UNIT II

HIGH FREQUENCY TRANSMISSION LINES

Transmission line equation at radio frequency-Line of zero dissipation - voltages and currents on the dissipation less line - standing waves – Nodes – Standing wave ratio - Input impedance of the dissipation less line -open and short circuited lines - power and impedance measurement on lines. Reflection losses – Measurement of VSWR and wavelength.

2.1. TRANSMISSION LINE EQUATION AT RADIO FREQUENCY

In the previous section, we have discussed the theory of transmission line, definitions of line parameters, general solution of a transmission line and physical significance of the equation. In this chapter, we shall discuss the line at a radio and power frequencies. For the radio frequency line working at a frequency of the range of megahertz and more than that, the standard assumptions are different than that studied in the previous chapter. For the radio frequency line of either open wire type or coaxial line type, the standard assumptions made for the analysis of the performance of the line are as follows.

1) At very high frequency, the **skin effect** is considerable. Hence it is assumed that the currents may flow on the surface of conductor. Then the internal inductance becomes zero.
2) It is observed that due to the skin effect, resistance R increases with $\sqrt{f}$. But the line reactance ωL increases directly with frequency f. Hence the second assumption is $\omega L >> R$.
3) The third assumption is that the line at radio frequency is constructed such that the leakage conductance G may be considered zero.

There are two considerations for the analysis of the line performances. First consideration is that R is slightly small with respect to ωL while the second one is that R is completely negligible as compared with ωL. If R is neglected completely, then such a line is termed as **zero dissipation line.** This concept is useful when the line is used for transmission of power at a high frequency and the losses are neglected completely. While if R is small, then such a line is termed as **small dissipation line**. In the applications where line is considered as a circuit element or properties of resonance are involved, this concept of small dissipation line is very much useful.

2.1.1 Parameters Of Open Wire Line:

In a transmission line the losses may be neglected and the transmission of power at high efficiency, R is negligible are the line is said to be Zero dissipation. The Open wire line parameters are inductance, Capacitance and resistance. The inductance of a open wire line becomes.

$$L = \frac{\mu_0}{2\pi} \ln\frac{d}{a} = \frac{4\pi \times 10^{-7}}{2\pi} \ln\frac{d}{a} \text{ H/m}$$
$$= 9.21 \times 10^{-7} \log\frac{d}{a} \text{ H/m}$$

ie., The current flowing on the surface is almost Zero. So the surface of the conductor in a skin of very small depth. The internal flux and internal inductance are then reduced nearly to Zero. The value of capacitance of a line is not affected by skin effect or frequency.

$$C = \frac{\pi \varepsilon V}{\ln \frac{d}{a}} \text{ F/m}$$
$$= \frac{27.7}{ln\frac{d}{a}} \mu\mu F/m$$
$$= \frac{12.07}{ln\frac{d}{a}} \mu\mu F/m$$

In the case of skin effect, the current flows over, the surface of the conductor in a thin layer, with a resultant reduction in effective cross section or an increase in resistance of a conductor. The effective thickness of the surface layer is,

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad m$$

Where μ - Permeability and σ - Conductivity
For copper $\mu = \mu_0 \; 4\pi \times 10^{-7}$, $\sigma = 5.75 \times 10^{-7}$/m at 20^0 C

$$\delta = \frac{0.0664}{\sqrt{f}}$$

The resistance of the round conductor of radius a meters to direct current is inversely proportional to the area $Rd_C = \frac{k}{\pi a^2}$

The Alternating current is, $R_{aC} = \frac{k}{2\pi a \delta k}$. The ratio of resistance to alternating current to resistance to direct current us,

$$\frac{Ra_c}{Rd_c} = \frac{k\pi a^2}{2\pi a\delta\, k}$$
$$= \frac{a}{2\delta} \qquad\qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$
$$= a\frac{\sqrt{\pi f \mu \sigma}}{2}$$

For copper, $\frac{Ra_c}{Rd_c} = 7.53\sqrt{f}$ Where a is in meters and f is in cycles per second. The resistance of an open wire line of copper, with spacing greater than 20 a is,

$$Ra_c = \frac{8.33 \times 10^{-8}\sqrt{f}}{a} \; \boldsymbol{\Omega/m}.$$

3.1.2. Parameters Of Coaxial Cable At Rf:

Here the skin effect the current flows on the outer surface of inner conductor and inner surface of the outer conductor. This Phenomenon eliminates flux linkage due to internal conductor flux and the inductance of a coaxial line.

$$L = \frac{\mu_0}{2\pi}\ln\frac{b}{a} = 2 \times 10^{-7}\ln\frac{b}{a} \text{ H/m}$$
$$= 460 \times 10^{-7}\log\frac{b}{a}$$

The capacitance of a co-axial cable not affected by frequency $C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}}$ F/m

$$= \frac{55.5\,\varepsilon_r}{\ln(\frac{b}{a})} \mu\mu \text{ f/m}$$

$$= \frac{24.14\ \varepsilon_r}{\ln\left(\frac{b}{a}\right)}\ \mu\mu\ f/m$$

The resistance of a copper co-axial line is $Ra_c = 4.16 \times 10^{-8}\sqrt{f}\left(\frac{1}{b}+\frac{1}{a}\right)$ /m

Where a & b are outer radius of inner conductor and inner radius of outer conductor. The quality of the insulating material measured in terms of power factor of the material. The shunt susceptance is, $Y = g + j\omega C$

And the power factor is express able from substance triangle. $Pf = \frac{g}{\sqrt{g^2+\omega^2C^2}}$

The conductance of usual good insulating material is very small so, $g << \omega C$.

$$Pf = \frac{g}{\omega C}$$

$$g = \omega C \times Pf$$

$\sqrt{g^2 + \omega^2 C^2}$

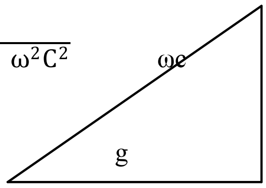


Fig 3: Loss Triangle

The quality of the dielectrics may also be expressed interest of dissipation factor. Dissipation factor is the ratio of energy dissipated to energy stored in a dielectric per cycle and is proportions to the tangent angle of Φ. For a good dielectric with small power factor g<<WC The dissipation factor and power factors are equal magnitudes.

3.2. LINE OF ZERO DISSIPATION

In general, the characteristic impedance (Z_0) and propagation constant (γ) of a line are given by,

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} \quad \text{... (1)}$$

and

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)} \quad \text{... (2)}$$

According to the standard assumptions for line at a high frequency,

$j\omega L >> R$ and $j\omega C >> G$

$$\therefore \quad Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \quad \text{... (3)}$$

As the value of characteristic impedance is real and resistive, it is represented by symbol R_0,

$$\therefore \quad \boxed{Z_0 = R_0 = \sqrt{\frac{L}{C}}} \quad \text{... (4)}$$

Similarly the propagation constant γ is given by,

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

$$\therefore \quad \gamma = 0 + j\,\omega\sqrt{LC} \quad \text{... (5)}$$

But $\gamma = \alpha + j\beta$

Hence at high frequencies,

$$\alpha = 0 \text{ and}$$

$$\beta = \omega\sqrt{LC} \quad \text{radian/m} \qquad \dots (6)$$

Then the velocity of propagation is given by,

$$v = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \quad \text{m/sec} \qquad \dots (7)$$

From equation (7), the velocity of propagation for open wire dissipationless line, separated by air, is same as the velocity of light in space.

3.3. VOLTAGES AND CURRENTS ON THE DISSIPATION LESS LINE

Consider a transmission line of length l and terminals in Z_R as shown in figure

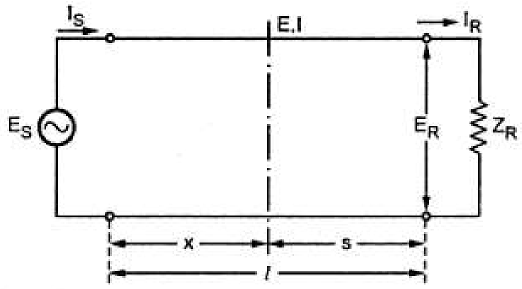


Fig 3.1 Transmission line of length l

The expression for voltage E and current T at a distance S from the receiving end in terms of receiving end parameters.

$$E = E_R \cos h\gamma S + I_R Z_0 \, Sinb \, h \, \gamma s \quad \dots\dots\dots\dots\dots(1)$$

But very high frequency $Z_0 = R_0$ & $\gamma = j\beta$

$$\therefore \quad (1) \qquad \Rightarrow E = E_R \; Cosh\,(j\beta s) + I_R \, Z_0 \; Sin\,h\,(j\beta s)$$

$$= E_R \; Cos\,\beta s + I_R \, Z_0 \; Sin\beta s \quad \dots\dots(2)$$

Similarly for current at a point distance away from receiving end is given by,

$$I \quad = \quad I_R \; Cos\,\beta s + \frac{E_R}{R_0} \; Sin\beta s \quad \dots\dots(3)$$

But $\beta = \frac{2\pi}{\lambda}$

$$\therefore \; E = E_R \, Cos\,\frac{2\pi s}{\lambda} + jI_R R_0 \, Sin\,\frac{2\pi s}{\lambda} \quad \dots\dots\dots\dots\dots (4)$$

$$I = I_R \, Cos\,\frac{2\pi s}{\lambda} + j\frac{E_R}{R_0} \, Sin\,\frac{2\pi s}{\lambda} \quad \dots\dots\dots\dots\dots(5)$$

From the equation (4) & (5) it is clear that the Voltage and current distribution is the sum of cosine & Sine distributions. Now considered two different conditions at the receiving end.

i. When a line is open circuited at the receiving and $I_R = 0$

$$E_{OC} = E_R \, Cos \, \frac{2\pi s}{\lambda}$$

$$I_{OC} = j\frac{E_R}{R_0} \, Sin \, \frac{2\pi s}{\lambda}$$

From the above eqn. it is clear that current and voltage are in quadrature. The magnitudes of voltage and current distributions for an open circuited line 3/2 wavelength long. Δt every $\frac{\lambda}{4}$ distance voltage charges from maximum to minimum or vice versa.

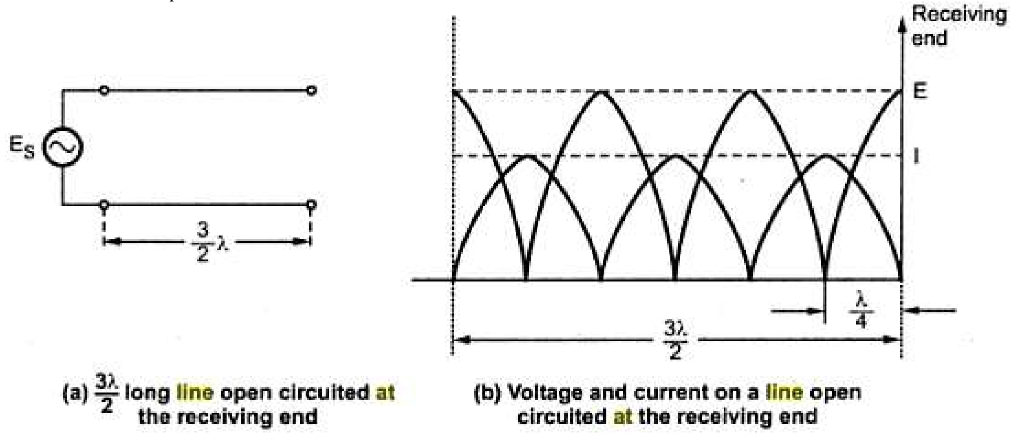


Fig 3.2 open circuited at the receiving end

ii. When the line is short circuited at the receiving end ER = 0. The voltage and current at a point, distance S away from the load end are given by,

$$E_{SC} = jI_R R_0 Sin \, \frac{2\pi s}{\lambda}$$

$$I_{SC} = jI_R R_0 Cos \, \frac{2\pi s}{\lambda}$$

Then the magnitude of voltage and current distributions for a short circuited line 3/2 wavelength long as shown in the figure.

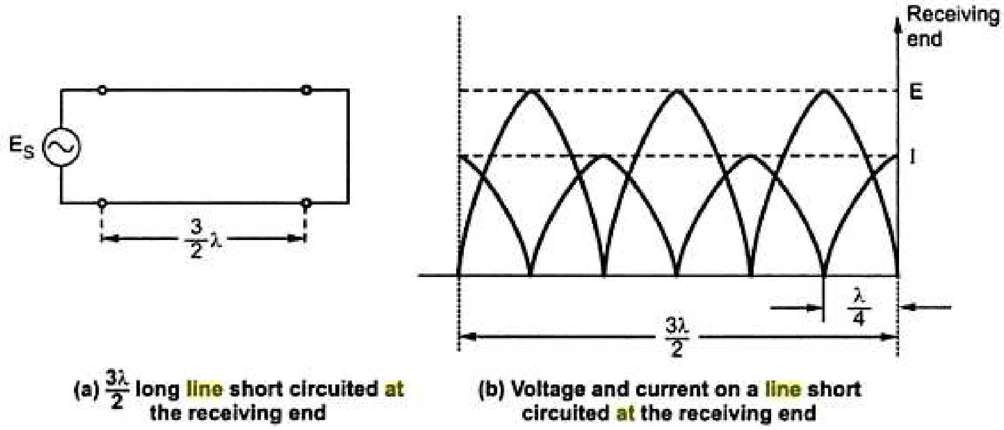


Fig 3.3. short circuited at the receiving end

iii. When a line is terminated an impedance ZR = R0 the reflection coefficient is given by,

$$K = \frac{z_R - R_0}{z_R + R_0} = \frac{R_0 - R_0}{R_0 + R_0} = 0$$

The reflection wave is absent. Then the voltage and current on the line is given by,

$$E = E_R e^j \beta s$$

&

$$I = I_R e^j \beta s$$

∴ The both voltage and current have constant magnitude with Zero attention only continuous varying phase angle along the line. The voltage and current distributions are,

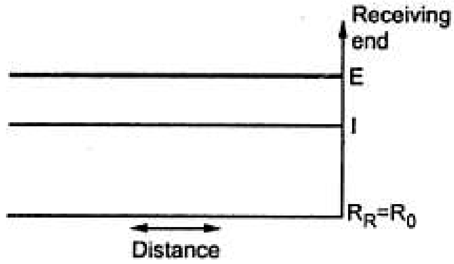


Fig 3.4: Voltage & current on a line terminated with R_0

3.4. STANDING WAVES NODES – STANDING WAVE RATIO

If voltage magnitudes are measured along the length of a line terminated in load other than R_0 the potted values will appear as in figure. Current magnitudes might be plotted and would be similar expect for a $\lambda/4$ shift in position of maxima and minima.

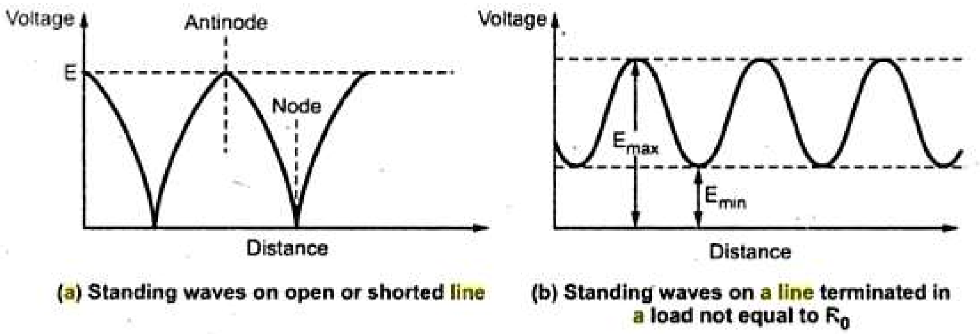


Fig 3.5. Standing waves on a line

Nodes:

Nodes are points of Zero voltage and current on the standing wave system antinodes are points of maximum voltage or current. i.e., The point along the line where magnitude of voltage or current is Zero (minimum Voltage or current) are called nodes. The points along the line where magnitude of voltage or current is maximum are called Antinodes or Loops. The nodes & antinodes are as shown in above figure. When the line is terminated in Ro the standing waves are absent, such a line is called smooth line.

Standing wave Ratio

A line terminated in R_0 has no standing waves and thus no nodes, antinodes are called smooth line. The ratio of the maximum to minimum magnitudes of current or voltage on a line having standing wave is called the standing wave ratio S.

$$S = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}}$$

S is always greater than 1 and when it is equal to 1 the line is correctly terminated and there is no reflection. So the values lives between 1 &∞. The ratio of E_{max} to E_{min} is referred as voltage standing wave ratio (VSWR) and the ratio of I_{max} to I_{min} is referred as current standing wave ratio (ISWR). The maxima of voltage along the line occur at points at which the incident and reflected waves are in phase and add directly

$$|E_{max}||E_{inc}| + |E_{ref}|$$

Similarly the voltage minima occur at points at which the incident and reflected waves are out of phase and subtracted directly.

$$|E_{max}||E_{inc}| + |E_{ref}|$$

Then the standing wave ratio is given by,

$$S = \left|\frac{E_{max}}{E_{min}}\right|$$

$$= \frac{|E_{inc}| + |E_{ref}|}{|E_{inc}| - |E_{ref}|}$$

$$= \frac{1 + \frac{|E_{ref}|}{|E_{inc}|}}{1 - \frac{|E_{ref}|}{|E_{inc}|}}$$

But the ratio of reflected to incident voltage is defined as reflection coefficient K

$$|K| = \frac{|E_{ref}|}{|E_{inc}|}$$

The standing wave ratio then may be defined interns of the reflection coefficient.

$$S = \frac{1 + |K|}{1 - |K|}$$

$$\therefore \quad |K| = \frac{S - 1}{S + 1}$$

$$(or)$$

$$|K| = \frac{|E_{max}| - |E_{min}|}{|E_{max}| + |E_{min}|}$$

It is possible to calculate values of |K| and S from measurements of maximum and minimum voltage on the line.

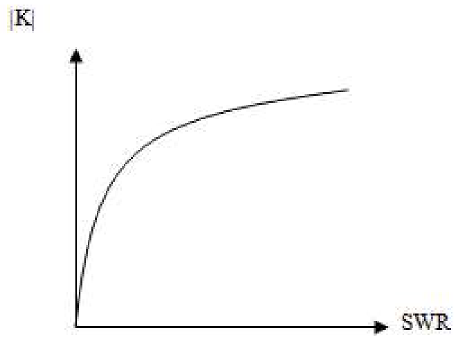


Fig 3.6 SWR Vs |K|

$$S = \frac{1+|K|}{1-|K|} = \frac{1+\left(\frac{R_R - R_0}{R_R + R_0}\right)}{1-\left(\frac{R_R - R_0}{R_R + R_0}\right)}$$

$$S = \frac{R_R}{R_0} \quad (for\ R_R > R_0)$$

$$S = \frac{R_R}{R_0} \quad (for\ R_R < R_0)$$

3.5. INPUT IMPEDANCE OF OPEN AND SHORT-CIRCUITED LINE:

The input impedance of dissipation less line is,

$$Z_s = R_0 \left(\frac{Z_R + jR_0 \tan\beta_s}{R_0 + jZ_R \tan\beta_s}\right)$$

For Short Circuited line $Z_R = 0$

$$Z_{sC} = R_0 \left(\frac{jR_0 \tan\beta_s}{R_0}\right)$$

$$Z_{sC} = jR_0 \tan\beta_s$$

Sub $\beta = \frac{2\pi}{\lambda}$ $\quad \therefore \ Z_{sC} = jR_0 \tan\frac{2\pi}{\lambda} s$

The variation of ${}^{Z_R}/_{R_0} = {}^{X}/_{R_0}$ with length of line S may be plotted below. To calculate the input impedance of open- circuit line, Another form of input impedance is,

$$Z_{sC} = R_0 \left(\frac{1 + j\frac{R_0}{Z_R}\tan\beta_s}{\frac{R_0}{Z_R} + \tan\beta_s}\right)$$

For an open circuited line $Z_R = \infty$

$$Z_{0C} = R_0 \left[\frac{1}{j \tan\beta_s}\right]$$

$$= \frac{-jR_0}{\tan\beta_s} = -jR_0 \operatorname{Cot} \frac{2\,\overline{\wedge}}{\lambda} s$$

The variation of $\frac{Z_{0C}}{R_0} = \frac{X}{R_0}$ as the function of the length of line S.

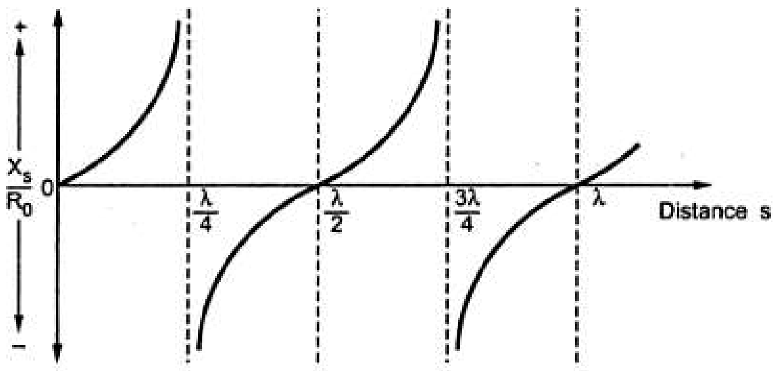


Fig 3.7 Short circuit

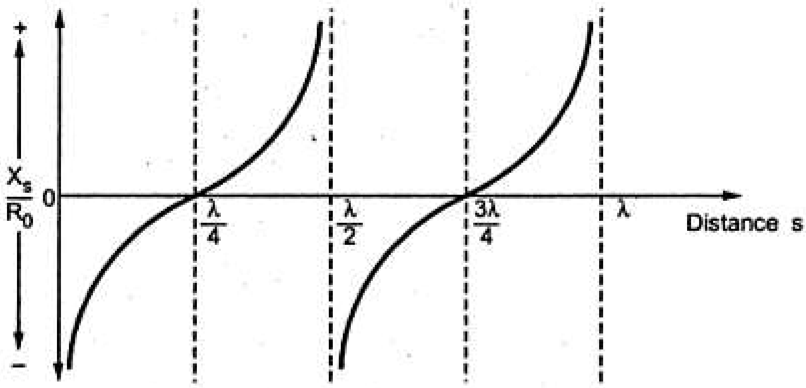


Fig 3.8 Open – Circuit

The Input impedance of open or short circuited line is a pure reactance. The Value of reactance is a repetitive function of length with a period of $S = \lambda/2$ as a result of tangent function. For the first quarter wave length, a short circuited line act as an inductance, where as an open circuited line appears as a capacitance.

3.6. POWER AND IMPEDANCE MEASUREMENT ON LINES

The expression for voltage and current on the dissipation less line are given by,

$$E = I_R \frac{Z_R + R_0}{2} 1 + |K| \llcorner \phi - 2\beta s$$

$$I = I_R \frac{Z_R + R_0}{2R_0} 1 - |K| \llcorner \phi - 2\beta s$$

When incident and reflected voltage waves are in phase then at that point voltage is maximum.

$$E_{max} = \frac{I_R |Z_R + R_0|}{2R_0} (1 - |K|)$$

For current

$$I_{max} = \frac{I_R |Z_R + R_0|}{2R_0} (1 + |K|)$$

Hence,

$$\frac{E_{max}}{I_{min}} = R_0$$

Since a change to the values at voltage and current minima requires only the reversal of phase of the reflected waves or a minus sign in front of |K|,

$$\frac{E_{min}}{I_{min}} = R_0$$

$$I_{min} = \frac{I_R}{2R_0} |Z_R + R_0| (1 - |K|)$$

$$\frac{E_{max}}{I_{min}} = R_0 \left[\frac{1 + |K|}{1 - |K|}\right] = SR_0$$

$$\frac{E_{max}}{I_{min}} == SR_0 = R_{max}$$

Similarly,

$$E_{min} = \frac{I_R}{2}|Z_R + R_0|(1 - |K|)$$

$$\frac{E_{min}}{I_{max}} = R_0\left[\frac{1 - |K|}{1 + |K|}\right] = \frac{R_0}{S}$$

$$= R_{min}$$

∴ The maximum power

$$P = \frac{E^2{}_{max}}{R_{max}}$$

Minimum power

$$P = \frac{E^2{}_{min}}{R_{min}}$$

$$P2 = \frac{E^2{}_{max}}{R_{max}} \times \frac{E^2{}_{min}}{R_{min}}$$

$$P = [|E_{max}||E_{min}|]R_0$$

The impedance is minimum at a point where voltage is also minimum.

$$Z_S = R_{min} = \frac{R_0}{S}$$

At any point on a line the impedance is,

$$Z_s = R_0\left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda}s\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda}s\right)}\right]$$

At the point of voltage minimum, distance S away from the load.

$$\frac{R_0}{S} = R_{in} = R_0\left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda}s\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda}s\right)}\right]$$

Solving equation. $$R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda}s\right) = S\left[Z_R + jR_0 \tan(\beta_S)\right]$$

$$-SZ_R + jZ_R \tan\left(\frac{2\pi}{\lambda}s^1\right) = -R_0 + jR_0 \tan(\beta_S)$$

$$-Z_R + jZ_R \tan\left(\frac{2\pi}{\lambda}s^1\right) = -R_0 + jR_0 \tan\left(\frac{2\pi}{\lambda}s^1\right)$$

$$Z_R = R_0\left[\frac{1 - j\tan\left(\frac{2\pi}{\lambda}s^1\right) S}{S + j\tan\left(\frac{2\pi}{\lambda}s^1\right)}\right]$$

In this method measurement of voltage minimum is preferred over that of voltage maximum as voltage minimum measurement is possible with greater accuracy.

PROBLEMS

Example 2.1 *A line with zero dissipation has R = 0.006 Ω/m, L = 2.5μH / m and C = 4.45pF/m. If the line is operated at 10 MHz find*

i) Ro ii) α iii) β iv) v, v) λ

Solution : Given R = 0.006 Ω/m, L = 2.5×10^{-6} H/m, C = 4.45 pF/ m, f = 10 MHz. At f = 10MHz, $\omega L = 2\pi f L = 2\times\pi\times10\times10^6\times2.5\times10^{-6} = 15.708\ \Omega$. Hence ωL >> R at 10 MHz. So according to standard assumption for the dissipationless line, we can neglect R.

i) The characteristic impedance is given by,

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.5\times10^{-6}}{4.45\times10^{-12}}} = \mathbf{749.53\ \Omega}$$

ii) The propagation constant is given by,

$$\gamma = \alpha + j\beta = 0 + j\omega\sqrt{LC}$$

Hence $$\gamma = \alpha + j\beta = 0 + j(2\times\pi\times10\times10^6)\sqrt{2.5\times10^{-6}\times4.45\times10^{-12}}$$

$$\therefore \quad \gamma = \alpha + j\beta = \mathbf{0 + j\ 0.2095\ per\ m}$$

∴ Attenuation constant = α = **0**

Phase constant = β = **0.2095 rad/m**

iii) The velocity of propagation is given by,

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5\times10^{-6}\times4.45\times10^{-12}}} = \mathbf{2.998\times10^8\ m/sec}$$

iv) The wavelength is given by,

$$= \frac{2\pi}{\beta} = \frac{2\pi}{0.2095} = \mathbf{29.9913\ m}$$

Example 2.2 : *A line having characteristic impedance of 50 Ω is terminated in load impedance* (75 + j75) Ω. *Determine the reflection coefficient and voltage standing wave ratio.*

Solution : The reflection coefficient is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{(75+j75)-50}{(75+j75)+50} = \frac{25+j75}{125+j75}$$

$$\therefore \quad K = \frac{79.056\angle 71.56^\circ}{145.7738\angle 30.96^\circ} = \mathbf{0.5423\angle 40.6}$$

The voltage standing wave ratio is given by,

$$VSWR = SWR = S = \frac{1+|K|}{1-|K|} = \frac{1+0.5423}{1-0.5423}$$

$$\therefore \quad S = \mathbf{3.369}$$

Example 2.3 : *A line with characteristic impedance of $692 \angle -12°$ is terminated in 200 Ω resistor. Determine K and S.*

Solution : Given

$$Z_0 = 692 \angle -12° \, \Omega = 678.878 - j\,143.87$$

$$Z_R = 200 \, \Omega$$

The reflection coefficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{200 - (678.878 - j\,143.87)}{200 + (678.878 - j\,143.87)} = \frac{-467.878 + j\,143.87}{878.878 - j\,143.87}$$

$$\therefore \quad K = \frac{498.1 \angle 163.21}{890.57 \angle -9.29}$$

$$\therefore \quad K = \mathbf{0.559 \angle 172.5°}$$

Then the standing wave ratio is given by,

$$S = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.559}{1 - 0.559} = \mathbf{3.535}$$

Example 2.4 : *Design a quarter wave transformer to match a load of 200 Ω to a source resistance of 500 Ω. Operating frequency is 200 MHz.*

Solution : For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_S = \frac{R_0^2}{Z_R}$$

The source impedance $Z_S = 500 \Omega$

load impedance = $Z_R = 200 \, \Omega$

$$\therefore \quad 500 = \frac{R_0^2}{200}$$

$$\therefore \quad R_0^2 = (500)\,(200)$$

$$\therefore \quad R_0^2 = 100000$$

$$\therefore \quad R_0 = 316.22 \, \Omega$$

The frequency of operation is

$$f = 200 \text{ MHz}$$

Hence the wavelength is given by,

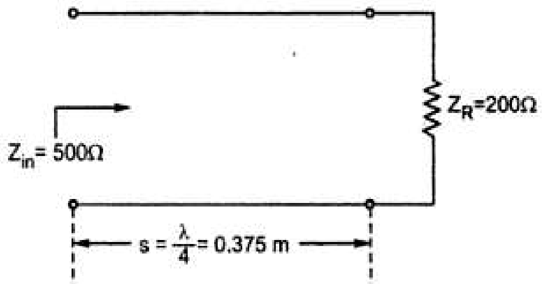
$$f.\lambda = c$$

$$\therefore \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ m}$$

∴ The length of quarter wave line is given by,

$$= \frac{\lambda}{4} = \frac{1.5}{4} = 0.375 \text{ m}$$

Hence the quarter wave line is as shown below in the Fig.



Example 2.5 : *Determine a length and impedance of a quarter wave transformer that will match a 150Ω load to a 75Ω line at a frequency of 12 GHz. Derive formula used.*

Solution : Given : $Z_R = 150\ \Omega$, $R_0 = 75\ \Omega$, f = 12 GHz

For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_S = \frac{R_0^2}{Z_R} \quad \left(\text{or } R_0' = \sqrt{Z_S \cdot Z_R}\right)$$

$$\therefore \quad Z_{in} = Z_S = \frac{(75)^2}{150} = 37.5\ \Omega$$

Thus R_0' of the matching section is 37.5 Ω

The operating frequency is 12 GHz.

∴ The wavelength can be calculated as,

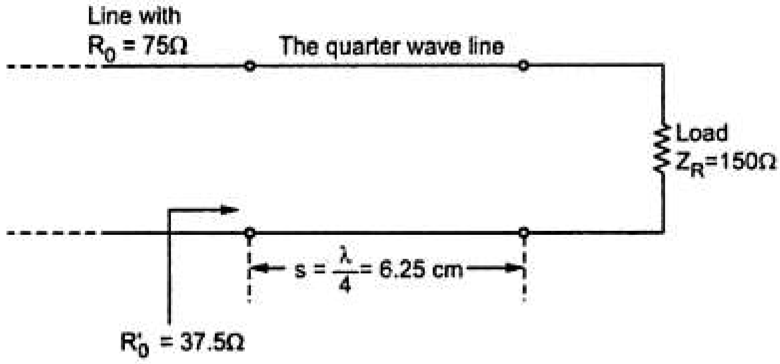
$$f.\lambda = c$$

$$\therefore \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{12 \times 10^9} = 0.025 \text{ m} = 25 \text{ cm}$$

Hence the length of quarter wave line is given by

$$s = \frac{\lambda}{4} = \frac{0.025}{4} = 6.25 \text{ cm}$$

Thus the quarter wave transformer is as shown in the Fig.



Example 2.6 : *A certain R.F. transmission line is terminated in pure resistive load. The characteristic impedance of the line is 1200 Ω and the reflection coefficient was observed to be 0.2. Calculate the terminating load, which is less than characteristic impedance.*

Solution : At radio frequencies, $S = \frac{R_0}{R_R} = \frac{1200}{R_R}$ for resistive load

$$K = \frac{s-1}{s+1} = 0.2$$

$$\therefore \qquad K = \frac{\frac{1200}{R_R}-1}{\frac{1200}{R_R}+1} = 0.2$$

Simplifying for R_R, we get

$$R_R = 800\ \Omega$$

Example 1: A line with zero dissipation has R=0.006 **Ω**/m, L = 2.5μH/M and C = 4.45 PF/m. If the line is operated at 10MHz find i) R_0, ii) $\propto$ iii) βiv) ϑ v) λ.

Solution: ***Given:*** R = 0.006 **Ω**/m L = 2.5×10^{-6}H/m, C = 4.45 PF/m, f = $10\times 10^{-6}H_z$

$\therefore$ ωL $= 2\pi fL = 15.708\Omega$ ωL>>R at 10 MHz. So neglect R

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}\sqrt{\frac{2.5\times10^{-6}}{4.45\times10^{-12}}} = 749.53\Omega$$

$$\gamma = \propto + j\beta$$

$= 0 + j\omega\sqrt{LC}$

$= 0 + j\,(2\pi 10\text{ M})\sqrt{2.5\times10^{-6}\times 4.45\times10^{-12}}$

$= 0 + j.2095$ per meter

$$\therefore \quad \propto = 0 \qquad \beta = 0.2095 \text{ rad/m}$$

$$\vartheta = \sqrt{\frac{1}{\text{LC}}} = \frac{1}{\sqrt{2.5 \times 10^{-6} \times 4.45 \times 10^{-12}}} = 2.998 \times 10^{8} \text{ m/sec}$$

$$\gamma = \frac{2\pi}{0.2095} = 29.9913 \text{ m}$$

Example 2. A certain transmission line, working at radio frequencies, has following constants L = 9µH/M and C = 16 PF/m. The line is terminated in a resistive load of 1000 **Ω**. Find the reflection co-efficient and standing wave ratio.

Solution: The Characteristic impedance of line is given by,

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{9 \times 10^{-6}}{16 \times 10^{-12}}} = 750\ \Omega$$

The reflection co-efficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{1000 - 750}{1000 + 750} = 0.1428$$

The standing wave ratio S is given by,

$$S = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.1428}{1 - 0.1428} = 1.333$$

Example 3. A certain RF transmission line is terminated in pure resistive load. The characteristic impedance of the line is 1200**Ω** and the reflection co-efficient was observed to be 0.2. Calculate the terminating load, which is less than characteristic impedance

Solution At radio frequencies

$$S = \frac{R_0}{R_R} = \frac{1200}{R_R} \quad for\ resistive\ load$$

$$K = \frac{S - 1}{S + 1} = 0.2$$

$$K = \frac{\frac{1200}{R_R} - 1}{\frac{1200}{R_R} + 1} = 0.2$$

$$R_R = 800\Omega$$

Example 4. Calculate Standing wave ratio and reflection co-efficient on a line having Z_0 = 300**Ω** and terminated in $Z_R = 300 + j400$.

Solution: The reflection co-efficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{(300 + j400) - (300)}{(300 + j400) + 300} = \frac{j400}{600 + j400}$$

$$= \frac{400 \angle 90°}{721.11 \angle 33.69°}$$

$$= 0.5547 \angle 56.31^0$$

The standing wave ratio S is given by

$$S = \frac{1 + |K|}{1 - |K|}$$

$$= \frac{1 + 0.5547}{1 - 0.5547}$$

$$S = 3.4913$$

Example 5. Determine length and location of a single short circuited stub to produce an impedance match on a transmission line with R_0 of 600**Ω** and terminated in 1800 **Ω**.

Solution: ***Given*** $R_0 = 600\Omega$ $Z_R = 1800\Omega$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{1800 - 600}{1800 + 600} = \frac{1200}{2400} = 0.5\angle\ 0°$$

Hence

$$S = \frac{\emptyset + \pi - cos^{-1|K|}}{2\beta}$$

$$= \frac{\emptyset + \pi - cos^{-1|K|}}{2(\frac{2\pi}{\lambda})}$$

$$= \frac{0 + \pi - cos -1\ (0.5)}{4\pi} . \lambda$$

$$= 0.166\ \lambda$$

$$L = \lambda/2\pi\ tan - 1 \left[\frac{\sqrt{1 - |K|^2}}{2\ |K|}\right]$$

$$= \lambda/2\pi\ tan - 1 \left[\frac{\sqrt{1 - 0.5^2}}{2 \times 0.5}\right]$$

$$= 0.1135\ \lambda$$

$$Case\ 2: S1 = \frac{\emptyset + \pi + cos^{-1|K|}}{2\beta}$$

$$= \frac{0 + \pi\ cos -1\ (0.5)}{4\pi} . \lambda$$

$$= 0.033\ \lambda$$

$$L1 = \lambda/2\pi\ tan - 1 \left[\frac{\sqrt{1 - |K|^2}}{-2\ |K|}\right]$$

$$= \lambda/2\pi\ tan - 1 \left[\frac{\sqrt{1 - 0.5^2}}{-2 \times 0.5}\right]$$

$$= \lambda/2\pi(\pi\ tan - 1\ 0.866)$$

$$= 0.386\ \lambda$$

Example 6. Determine a length and impedance of a quarter wave transformer that will match a 150 Ω load to 975 Ω line at a frequency of 12GHz. Derive formula used.

Solution: Given $Z_R = 150\ \Omega$, $R_0 = 75\ 150\ \Omega$, $f = 12\ GH_z$

For a quarter wave transformer,

$$Z_{in} = Z_s = \frac{R_0^2}{ZR}(or)R_0^1 = \sqrt{Z_{es}.Z_R})$$

$$Z_{in} = Z_s \frac{75^2}{150} = 37.5\ \Omega$$

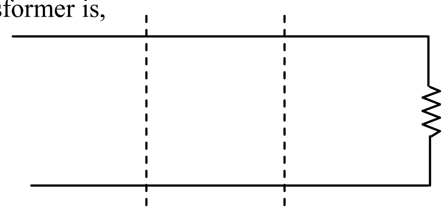
Thus R_0^1 of the matching section is 37.5 Ω. The operating frequency is 12GHZ

∴ The wavelength can be calculated as, f. λ = c

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{12 \times 10^9} = 0.025 \text{ m} = 25 \text{ cm}$$

Hence the length of quarter wave line is, $S = \frac{\lambda}{4} = \frac{0.025}{4} = 6.25$ cm

Thus the quarter wave transformer is,



Example 7: A line having characteristic impedance of 50Ω is terminated in impedance (75 + j75) Ω. Determine the reflection coefficient and voltage standing ratio.

Solution: The reflection coefficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{(75+j75)-50}{(75+j75)+50} = \frac{25+j75}{125+j75}$$

$$\therefore K = \frac{79.056 \angle 71.56°}{145.7738 \angle 30.96°} = 0.5423 \angle 40.6$$

The voltage standing wave ratio is given by, VSWR = SWR = $S = \frac{1+|k|}{1-|k|} = \frac{1+0.5423}{1-0.5423}$

S = 3.369

Example 8: A line with characteristic impedance OF 629 ∠ -20° is terminated in 200 Ω resistor. Determine K and S.

Solution: Given

$$Z_0 = 629 \angle -20° \ \Omega = 678.878 - j143.87 \quad Z_R = 200 \ \Omega$$

The reflection coefficient is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{200 - (678.878 - j143.87)}{200 + (678.878 - j143.87)} = \frac{-467.878 + j143.87}{-467.878 - j143.87}$$

$$K = \frac{498.1 \angle 163.21}{890.57 \angle -9.29}$$

$$K = 0.0559 \angle 172.5°$$

Then the standing wave ratio is given by,

$$S = \frac{1 + |k|}{1 - |k|} = \frac{1 + 0.559}{1 - 0.559} = 3.535$$

Example 9: A land of admittance $\frac{Y_R}{G_0} = 1.25 + j0.25$. Find the length and location of single stub turner short circuited.

Solution: The normalized load admittance is given as, $\frac{Y_R}{G_0} = 1.25 + j0.25$

$$\therefore \quad \frac{1/Z_R}{1/R_0} = (1.25 + j0.25)$$

$$\therefore \qquad \frac{R_0}{Z_R} = (1.25 + j0.25)$$

The reflection coefficient is given by, $K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R - R_0}{Z_R + R_0} = \frac{1 - \frac{R_0}{Z_R}}{1 + \frac{R_0}{Z_R}}$

$$K = \frac{1 - (1.25 + j0.25)}{1 + (1.25 + j0.25)} = \frac{-0.25 - j\,0.25}{2.25 + j\,0.25}$$

$$= \frac{0.3536 \angle -135°}{2.2638 \angle 6.34°}$$

$$= 0.1561 \angle -141.34°$$

$$= 0.1561 \angle -2.466°$$

Calculating value of $\cos^{-1}(|K|)$ $\qquad \therefore \quad \cos^{-1}(|K|) = \cos^{-1}(0.1561) = 1.414$

Calculation for the length and location of stub:

Case (1) $\qquad S_1 = \frac{\phi + \pi - cos^{-1}(|K|)}{2\beta}$

$$= \frac{\phi + \pi - cos^{-1}(|K|)}{2\left(\frac{2\pi}{\lambda}\right)}$$

$$= \frac{-2466 + \pi - 1.414}{\lambda 4\pi} .\lambda$$

$$= -0.0587\,\lambda$$

Length of the stub, $\qquad L = \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1 - |K|^2}}{2|K|}\right]$

$$= \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1 - (0.3535)^2}}{2(0.3535)}\right] = 0.1469\,\lambda$$

***Case* (2)** : $S_1 = \frac{\phi + \pi - cos^{-1}(|K|)}{2\beta}$

$$= \frac{\phi + \pi - cos^{-1}(|K|)}{4\pi}$$

$$= \frac{-2466 + \pi + 1.414}{4\pi} .\lambda$$

$$= 0.1662\lambda$$

Length of the stub, $\qquad L = \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1 - |K|^2}}{-2|K|}\right]$

$$= \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1 - (0.3535)^2}}{-2(0.3535)}\right]$$

$$= \frac{\lambda}{2\pi} tan^{-1}(-1.3231)$$

$$= \frac{\lambda}{2\pi}\left[\pi - tan^{-1}(1.013231)\right]$$

$$= 0.353\lambda$$

Example 10: A lossless transmission line with Z_0 = 75Ω and of electrical length l = 0.3λ is terminated with load impedance of Z_R = (40 + j20) Ω. Determine the reflection coefficient at load SWR of line, input impedance of the line.

Solution: Given $Z_0 = R_0 = 75\Omega$ $Z_R = (40 + j20)$

$$\text{The reflection coefficient is given by,} \quad K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{(40 + j20) - 75}{(40 + j20) + 75}$$

$$= \frac{-35 + j20}{115 + j20}$$

$$= \frac{40.311 \angle 29.74°}{116.726 \angle 9.86°}$$

$$= 0.3453 \angle 19.88°$$

$$\text{The standing wave ratio is given by,} \quad S = \frac{1 + |K|}{1 - |K|}$$

$$= \frac{1 + 0.3453}{1 - 0.3453}$$

$$= 2.0548$$

$$\text{The input impedance of the line is given by,} \quad Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi s}{\lambda}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi s}{\lambda}\right)}\right]$$

$$= 75 \left[\frac{(40 + j20 + j(-230.82)\frac{2\pi \times 0.3\lambda}{\lambda}}{75 + (-j123.1) + (+61.55)\frac{2\pi \times 0.3\lambda}{\lambda}}\right]$$

$$= 75 \left[\frac{40 + j20 + j(-230.82)}{75 + (-j123.1) + (+61.55)}\right]$$

$$= 75 \left[\frac{40 - j210.82}{136.55 - j123.1}\right]$$

$$= 75 \left[\frac{241.581 \angle -79.25°}{183.84 \angle -42.03°}\right]$$

$$= 75\,(1.167 \angle -37.222°)$$

$$= (69.7 - j52.95)\,\Omega$$

Example 11: A lossless RF line has Z_0 of 600 Ω and is connected to a resistive loan of 75 Ω. Find the position and length of short circuited stub of same construction line which would enable the main length of a line to be correctly terminated 150 MH_z.

Solution: Given f = 50 MH_z $R_o = 600\ \Omega$ $Z_R = 75\ \Omega$

$$\text{Calculating } \lambda \text{ first,}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{150 \times 10^6} = 2m$$

$$\text{The reflection coefficient is given by}$$

$$K = \frac{Z_R - R_0}{Z_R + Z_0} = \frac{Z_R - R_0}{Z_R + R_0}$$

$$K = \frac{75-600}{75+600} = \frac{-525}{675} = -0.7777$$

$$K = 0.7777 \angle \pi^c = 0.7777 \angle 180°$$

The two possible locations of stub are as follows.

Case: 1 $$S_1 = \frac{\phi + \pi - cos^{-1}(|K|)}{2\beta}$$

But $\beta = \frac{2\pi}{\lambda}$ $$S_1 = \frac{\phi + \pi - cos^{-1}(|K|)}{4\pi}.\lambda$$

$$= \frac{\pi + \pi - cos^{-1}(0.7777)}{4\pi}(2) = 0.8918\, m$$

The length of the stub is given by $$L = \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1-|K|^2}}{2|K|}\right]$$

$$\therefore L = \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1-(0.7777)^2}}{2(0.7777)}\right] = 0.122m$$

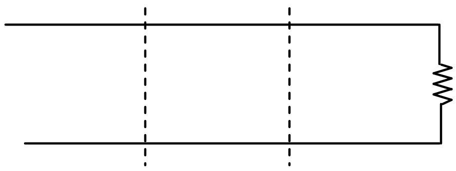
Case 2: $$S_1 = \frac{\phi + \pi + cos^{-1}(|K|)}{4\pi}.\lambda = \frac{\pi + \pi - cos^{-1}(0.7777)}{4\pi} \times 2$$

$$= 1.108\, m$$

The length of the stub is given by $$L = \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1-|K|^2}}{-2|K|}\right]$$

$$= \frac{\lambda}{2\pi} tan^{-1}\left[\frac{\sqrt{1-(0.7777)^2}}{-2(0.7777)}\right]$$

Thus the quarter wave transformer is as shown in the Fig:



Example 12: A should line experiment performed with the following results: Distance between successive minima is 21 cm, distance of first voltage minimum from load is 0.9 cm. SWR of line is 2.5. If $Z_0 = 50\ \Omega$, determine load impedance.

Solution: *The distance between two successive voltage minima is given by,* $\frac{\lambda}{2} = 21$

$$\lambda = 42\, cm = 42 \times 10^{-2}\, m = 0.42\, m$$

The frequency of operation is given by, $f\lambda = c$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{42 \times 10^{-2}} = 0.7152\, GHz$$

The standing impedance in terms of S and s'(i.e. distance of first voltage minimum from load) is given by

$$Z_L = R_0\left[\frac{1 - jS\tan\left(\frac{2\pi s'}{\lambda}\right)}{S - j\tan\left(\frac{2\pi s'}{\lambda}\right)}\right] = 50\,\frac{1 - j(2.5)tan\left(\frac{2\pi \times 0.9}{42}\right)}{2.5 - jtan\left(\frac{2\pi \times 0.9}{42}\right)}$$

$$= 50\left[\frac{1 - \mathrm{j}(0.25)(0.1355)}{2.5 - \mathrm{j}(0.1355)}\right] = 50\left[\frac{1 - \mathrm{j}0.033875}{2.5 - \mathrm{j}0.1355}\right]$$

$$= 50\left[\frac{1\angle -1.9401}{2.5\angle -3.102}\right] = 50\,[0.4\angle 1.1622°$$

$$=20\angle 1.1622°\,\Omega$$

Example 13 : Design a quarter wave transformer to match a load of source resistance of 500 Ω. Operating frequency is 200 MH_Z.

Solution: For a quarter wave transformer, the input impedance is given $Z_{in} = Z_S = \frac{R_0^2}{Z_R}$

The source impedance Zs = 500 Ω. Load impedance ZR = 200 Ω

$$500 = \frac{R_0^2}{Z_R}$$

$$\therefore \frac{R_0^2}{Z_R} = (500)(200)$$

$$\therefore \frac{R_0^2}{Z_R} = 100000$$

$$\therefore \quad R_0 = 316.222\Omega$$

The frequency of operation is $f = 200\,MHz$

Hence the wavelength is given by $\therefore \quad \lambda = \frac{c}{f} = \frac{3\times 10^8}{42\times 10^6} = 1.5m$

$\therefore$ *The length of quarter wave line is given by,* $s = \frac{\lambda}{4} = \frac{1.5}{4} = 0.375$

Example 14: Determine a length and impedance of a quarter wave transformer that will match a 150 Ω load to 1 75 Ω line at a frequency of 12 GHz. Derive formula used.

Solution: Given, ZR =150 Ω, R_0 = 1 75 Ω, f =m12 GHz

For a quarter wave transformer, the input impedance is given by,

$$Z_{in} = Z_s = \frac{R_0^2}{Z_R}(Or)\left(R_0^1 = \sqrt{Z_s.Z_R}\right)$$

$$Z_{in} = Z_s = \frac{(75)^2}{150} = 37.5\boldsymbol{\Omega}$$

Thus R_0^1 *of the matching section is* 37.5 Ω*. The operating frequency is* 12 *GHz*

$\therefore$ *The wavelength can be calculated as,*

$$f.\lambda \quad = c$$

$$\therefore \quad \lambda = = \frac{3\times 10^8}{12\times 10^9} = 0.025\,m = 25m$$

Hence the length of quarter wave line is given by $S = \frac{\lambda}{4} = \frac{0.025}{4} = 6.2\,cm$

SUMMARY

- The reflection co-efficient is K = $\frac{Z_R - Z_0}{Z_R + R_0}$
- When the line is terminated in R_0 the standing waves
- Standing Wave ratio S =$\frac{E_{max}}{E_{min}}$

In terms of reflection coefficient S = $\frac{1+|K|}{1-|K|}$ (or) $|K| = \frac{S-1}{S+1}$

- $|K| = \frac{|E_{max}|-|E_{min}|}{|E_{max}|+|E_{min}|}$
- The power equation of P =$|I_{max}| - |EI_{min}|R_0$ (or) $\frac{E2_{max}}{R_{min}}$
- Quarter wave line Zin = $\frac{R_0^2}{Z_R}$; l = $\lambda/4$
- For a dissipation less line $Z_0 = R_0 = \sqrt{\frac{L}{C}}$ ωL>> R
- $\gamma = \propto + j\beta$; $\propto = 0$; $\beta = \omega\sqrt{LC}$

  Velocity $= \vartheta = \frac{1}{\sqrt{LC}}$
- When the line is open circuited at the receiving end $I_R = 0$

  $E_{OC} = E_R \cos\frac{2\pi}{\lambda} S$

  $I_{OC} = j\frac{E_R}{R_0}\sin\frac{2\pi}{\lambda} S$
- Nodes are points of Zero voltage and current on the standing wave system.
- Antinodes are points of maximum voltage or current.

PART A

1. **State the assumptions for the analysis of the performance of the radio frequency line.**
   - Due to the skin effect ,the currents are assumed to flow on the surface of the conductor. The internal inductance is zero.
   - The resistance R increases with Ö f while inductance L increases with f . Hence wL>>R.
   - The leakage conductance G is zero
2. **State the expressions for inductance L of a open wire line and coaxial line.**
   - For open wire line ,
     L=9.21*10-7(μ/μr +4ln d/a)=10-7(μr+9.21log d/a) H/m
   - For coaxial line,
     L = 4.60*10-7[log b/a]H/m
3. **State the expressions for the capacitance of a open wire line**
   For open wire line ,
   C=(12.07)/(ln d/a)μμf/m
4. **What is dissipationless line?**
   A line for which the effect of resistance R is completely neglected is called dissipationless line .
5. **What is the nature and value of Z_0 for the dissipation less line?**
   For the dissipation less line, the Z_0 is purley resistive and given by, Z_0=R_0 = √ L/c
6. **State the values of** a **and** b **for the dissipation less line.**
   α=0 and β=w √LC