|}$$

$$= \frac{748.3314}{|880.94 - j162.22|} = \frac{748.3314}{895.7514} = 0.83542$$

$$K_R = \sqrt[2]{\frac{Z_R Z_0}{|Z_{R+} Z_0|}} = \sqrt[2]{\frac{400 \times 700}{|400 + j0 + 680.94 - j162.22°|}} = \frac{1058.3}{|1080.94 - j162.22|}$$

$$= \frac{1058.3}{1093.044} = 0.9682$$

$$K_{sR} = \sqrt[2]{\frac{Z_g Z_R}{|Z_g + Z_R|}} = \sqrt[2]{\frac{200 \times 400}{|200 + j0 + 400 + j0|}} = \frac{565.685}{600}$$

$= 0.9428$

$$Insertion\ loss = 20\left[log\frac{1}{K_s} + log\frac{1}{K_R} - log\frac{1}{K_{sR}} + 0.4343 \times \propto l\right] dB$$

$$= 20\left[log\frac{1}{0.83542} + log\frac{1}{0.9682} - log\frac{1}{0.9428} + 0.4343 \times 0.00712 \times 200\right]$$

$$= 13.7\ dB$$

Example1.16: A transmission line has $Z_0 = 745\angle -12^0\ \Sigma$ and is terminated in $Z_R = 200\ \Omega$. Calculate the reflection loss and return loss in dB.

Solution: The reflection factor

$$K = \sqrt[2]{\frac{Z_R Z_0}{|Z_{R+} Z_0|}} = \sqrt[2]{\frac{200 \times 745}{|100 + j0 + 728.72 - j154.894|}} = \frac{1058.3}{|828.72 - j154.894|}$$

$$= 843.0711 = 0.6475$$

$$\therefore Reflection\ loss = 20\ log\frac{1}{|K|} + 20\ log\frac{1}{0.6475} = 3.7751\ dB$$

$$= 20\ log\left|\frac{828.72 - j154.894}{-628.72 + j154.894}\right| = 20\ log\left|\frac{843.0711}{647.519}\right| = 2.2922\ dB$$

SUMMARY

TRANSMISSION LINE PARAMETERS:

1. Characteristic Impedance, $Z_0 = \sqrt{\frac{Z}{Y}}\ \boldsymbol{\Omega}$
2. Propagation Constant $\gamma = \sqrt{ZY}$
   Where, the series constant $Z = R + j\omega L\ \Omega$ / unit length

the series constant Y = G + $j\omega C$ ℧ / unit length

General Solution of Transmission Line:

i) $E = \frac{E_R}{2Z_R}[(Z_0 + Z_R)e^{\gamma s} + ((Z_R - Z_0)e^{-\gamma s}]$

$I = \frac{I_R}{2Z_0}[(Z_0 + Z_R)e^{\gamma s} + ((Z_0 - Z_R)e^{-\gamma s}]$

ii) $E = E_R \text{Cosh}\gamma s + I_R Z_0 \text{Sinh}\gamma s$

$I = I_R \text{Cosh}\gamma s + \frac{E_R}{2Z_0} \text{Sinh}\gamma s$

Input impedance of a Transmission Line:

$$Z_s = \frac{Z_0[\, Z_R + Z_0\, tanh\gamma l]}{Z_0 + Z_R tanh\gamma l}$$

$$Z_s = Z_0 \left[\frac{e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l}}{e^{\gamma l} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l}} \right]$$

Wavelength:

$\lambda = \frac{2\pi}{\beta}$ Where, β = phase constant

Velocity of propagation:

$$V_p = \frac{\omega}{\beta}$$

Insertion Loss:

$$Reflection\ factor\ Ks\ is, Ks = \frac{2\sqrt{Z_G Z_0}}{|Z_R + Z_0|}$$

$$KR = \frac{2\sqrt{Z_R Z_0}}{|Z_R + Z_0|}$$

Insertion Loss in Nepers:

$$= ln\frac{1}{K_S} + ln\frac{1}{K_R} - ln\frac{1}{K_{SR}} + \propto l$$

Insertion Loss In dB:

$$= 20\ (\text{Log}\frac{1}{K_S} + log\frac{1}{K_R} - log\frac{1}{K_{SR}} + 0.4343 \propto l)$$