UNIT III
IMPEDANCE MATCHING INHIGH FREQUENCY LINE

Impedance matching: Quarter wave transformer - Impedance matching by stubs - Single stub and double stub matching - Smith chart - Solutions of problems using Smith chart - Single and double stub matching using Smith chart.

3.1. IMPEDANCE MATCHING: QUARTER WAVE TRANSFORMER

The generalized expression for the input impedance of the line is given by,

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\beta_S)}{R_0 + jZ_R \tan(\beta_S)}\right]$$

Rearranged the terms,

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\beta_S)} + jR_0}{\frac{R_0}{\tan(\beta_S)} + jZ_R}\right]$$

β= 2π/λ

$$= R_0 \left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda}s\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{\lambda}s\right)} + jZ_R}\right]$$

For quarter – wave line S = $\lambda/_4$

$$= R_0 \left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{4}\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{4}\right)} + jZ_R}\right]$$

$$= R_0 \left[\frac{\frac{Z_R}{\tan \pi/2} + jR_0}{\frac{R_0}{\tan \pi/2} + jZ_R}\right]$$

$$Z_{in} = R_0 \left[\frac{jR_0}{jZ_R}\right]$$

$$Z_{in} = R_0 \left[\frac{R_0^2}{Z_R}\right]$$

This equation is similar to the equation for impedance matching using transformer.

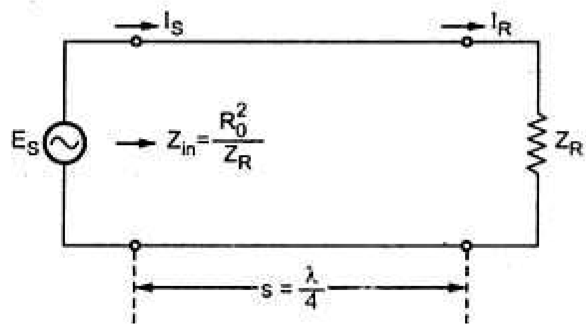


Fig 3.1: The quarter – wave line

Thus a quarter wave line may be used as a transfer for impedance matching of load Z_R with input impedance $Z_{in} = Z_R$. For matching impedance Z_R& Z_{in}, the line characteristic impedance R_0 may be selected such that conditions.

$$R_0 = \left|\sqrt{Z_R . Z_{in}}\right|$$

A quarter wave line can transform a low impedance into a high impedance and vice versa, thus it can be considered as an impedance inverter. The quarter wave matching section has number of application. One of the important applications is impedance transformation in coupling a transmission line to a resistive load such as an antenna.

If the antenna resistance is R_A and the characteristic impedance of the line is R_0, then the quarter wave impedance matching section is designed such that its characteristic impedance $R_0$1 transforms antenna resistance R_A to the characteristic impedance of line R_0 given by

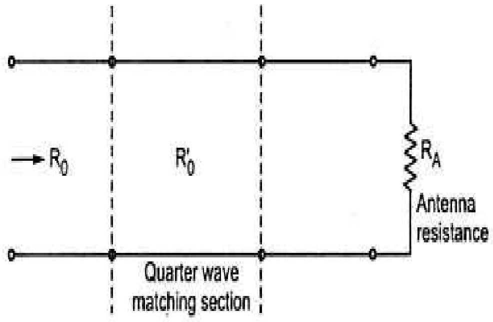


Fig 3.2: The quarter wave line matching section

Another application of a quarter wave section is in the line with load which is not pure resistive. Under such a condition Voltage is minimum and current is maximum. The characteristic impedance of the impedance matching,

$R_0^1 = \sqrt{R_A . \left(\frac{R_0}{S}\right)} = R_0\sqrt{\frac{1}{S}}$ For step down impedance

$$R_0^1 = \sqrt{R_0 (SR_0)} = R_0\sqrt{S}$$

The quarter wave line may be used to provide mechanical support to the open wire line or centre conductor of a co-axial cable. Such a line as mechanical support is shorted at ground. As quarter wave line is shorted at ground, its input impedance is very high. So the signal on line passes to the receiving end, without any losses due to this mechanical support. Thus the lines act as an insulator at this point. Hence such a line is referred as copper insulator.

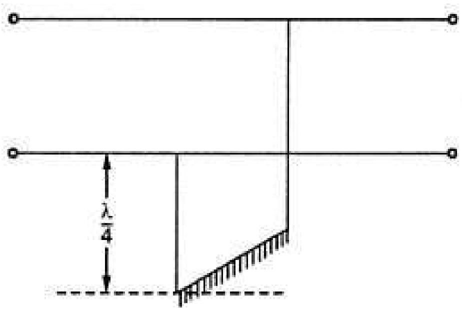


Fig 3.3: A quarter wave line used as a mechanical support to an open wire line

3.2. IMPEDANCE MATCHING BY STUBS

A section of transmission line can be used as matching section by inserting them between load and the source. It is also possible to connect sections of open and short circuited line called stub in shunt with the line at some points or points to affect impedance matching. This is called stub matching. Another mean of accomplishing impedance matching is the use of an open or short circuited line of suitable length, called stub at a designated distance from the load. This is called stub matching.

The advantages are:

i. The length and Characteristic impedance of the line remains unaltered.
ii. Adjustable susceptance are added in shunt and the line.

Two types of stub matching are,

- Single Stub Matching
- Double Stub Matching

3.3. SINGLE STUB AND DOUBLE STUB MATCHING

In single stub matching technique, a stub of suitable length is connected in parallel with line at certain distance from load short circuited stub. Since connect the stub in parallel with the main line, it is easier to deal with the admittance as they can be added up.

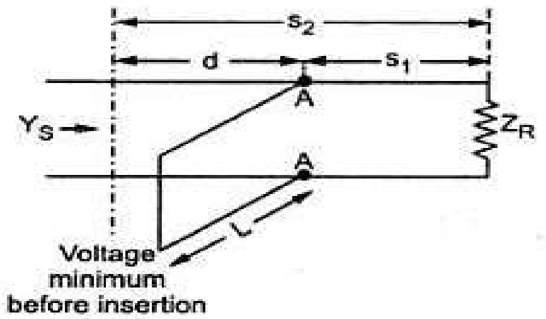


Fig 3.4: single stub matching technique

Consider a transmission line having characteristic admittance Y_0 terminated in a pure conductance Y_R. When $Y_R = Y_0$ standing wave occur. If a suitable susceptance obtained by using appropriate length of a short circuited or open circuited stub is added in shunt so as to obtain anti-resistance with the susceptance already existing, then up to that point, matching. The connected stub should provide susceptance equal in magnitude but opposite in phase as compared with the susceptance already existing. Then up to that point, matching has been achieved. It is desirable that the stub be located as near the load as possible. Also the characteristic admittance of the stub so connected in shunt should be same as that of the main line.

The second standard form of input impedance is,

$$Z_{in} = R_0 \left[\frac{Z_R\, Cos\, h\gamma l + Z_0 Sin\, h\gamma l}{Z_0\, Cos\, h\gamma l + Z_R Sin\, h\gamma l}\right]$$

$$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 tan\, h\gamma l}{Z_0 + Z_R tan\, h\gamma l}\right]$$

Converting impedance to admittance,

$$Y_{in} = Y_0 \left[\frac{Y_R + Y_0 tan\, h\gamma l}{Y_0 + Y_R tan\, h\gamma l}\right]$$

$\gamma = \propto + j\beta$ (for lossless line $\propto = 0$) tan hjθ = j tanθ

$$= Y_0 \left[\frac{Y_R + Y_0 j\, tan\, \beta l}{Y_0 + Y_R j\, tan\, h\beta l}\right]$$

Normalizing,

$$\frac{Y_{in}}{Y_0} = \left[\frac{Y_R + j tan\, \beta l}{1 + j\, Y_R tan\, \beta l}\right]$$

$$= \left[\frac{Y_R + j tan\, \beta l}{1 + j\, Y_R tan\, \beta l}\right] \times \frac{1 - j\, Y_R\, tan\, \beta l}{1 - j\, Y_R\, tan\, \beta l}$$

$$= \frac{Y_r - jy_r^2\, tan\, \beta l + j\, tan\, \beta l + jy_r^2 tan^2\, \beta l}{1 + y_0^2\, tan^2\, \beta l}$$

$$\frac{Y_{in}}{Y_0} = \frac{Y_r [\, tan^2\, \beta l + j\, tan\, \beta l + (-1 y_r^2\,)}{1 + \, y_r^2\, tan^2\, \beta l}$$

For no reflection $Y_{in} \; = \; Y_0$ (or) $\frac{Y_{in}}{Y_0}$ =1 ie.,$Y_{in} \; = 1 \;\; 1 + j_0 = 0$

Thus the stub has to be located at a point where the real part of Y_{in} is equal to unity

$$\frac{Y_r\, [\, 1 + \, tan^2\, \beta l]}{1 + \, y_0^2\, tan^2\, \beta l} = 1$$

$$Y_r\, [\, 1 + \, tan^2\, \beta l] = [1 + \, y_r^2\, tan^2\, \beta l\,]$$

$$tan^2\, \beta l\, (Y_r \, - \, y_r^2) = 1 - \, y_r^2$$

$$tan^2\, \beta l = \frac{1 - \, Y_r}{Y_r\, \; (1 - \, Y_r\,)} = \frac{1}{Y_r}$$

$$tan\beta l = \sqrt{\frac{1}{Y_r}} = \sqrt{\frac{Y_0}{Y_R}}$$

This equation gives the location of stub admittance can be simplified as,

$$\beta ls = tan^{-1} \sqrt{\frac{Y_0}{Y_R}}$$

$$\frac{2\pi}{\lambda} ls \, = \, tan^{-1} \sqrt{Y_0/Y_R}$$

$$ls \, = \, \frac{\lambda}{2\pi} tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

The susceptance added by the stub will be

$$b_s \, = \, \frac{Y_0 - \, Y_R}{Y_0} \sqrt{\frac{Y_0}{Y_R}}$$

The desired length of the stub l_t to be yield b_s is,

$$bs \, = \, \frac{\lambda}{2\pi} tan^{-1} \sqrt{\frac{Z_R Z_0}{Z_R - Z_0}}$$

Disadvantages of Single stub matching:

This is useful for a fixed frequency only because as the frequency changes, the location of the stub will have to be changed. For final adjustment, the stub has to be moved along the line slightly. This is possible only in open wire and therefore on co axial line, single stub matching may become inaccurate in practice.

3.4. DOUBLE STUB MATCHING:

The disadvantages of single stub matching technology is overcome by double stub matching technology. Here two different short circuited stubs of lengths l_1& l_2 are used for impedance matching

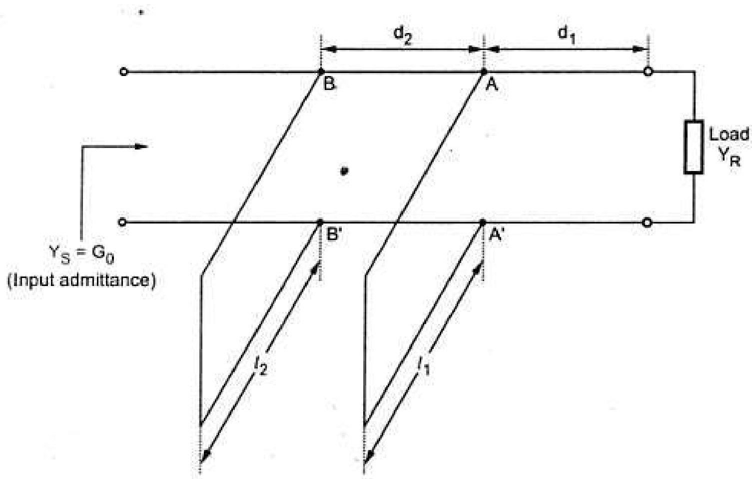


Fig 3.5: double stub matching technique

Let sub 1 be located at point AA[1], at a distance d, from the load. Let the length of the stub 1 be l. Similarly let stub 2 be located at point BB[1]at a distance d_2 away from stub1. Let the length of the stub 2 be l_2.

The input impedance of dissipation less line is

$$Z_S = Z_0\left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan h\beta l}\right]$$

But $Z_S = {}^1/_{Y_S} = , Z_0 = {}^1/_{Y_0} , \quad Z_R = {}^1/_{Y_R}$

$$\therefore = \frac{1}{Y_S} = \frac{1}{Y_0}\left[\frac{\frac{1}{Y_R} + j\frac{1}{Y_0}\tan \beta l}{\frac{1}{Y_0} + j\frac{1}{Y_R}\tan \beta l}\right]$$

$$\frac{Y_0}{Y_S} = \left[\frac{1 + j\frac{Y_R}{Y_0}\tan \beta l}{\frac{Y_R}{Y_0} + j\tan \beta l}\right]$$

Taking reciprocal$\frac{Y_R}{Y_0} + j \tan \beta l$ $\quad \frac{Y_S}{Y_0} = \frac{Y_R}{Y_0} + j\tan \beta l / 1 + j \tan \beta l$

$$Y_r = j\tan \beta l / 1 + Y_r \tan \beta l$$

Where Y_S - normalised input admittance and Y_r - normalised load admittance

$$Y_S = \frac{(Y_r + j\tan \beta l)}{1 + j\,Y_r \tan \beta l} \times \frac{1 - j\,Y_r \tan \beta l}{1 - j\,Y_r = \tan \beta l}$$

$$Y_S = \frac{Y_r^2 \tan \beta l + j\tan \beta l + Y_r = \tan^2 \beta l}{1 + Y_r^2 \tan \beta l}$$

$$Y_S = \frac{Y_r(1 + \tan^2 \beta l) + j\tan \beta l(1 - Y_r^2)}{1 + Y_r^2 \tan \beta l}$$

Stub1 is located at point AA[1] at a distance l = d, from the load. $Y_S = g_i + jb_i$

When a stub 1 having a susceptance jb_i is added at this point, the new admittance value will be, $Y_s^1 = g_i$. The point BB[1] should located such that the normalized admittance at this point is given by, $Y_B^1 = 1 + jbe$. Finally the length of stub 2 is adjusted such that susceptance of the stub 2 is $\mp jb_2$. Practically the two stubs must be separated by a distance $\frac{\lambda}{16}, \frac{\lambda}{8}, \frac{3\lambda}{16}, \frac{3\lambda}{8}$ etc. But the most commonly used separation between the two stubs.

3.4.1. STEPS TO SOLVE A STUB MATCHING PROBLEM

Steps To Solve A Single-Stub Matching Problem

Goal: Design a single-stub matching network such that $Y_{IN} = Y_{STUB} + Y_A = Y_0$

1. Convert the load to a normalized admittance: Y_L=g+jb
2. Transform Y_L along constant Γ towards generator until Y_A = 1 + jb. This matches the network's conductance to that of the transmission line and determines d_{stub}
3. Find Y_{stub} = -jb on Smith Chart
4. Transform Y_{STUB} along constant Γ towards load until we reach P_{SC} (for short-circuit stub) or P_{OC} (for open-circuit stub) This cancels susceptance from (2) and determines L_{STUB}

Steps To Solve A Double-Stub Matching Problem

Goal: Design a double-stub matching network such that $Y_{IN,A} = Y_0$

1. Convert the load to a normalized admittance: Y_L=g+jb
2. Transform Y_L along constant Γ towards generator by distance dA to reach $Y_A = g_A + jb_A$
3. Draw auxillary circle (pivot of g=1 circle by distance dB)
4. Add susceptance (b) to Y_A to get to $Y_{IN,A}$ on auxillary circle. The amount of susceptance added is equal to $-b_{SA}$, the input susceptance of stub A.
5. Find $Y_{SA} = -jb_{SA}$ Determine L_A by transforming Y_{SA} along constant Γ towards load until we reach P_{SC} (for short-circuit stub) or P_{OC} (for open-circuit stub).
6. Transform $Y_{IN,A}$ along constant Γ towards generator by distance dB to reach Y_B on auxillary circle. The susceptance of Y_B (b_B) is equal to $-b_{SB}$, the input susceptance of stub B.
7. Find $Y_{SB} = -jb_{SB}$ Determine L_B by transforming Y_{SB} along constant Γ towards load until we reach P_{SC} (for short-circuit stub) or P_{OC} (for open-circuit stub).

3.5. SMITH CHART

The main draw book of the circle diagram is that the constant S Circles and constant βs. Circle are not concentric., Hence it is difficult to interpolate these circles. Also diagram can be used for the limited range of the impedance values with practical chart size. Hence P.H. Smith developed a new chart which overcome all the drawback called chart and it is extensively used for the analysis of transmission line problems. In a circle diagram the values of resistive and reactive components are represented in the rectangular from which extends to infinity. But the Smith chart, the infinite resistance and reactance components are transformed to an area inside a circle. Hence Smith chart is also known as circular chart.

"Smith Chart is a special polar diagram containing constant resistance circles, constant reactance circles, circles of constant standing wave ratio and radius lines representing line-angle loci; used in solving transmission line and waveguide problems" The basic difference between circle diagram and smith chart is that in the circle diagram the impedance is represented in a rectangular form while in the smith chart the impedance is represented in a circular form. Smith Chart is based on two sets of orthogonal circles. The tangents drawn at the points of intersection of two circles would be mutually perpendicular one set of circles represent the rate of the resistive component(R) of the line impedance to the characteristic impedance (Zo) of the line, which for a lossless line is purely resistive. The second set of circles represents the ratio of the reactive component(X) of the line impedance to the characteristic impedance (Zo) of the line.

3.5.1. Construction of Smith Chart:

The reflection co-efficient

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R/Z_0 - 1}{Z_R/Z_0 + 1}$$

Z_R - terminating impedance and Z_0 - Characteristic impedance

Let $\frac{Z_R}{Z_0}$ = normalised terminating impedance Z_r

$$\therefore K = \frac{Z_r - 1}{Z_r + 1}$$

$$Z_r - 1 = K(Z_r + 1)$$

$$Z_r (1 - K) = K + 1)$$

$$Z_r = \frac{1 + K}{1 - K}$$

Z_r and K are complex quantities. Hence Z_r can be represented as R +jx and k can be represented as $K_r + jK_x$

$$\therefore R + jX = \frac{1 + K_r + j K_x}{1 - K_r - j K_x}$$

$$= \frac{(1 + K_r) + j K_x}{(1 - K_r) - j K_x} \times \frac{(1 - K_r) + j K_x}{(1 - K_r) + j K_x}$$

$$= \frac{1 - K_r + j K_x + K_r - Kr^e + jK_x K_{r+j K_x - j K_x K_r} - K_x^2}{(1 - K_r)^2 - K_x^2}$$

$$= \frac{1 - K_r^2 - K_x^2 + 2 jK_x}{(1 - K_r)^2 + K_X^2}$$

$$R + jX = \frac{1 - K_r^2 - K_x^2 + 2 jK_x}{(1 - K_r)^2 + K_X^2}$$

Equating the real and imaginary parts

$$R = \frac{1 - K_r^2 - K_x^2}{(1 - K_r)^2 - K_x^2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$X = \frac{2K_r}{(1-K_r)^2 + K_x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (1) will give a family of circles called R – circles. While equation (2) will give family of circles called X-circles.

The Constant R – Circles:

$$R = \frac{1 - K_r^2 - K_x^2}{(1-K_r)^2 - K_x^2}$$

R $(1 + K_r^2 - 2K_r + K_x^2) = (1-K_r)^2 - K_x^2$

$$K_r^2 = (R+1) + K_x^2(R+1) - 2K_r = 1 - R$$

$\{K_r^2 + K_r^2\}$ (R+1) - $2K_r = 1 - R$

$\div 1 + R \Rightarrow$

$$K_r^2 + K_x^2 - \frac{2K_r R}{R+1} = \frac{1-R}{1+R}$$

Adding $\frac{1}{1+R}$ and bothsides to make a perfect square we have

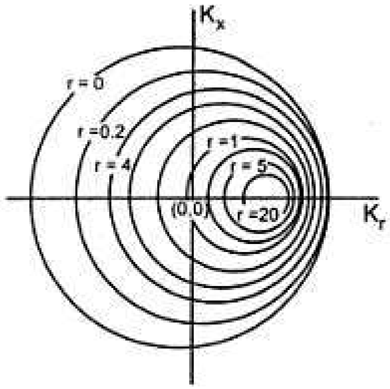


Fig 3.6. constant R – Circles

$$Kx^2 + K_r^2 + \frac{R^2}{(R+1)^2} - \frac{2kr_R}{R+1} = \frac{1-R}{1+R} = \frac{R^2}{(1+R)^2}$$

$$\Rightarrow Kx^2 + (kr - \frac{R}{1+R})^2 = \frac{(1-R)(1_R)+R^2}{(1+R)^2} = (\frac{1}{1+R})^2$$

Radius = $\frac{1}{1+R}$ and

centre = $\left(\frac{R}{1+R}, 0\right)$

All constant R – Circles touch the point (1,0) when R = ∞ r = 0 Hence R = ∞ represent a point.

The constant X – circles

Equation (2)

$$X = \frac{2K_r}{(1-K_r)^2 + K_x^2}$$

$$(1 - K_r)^2 + K_x^2 = \frac{2K_x}{x} = 0$$

$$(1 - K_r)^2 - K_x^2 - \frac{2K_x}{z} = 0$$

Adding $\left(\frac{1}{x}\right)^2$ on both sides

$$(1 - K_r)^2 + K_x^2 - \frac{2K_x}{x} + (\frac{1}{x})^2 = (\frac{1}{x})^2$$

$$(K_r - 1)^2 + \left(K_x - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

This equation represents a family of X circle with centre $(1,\frac{1}{x})$and radius $\frac{1}{x}$. All the circles touch the point (1,0).

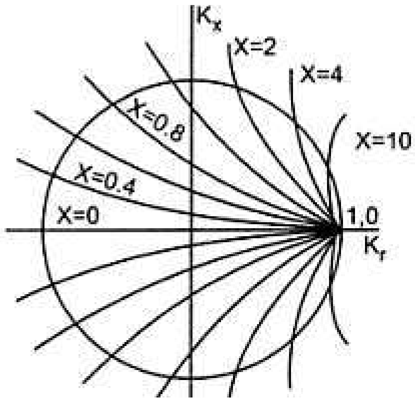


Fig 3.7. constant X – Circles

The complete smith is obtained by the super position of X – circles & R – Circles.

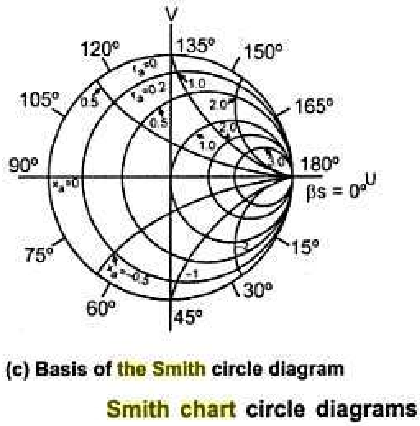


Fig 3.8. complete Smith Chart

3.5.1. PROPERTIES OF SMITH CHART

1. The smith chart may be used for impedance as well as for admittance.
2. The smith chart consists of constant x; - circles & constant x; circles super positioned on the chart. The values of r; and x; are normalized and they are gven by r;=$\frac{R}{R_0}$ and x; =$\frac{jxR}{R_0}$
3. The smith chart is based on the assumed that $1 - |K| \llcorner \phi - 2\beta_S = \mu + jV$. The maximum magnitude of U + jV is the maximum value of $|K|$ie, unity.
4. For properly terminated line and any length the impedance is represented by the point (1,0) which acts as the centre of the smith chart.
5. The outer rin of the chart is scaled into either degrees or wavelengths with an arrow. This arrow indicates the direction of travel along the line. The circles called βSscale of the chart which indicates the electric length of the line.
6. A complete revolution of 360^0 around the center the chart represents a distance of $\lambda/2$on the lines. The clockwise movement along the outer rim indicates the travel towards the source from load. Anti-clockwise movements along the rim indicates the travel towards the load from the source.$\lambda/2 = 360° \Rightarrow \lambda = 720°$
7. The Voltage minima occur to the left of the center of the chart along r;=axis while the voltage maxima occur to the right of the centre of the chart along r;-axis.
8. On the periphery of the smith chart, three scale are provided. Even though there are three scales the serve the same purpose. These scales are useful in determining the distance from the load or source in degree or wavelength.
9. At the right hand end of the horizontal axis the value of both R & X are infinity. Hence impedance is ∞. This indicates open circuit termination. At the left hand end of the horizontal axis, at A the value of R & X are zero. Hence impedance is 0 which indicates short circuit termination.
10. If the smith chart is used for impedance the inductance reactance is plotted upwards from the real axis while capacitive resistive is plotted downwards from the real axis. When the same chart is used for admittance the inductive susceptance is plotted downwards to the real axis and capacitive susceptance is plotted upwards on the real axis.

3.5.3. APPLICATIONS OF SMITH CHART:

i) Plotting an impedance:

Any complex impedance can be represented by a single point on the smith chart. This point is the intersection of constant r; circle and x; circle. Before plotting the point it has to normalised.

Consider $Z_R = R_R + jX_R$. (60 + j40) : $R_0 = 50\ \Omega$

∴ The normalised impedance

$$Z_r = \frac{Z_R}{R_0} = \frac{60 + j40}{50}$$

$$= 1.2 + j0.8$$

Point P is intersection of r; = 1.2 circle and X; = 0.8 circle. Point P indicate the normalised impedance on the chart.

ii. Measurement of VSWR:

After plotting the normalised impedance joint this point P and the centre of the chart O by the line to give OP. With O as center and OP as radios draw a full circle called the VSWR Circle. This circle intersects the real axis at two points A & B. A to the right half of O and B to the left half of O. The scale reading between OA (right half of O) gives the VSWR for a given line. From the chart the value of VSWR is 2.1 approximately.

iii. Measurement of reflection coefficient K:

External OP to intersect the innermost circle at C. This point indicate angle of K. At point C the angle of K is **ϕ**= 55.5 (approximately).At the bottom of the chart the measure of OP from O onwards will give the magnitude of K. In this example $|K|$ = 0.38. This K = 0.38 $\angle$ 55.5°

iv. Location of voltage maximum and minimum:

The intersection of S circle with horizontal axis on the left of the chart center represent voltage minima the center corresponds to voltage maxima.

v. Impedance to admittance conversion:

To find normalise admittance extend OP line downward to intersect the S circle at P', Y_R = R'(0.6-j0.4) is normalised admittance. The location of the first voltage minima and maxima can be read directly from the outer most wavelength circle of the smith chart. This V, will give the distance of Voltage minima from the load. In the graph it is equal to 0.174λ. Similarly the arcc'vv' will give the distance of first maxima from the load.

vi. **Measurements of input impedance of the line:**

When the load impedance is given the input impedance at any length of the line can be readily determined. Consider a load impedance expressed by a point P with O as center and OP as radius. S circle can be drawn produce OP to cut the wavelength circle at C'. In Order to find input impedance of length l. We have to go towards the generator l_2.Clockwise on the Smith Chart. Locate the point T on the wavelength scale at a distance of l/2 from Spt. Will be given input R on the S Circle. This point will give the normalised input impedance which than be multiplied by Z_0 will give in input impedance. The complete Smith Chart is obtained by the superposition of X- circle and R- Circle.

Vii) Determination of load impedance:

Smith chart can also be used to determine load impedance. If SWR and the distance of first voltage minima from the load is given. Voltage minima (V min) always lie at the left side of the horizontal axis at a distant $\frac{1}{s}$ from the center O of the chart. First Plot a point P at a distance$\frac{1}{s}$ from O with OP as radius draw a S. Circle. Move the given distance of V_{min} from A and locate the point Q on the wavelength chart. Join Q_o to cut the circle at R. The co-ordinate of R will give the normalised load impedance. When multiplied by Z_0 will give the desired load impedance.

viii. **Determination of input impedance and admittance of short circuited line:**

Since the input impedance or admittance is purely reactive, it has to be marked on the periphery of the smith chart. Each point on this circle is associated with a particular value in wavelengths. Hence the value read on this circle gives the normalised input impedance of a given short circuited line the starting point being x = 0. To obtain the normalised input admittance of such a line take the reciprocal of the input impedance or take the reading at the point which is metrically opposite to the impedance point.

ix. **Input impedance and admittance of an open circuited line:**

For the determination of the input impedance and admittance of an open circuit, the same procedure as above is followed except that the angle marking on the chart is shifted π/2 rad. Since the exists a phase difference of **π**/2 rad between impedance or admittance of the short circuited line and that of the open circuited line.

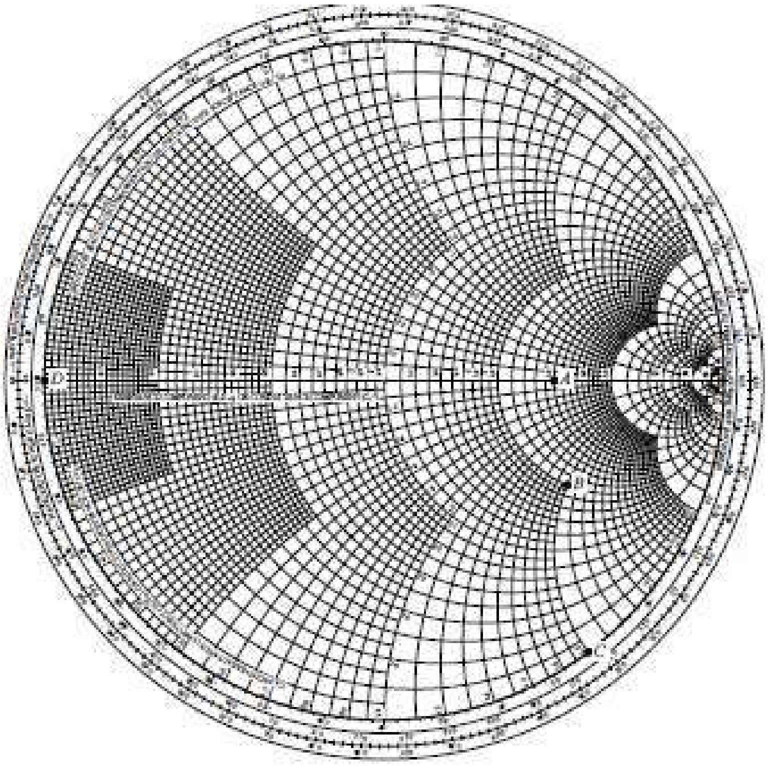


Fig 3.9 Smith chart

SMITH CHART PROBLEMS

1. Using Smith Chart plot the following normalized impedance
   i. 2-j ii. 1+j iii. 1+j2 iv. Matched filter

   Solution:

   i. R=2 , X=1 ii. R=1 X=1 iii. R=1 , X=2 iv. R=0 X=1

   (Ref. SMITH CHART. 1)

2. Using Smith Chart convert the following normalized impedance into admittance,
   i. 0.5+j0.3 ii. 2-j0.5

   Solution:

   Normalized impedance 0.5+j0.3 2-j0.5

   Diametrically opposite point will be a admittance.

   (Ref. SMITH CHART. 1)

3. A lossless line with characteristic impedance of 70Ω is terminated in an open circuit. Determine the sending end impedance for the following length of the line.
   i. λ/8 ii. λ/2 iii. λ/4

   ii. Solution:

   A point B on the Smith Chart represented open circuited moving towards clockwise to A' point that is a generator point.

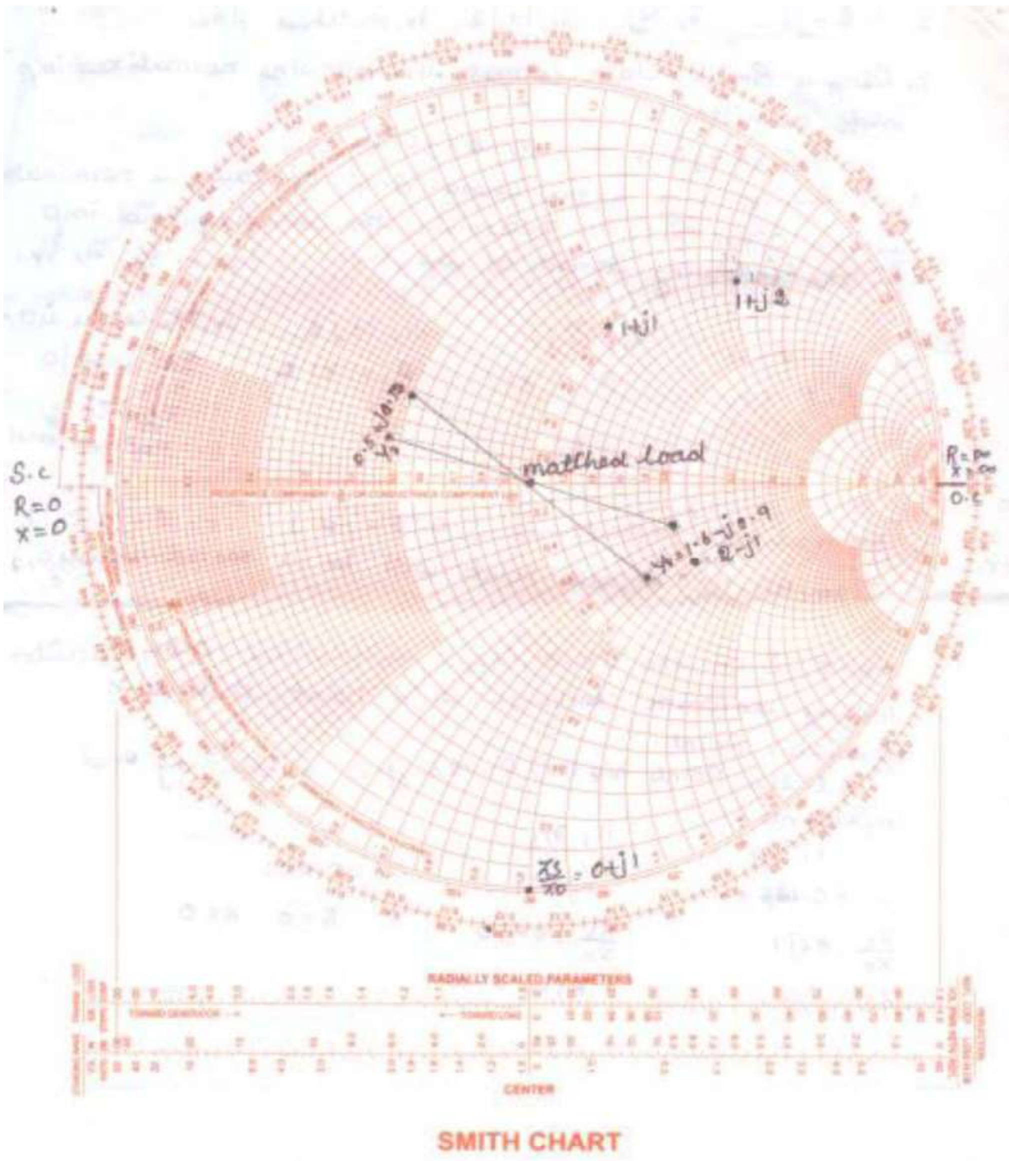
   At the point λ/8 (or) 0.12λ give a sending end impedance,

   iii. λ/8 =0.125λ ii. λ/2 =0.5λ iii. λ/4 =0

   $\frac{Z_s}{Z_0} = 0 + j$ $\frac{Z_s}{Z_0} = 0 + j0$ $\frac{Z_s}{Z_0} = 0 + j0$

   $Zs = j70\Omega$ $R = \infty, X = \infty$ $R = 0, X = 0$

   (Ref. SMITH CHART. 1)



SMITH CHART. 1

Example 3.1: A 30 m long lossless transmission line with the characteristic impedance of 50Ω operating at 2MHz. If the line is terminated in impedance 60+j40Ω. Calculate the reflection coefficient, Standing Wave Ratio, The input impedance if the velocity of the wave is V=0.6C ($C=3x10^8$m/s).

Solution:

Given: l=30m R_0=50Ω f=2MHz Z_R=60+j40Ω

The reflection coefficient

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R - R_0}{Z_R + R_0}$$

$$= \frac{60 + j40 - 50}{60 + j40 + 50}$$

$$= \frac{10 + j40}{110 + j40}$$

$$= 0.3523\angle 56°$$

Standing Wave Ratio,

$$S = \frac{1 + |K|}{1 - |K|}$$

$$S = \frac{1 + 0.3523}{1 - 0.3523} = 2.088$$

The input impedance,

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 tan\beta s}{R_0 + jZ_R tan\beta s}\right]$$

$$\beta = \frac{\omega}{V} = \frac{2\pi x 2x10^6}{0.6x3x10^8}$$

$$\beta l = \frac{0.666\pi x 180}{\pi} = 120°$$

$$Z_{in} = 50 \left[\frac{60 + j40\Omega + j50tan120°}{50 + j(60 + j40\Omega)tan120°}\right]$$

=23.97+j1.35Ω

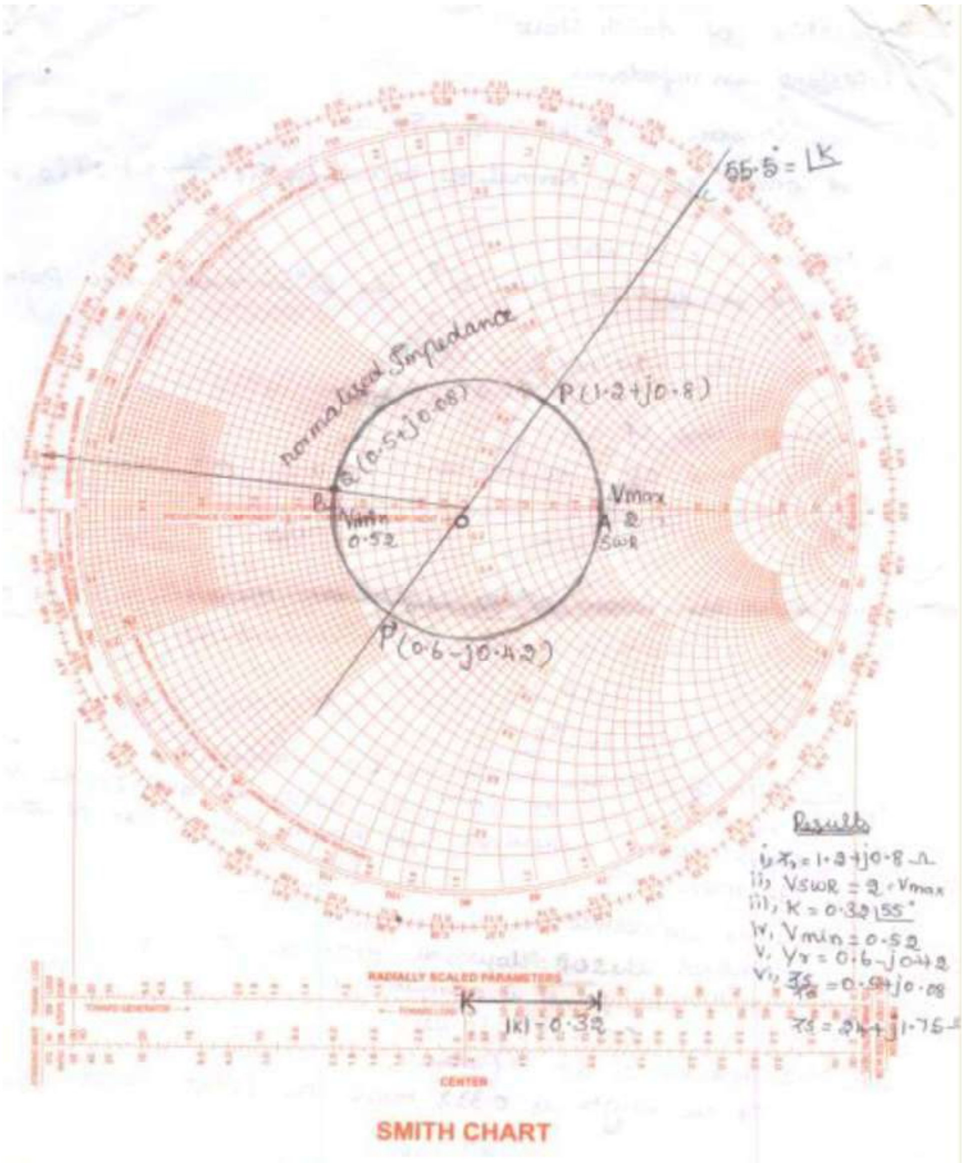
(Ref SMITH CHART. 2)

Procedure:

1. To plot the normalized impedance mark it A
2. Draw a circle OA as a radious
3. To determine VSWR (Maximum point along R axis)
4. Extend Zr to outside the chart, it will cut the degree scale at one point which is the angle of K.
5. To find the magnitude of K using the scale below the chart.
6. To determine the input impedance
7. $\lambda = \frac{V}{f} = \frac{0.6C}{2x10^6} = 90m$

8. $S = 30m = \frac{30}{90}\lambda = 0.333\lambda$
9. To determine the input impedance move the load impedance 0.333λ towards generator

$$\frac{Z_{in}}{Z_0} = 0.48 + j0.35$$
$$Z_{in} = 50(0.48 + j0.35)$$
$$Z_{in} = 24 + j1.75\Omega$$



SMITH CHART. 2

Example 3.2. VSWR on a loss less line is found to be 5 and successive voltage minima are 40Cm apart. The first voltage minima is observed to be 15Cm from the load. The length of a line of a line is 160 Cm and the characteristic impedance is 300Ω. Using Smith chart determine i. Load impedance ii. Sending End Impedance

Solution:

Given S=5 λ/2=40 λ=80

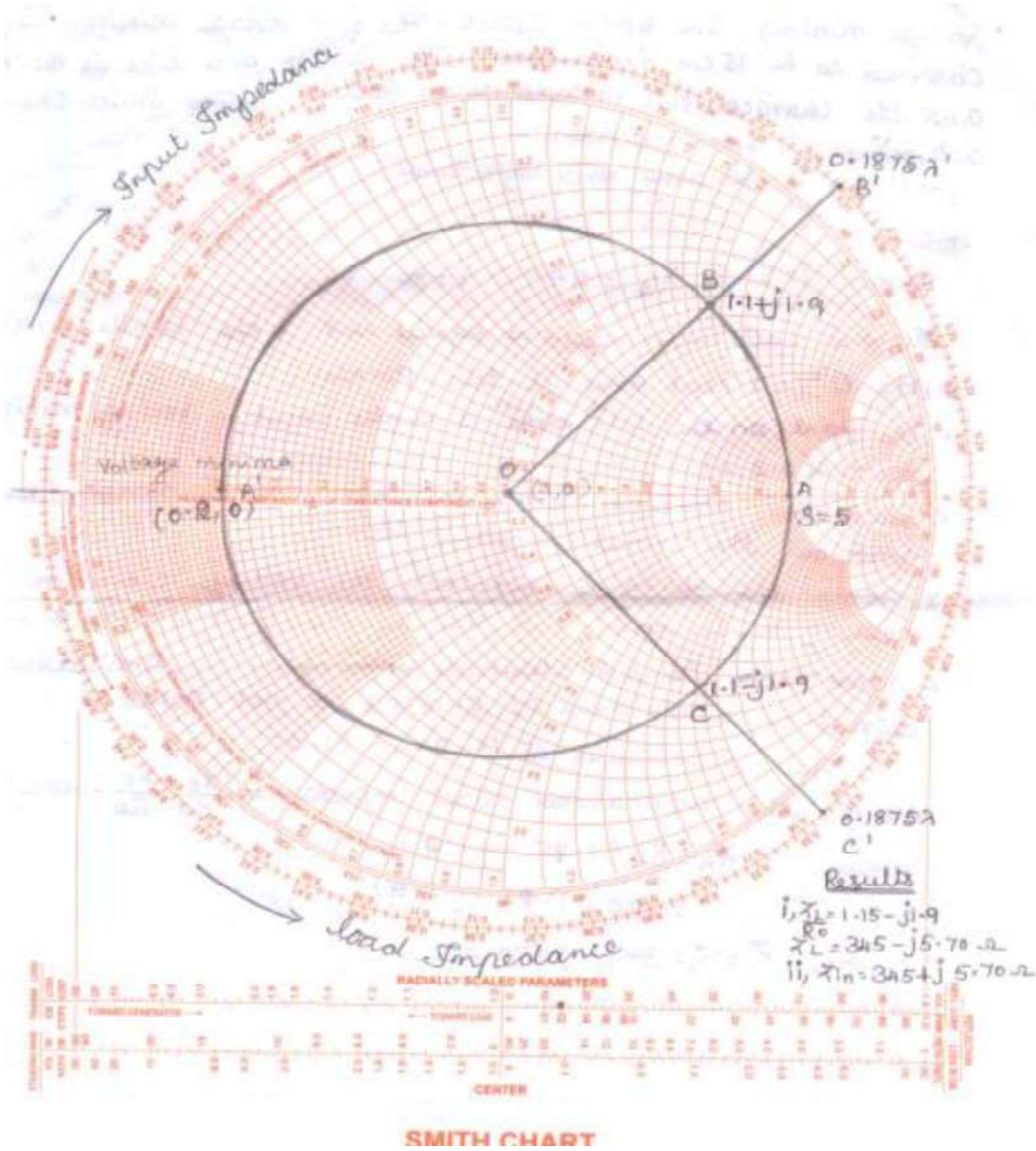
Procedure:

1. Draw the S circle with radious 5 and the center (1,0)
2. The circle cuts the real axis at two points.
3. The point on the LHS from a center point is voltage minima denote A(0.2,0)
4. Given minima is 15 Cm apart from load but the total length is 160Cm.
5. Hence the load is 15/80λ=0.1875λ.
6. To determine load impedance move from point A, towards load, at 0.1875λ for B.
7. The co ordinates ob B are (1.05,-1.9)
8. $\frac{Z_R}{Z_0} = \frac{Z_R}{Z_0} = 1.05 - j1.9$

$$Z_R = R_0(1.15 - j1.9)$$
$$Z_R = 300(1.15 - j1.9)$$
$$Z_R = 345 - j570\Omega$$

Similarly Zin=345+j570Ω

(Ref SMITH CHART. 3)



SMITH CHART. 3

Example 3.3: A lossless 50-Ω transmission line is terminated in a load with Z_L = (50+ j25) Ω. Use the Smith chart to find the following:

(a) The reflection coefficient Γ.

(b) The standing-wave ratio.
(c) The input impedance at 0.35λ from the load.
(d) The input admittance at 0.35λ from the load.
(e) The shortest line length for which the input impedance is purely resistive.
(f) The position of the first voltage maximum from the load.

Solution:

The normalized impedance

$$zL = \frac{(50 + j25)}{50}\ \Omega = 1 + j0.5$$

is at point Z-LOAD.

(a) $\Gamma = 0.24e^{j76.0}$ The angle of the reflection coefficient is read of that scale at the point θr .

(b) At the point SWR: S = 1.64.

(c) Zin is 0.350λ from the load, which is at 0.144λ on the wavelengths to generator scale.

So point Z-IN is at 0.144λ + 0.350λ = 0.494λ on the WTG scale.

At point Z-IN:

Zin = zin Z0 = (0.61− j0.022)×50 Ω = (30.5− j1.09) Ω.

(d) At the point on the SWR circle opposite Z-IN, Yin = yin Z0 = (1.64+ j0.06) 50 Ω = (32.7+ j1.17) mS.

(e) Traveling from the point Z-LOAD in the direction of the generator (clockwise), the SWR circle crosses

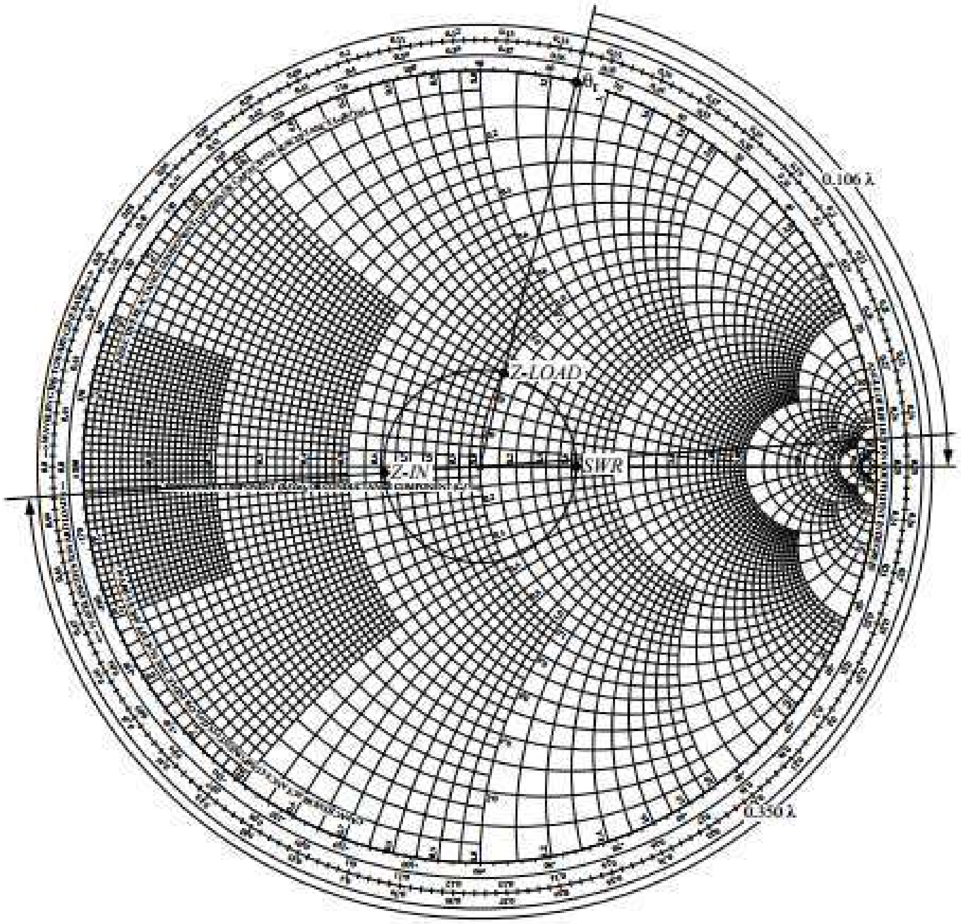
the xL = 0 line first at the point SWR. To travel from Z-LOAD to SWR one must travel

0.250λ −0.144λ = 0.106λ. (Readings are on the wavelengths to generator scale.)

So the shortest line length would be 0.106λ.

(f) The voltage max occurs at point SWR. From the previous part, this occurs at z = −0.106λ.

(Ref SMITH CHART. 4)



SMITH CHART. 4

Example 3.4: A RF transmission line has characteristic impedance of 55+j0Ω is terminated in a load impedance of 115+j75Ω what is the Reflection coefficient in polar form. At what distance from the load will first maxima and minima of voltage will occur? If the line is 1.183λ length what will be the input impedance? Work out the problem first numerically and then by Smith Chart.

Solution: Numerical method, Given Z0=55 Zr=115+j75Ω

Reflection coefficient ,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R - R_0}{Z_R + R_0}$$

$$K = \frac{115 + j75 - 55}{115 + j75 + 55}$$
$$K = \frac{60 + j75}{170 + j75}$$
$$= 0.5167\angle 27.53°$$

Standing Wave Ratio,

$$S = \frac{1 + |K|}{1 - |K|}$$
$$S = \frac{1 + 0.5167}{1 - 0.5167} = 3.139$$

Voltage maximal occurs at,

$$S' = S + \frac{\lambda}{4} = 0.288$$

The input impedance,

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 tan\beta s}{R_0 + jZ_R tan\beta s}\right]$$
$$Z_{in} = 55 \left[\frac{115 + j75 + j55\tan(\frac{2\pi}{\lambda} x1.18\lambda)}{55 + j(115 + j75) tan(\frac{2\pi}{\lambda} x1.18\lambda)}\right]$$
$$= 26.488 - j36.24\Omega$$

Using Smith Chart **(Ref SMITH CHART. 5)**

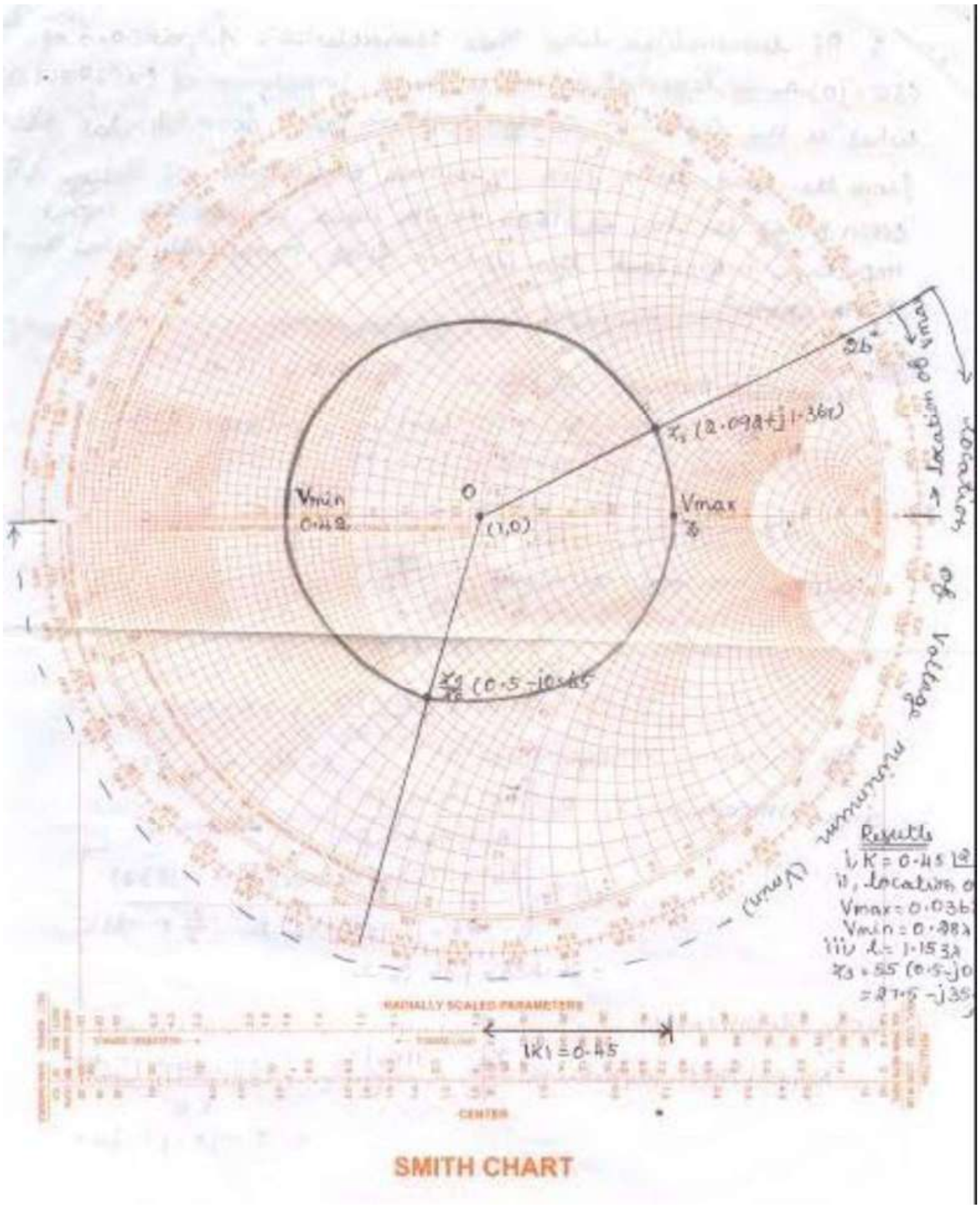
Procedure:

1. To plot the normalized impedance mark it A
2. Draw a circle OA as a radious
3. To determine VSWR (Maximum point along R axis)
4. Extend Zr to outside the chart, it will cut the degree scale at one point which is the angle of K.
5. To find the magnitude of K using the scale below the chart.
6. To determine the input impedance $S = 0.183\lambda$
7. To determine the input impedance move the load impedance 0.333λ towards load

$$\frac{Z_{in}}{Z_0} = 115 + j75$$
$$Z_{in} = \frac{115 + j75}{55}$$
$$Z_{in} = 2.09 + j1.367$$

The Sending end impedance is,

$$\frac{Z_s}{Z_0} = 0.5 - j0.65\Omega$$
$$Z_s = 55(0.5 - j0.65)$$
$$Z_s = 27.5 - j35.75\Omega$$



SMITH CHART. 5

SINGLE STUB MATCHING

Example 3.5: A load (50-j100)Ω is connected across 50Ω line. Design a short circuited stub to provide matching between the two at a frequency of 30 MHz using Smith Chart.

Solution:

Given $\boldsymbol{Z_R} = 50 - \text{j}100\Omega$ and $Z_0 = 50\Omega$

$$\frac{Y_R}{G_0} = \frac{50}{50 - \text{j}100}$$

$$= \frac{1}{1 - j2} = \frac{1 + j2}{5} = 0.2 + j0.4$$

Procedure:

1. Locate a point A on the chart indicating normalized admittance. It cuts the real axis at 5.8.
2. Intersection of S circle with unity conductance circle. Let it be point B. At point B the susceptance is +2.0 which is capacitance in nature. This point gives the position at which the stub is to be connected.
3. Extending line through A and B from origin βs=0.188λ from point B. Travelling from the load to the generator in clockwise direction.

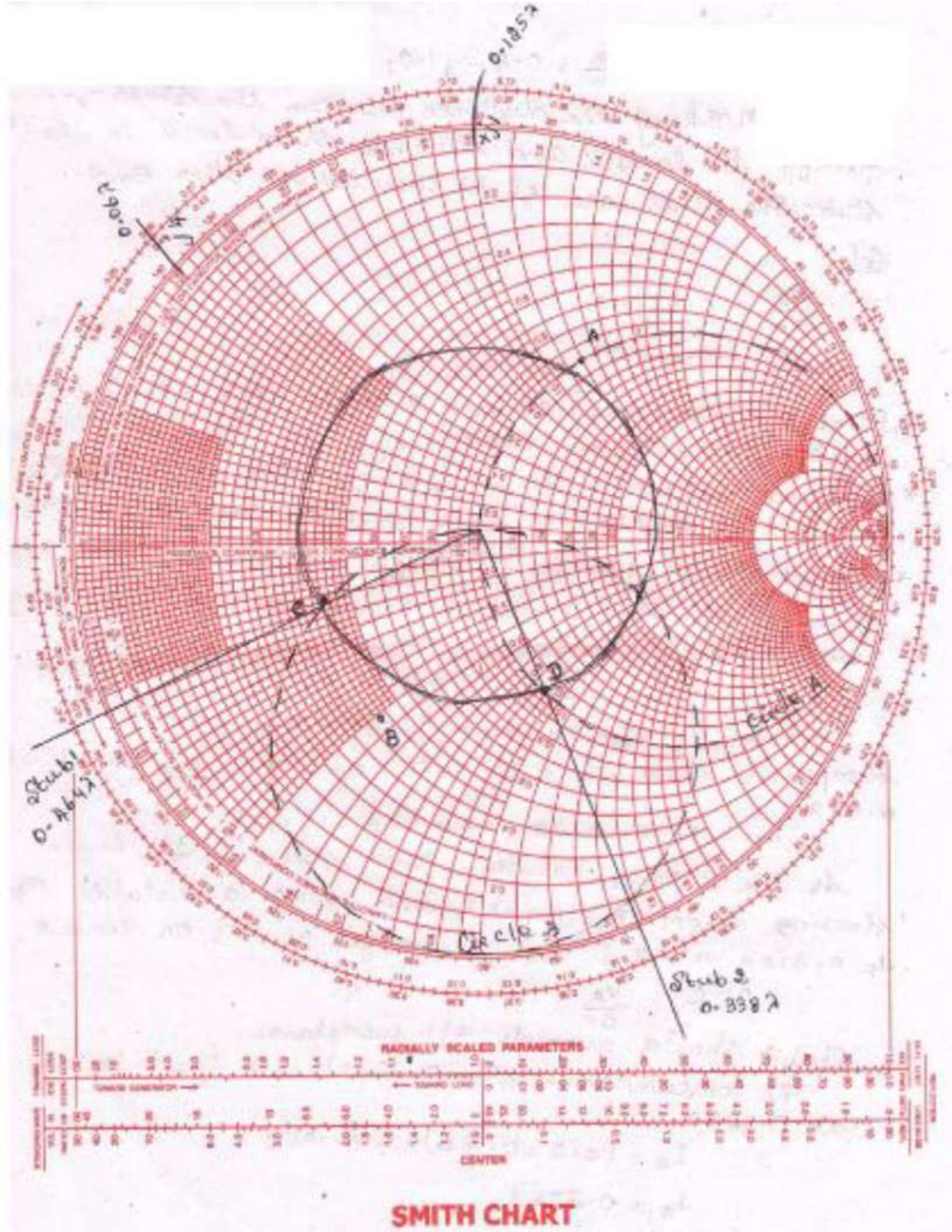
   $$S_1 = 0.188\lambda - 0.062\lambda = 0.126\lambda$$
4. As the line susceptance is capacitive the line must provide inductive susceptance of -2.0. Locate point C. The intersections with outer rim of chart gives bs=0.322λ.
5. From the short circuit end measuring the length

   L=0.322λ-0.25λ

   Hence location S_1=0.126λ

   Length ls= 0.072λ

(Ref. SMITH CHART. 6)



SMITH CHART. 6

DOUBLE STUB MATCHING

Example 3.6: Using double stub matching match a complex load of Z_R=18.75+j56.25 to a line with Z0=75Ω. Determine the stub lengths assuming a quarter wavelength spacing is maintained between two short circuited stubs.

Solution:

$$Z_L = \frac{Z_R}{Z_0} = \frac{18.75 + j56.25}{75} = 0.25 + j0.75$$

Procedure:

1. Mark normalised load impedance point
2. Draw impedance circle of S circle.
3. Identify normalized load admittance and load point.
4. Draw unity conductance circle (gi=1) that corresponds to stub 2 circleA.
5. Since spscing is $\frac{\lambda}{4}$, Draw circle B which is exactly 180° opposite to circle A this corresponds to stub 1.
6. Introduction of stubs will not change the conductance value instead it changes the suceptance.
7. So trace the same conductance circle from load point. Mark the point of intersection of the conductance circle with circle B (stub 1) is the location of stub 1.
   Here,

$$\frac{Y_L}{G_0} = 0.4 - j1.2 \;\; (normalized\ load\ admittance)$$

$$Y_{s1} = 0.4 - j0.5 \;\; (normalized\ admittance\ at\ point\ AA')$$

So change in succeptance due to the introduction of stub 1 is 1.2-0.5=0.7. So +j0.7 is added with

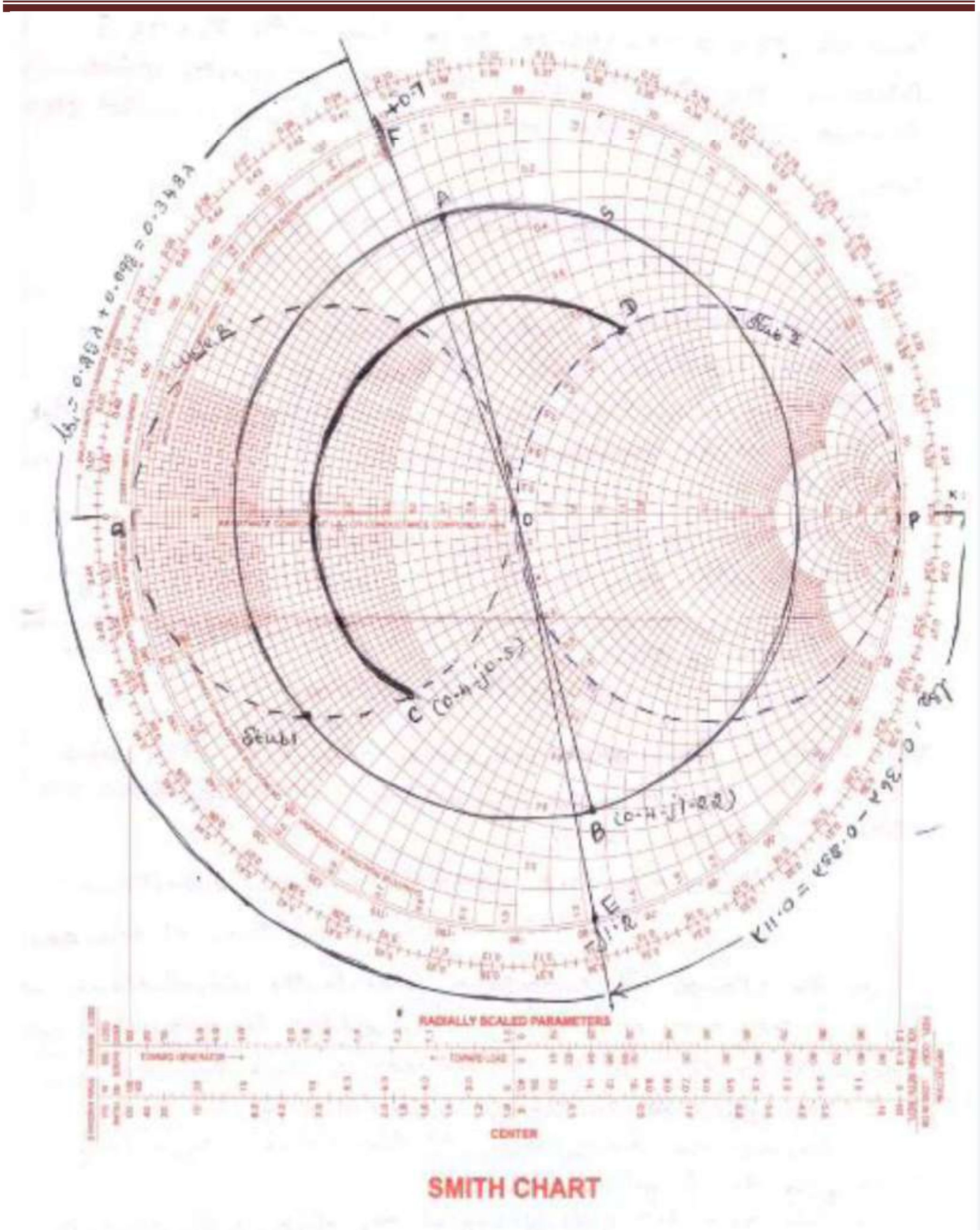
the original succeptance value.

8. Transfering this point (exactly 180°or $\frac{\lambda}{4}$) in clockwise direction (from stub 1 to stub 2 towards gennerator) marks the point of intersection with gi=1 circle (or A circle).
9. To find the lengths of the stubs (l_1 and l_2).
   i. Since the Succeptance of the first dtub lies below the horizontal line take it as (-j0.7) and find the point where susceptance is = +j0.7. The distance between short circuited end to the +j0.7 is length of stub 1. (l_1)
   ii. For stub 2 susceptance is +J1.2 so find that the point where susceptance i.e. –j1.2 and distance between short circuited end to –j12 is the length of second stub (l2).

Ls1=0.348λ

Ls2=0.11λ

(Ref. **SMITH CHART. 7**)



SMITH CHART. 7

Example 3.7: What length of the short circuited stubs must be attached to the line with $Z_0 = 75\Omega$. to achive maximum power transfer to a load of 37.5+j97.5Ω. Find VSWR.

Solution:

$$\frac{Z_R}{Z_0} = 0.5 + j1.3$$

$$\frac{Y_R}{Y_0} = \frac{\frac{1}{Z_R}}{\frac{1}{Z_0}} = \frac{Z_0}{Z_R} = 0.25 - j0.68$$

Procedure:

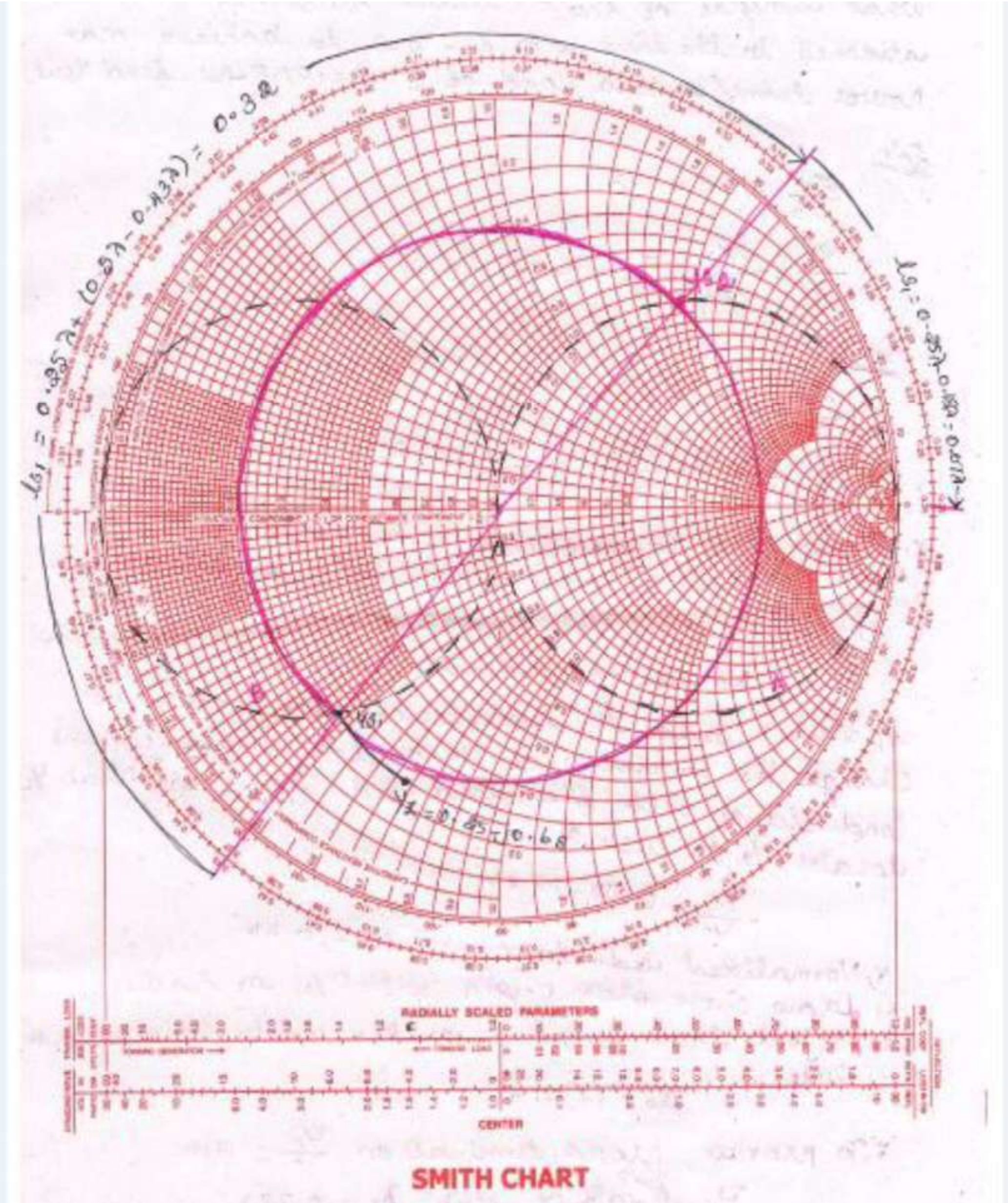
1. Draw a unit conductance circle. Let be the circle A.
2. The distance between two stub is λ/4=180° Then new circle B is drawn.
3. Normalized admittance $Y_L = 0.25 - j0.68$. Stub 1 adds the susceptance and it does not change the conductance. Hence a constant conductance circle from load admittance 1/L locate Y_L locates Y_{s1} on circle B.
$$\frac{Y_1}{Y_0} = 0.25 - j0.44$$
4. Draw circle from origin with OY_1 as radius. Project that Y_{s1} point on the unity conductance circle and
$$Y_{s2} = 1 + j1.7$$

To provide proper termination $\frac{Y_R}{Y_0} = 1 + j0$

5. The length of stub 1 $\lambda_{s1} = 0.32\lambda$

The length of stub 2 $\lambda_{s2} = 0.07\lambda$

(Ref. **SMITH CHART. 8**)



SMITH CHART. 8

Example 3.8: For a load of $\frac{Z_R}{Z_0} = 0.8 + j1.2$, design a double stub tuner matching the distance between the stubs $3\lambda/8$. Specify the length and distance from load to the first stub. The stubs are short circuited at other end.

Solution:

$$\frac{Z_R}{Z_0} = 0.8 + j1.2$$
$$\frac{Y_R}{G_0} = 0.4 - j0.6$$

Procedure:

1. Point A refers (0.8,1.2) and B refers (0.4,-0.6). circle A unity conductance circle.
2. Distance between stubs 3/8λ-0.375λ. The circle B is obtained by displacing each point on circle A by 0.375λ.
3. Stub 1 should change $\frac{Y_R}{G_0}$ to $\frac{Y_1}{G_0}$ which will lie on circle B. Stub 1 is shorted. Hence it cannot add any conductance. Travelling along constant conductance circle from A towards generator meet circle B at point C.

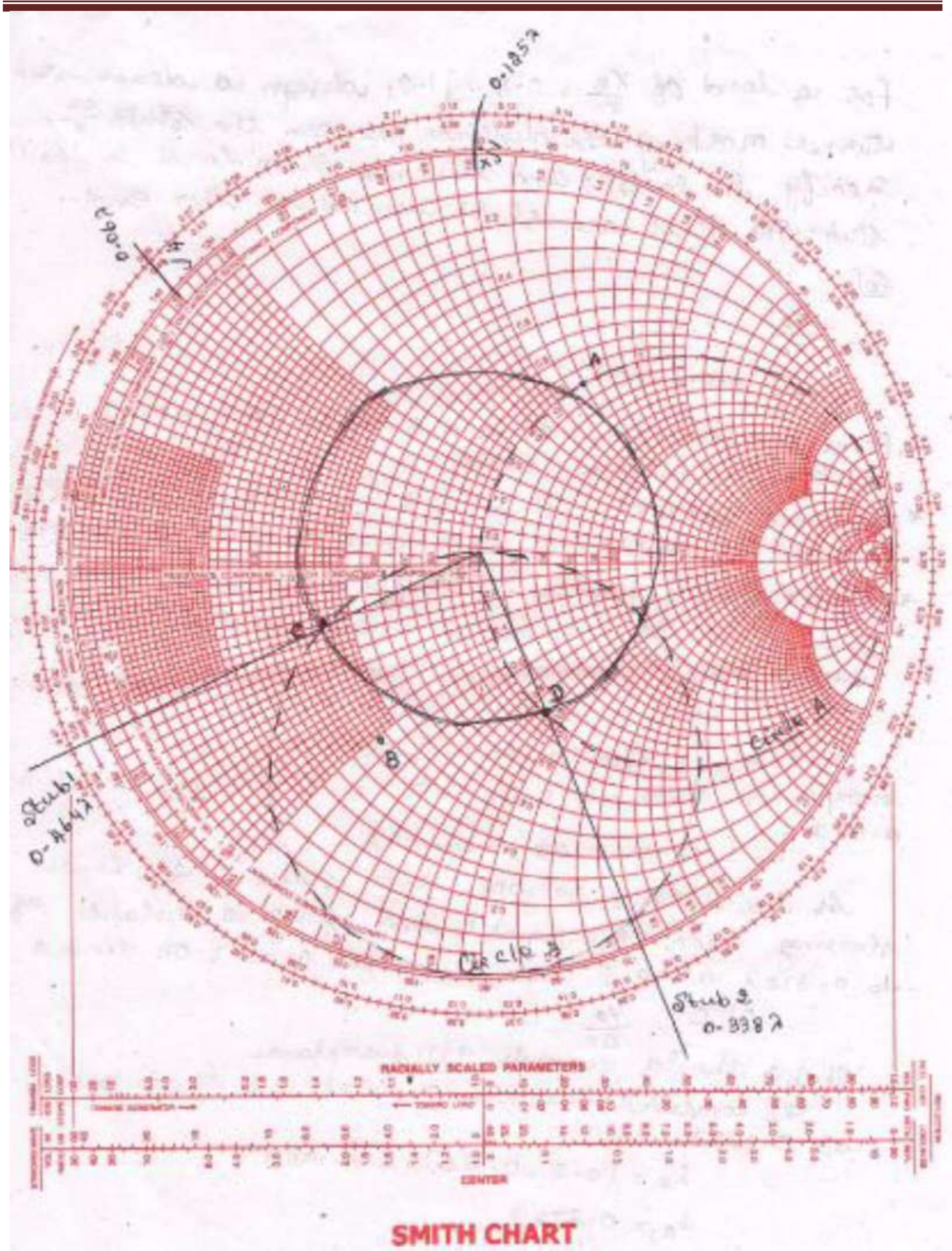
   At C $\frac{Y_1}{G_0} = 0.4 - j0.2$

   Hence stub 1 should add +j0.4. The constant susceptance circle of 0.4 cuts outer rim of chart at 0.06λ.

   The length is $L_1 = (0.5 - 0.25)\lambda + 0.06\lambda = 0.31\lambda$
4. As the distance between two stubs is 3/8λ=0.375λ, hence starting from the point 1, and going a distance equal to 0.375λ along S circle, Point D on circle A.
5. At C $\frac{Y_2}{G_0} = 1 - j1$. Stub 2 should provide +j1 susceptance. The constant susceptance circle cuts the outer rim at 0.125λ.

$$L_2 = (0.5 - 0.25)\lambda + 0.125\lambda$$
$$L_2 = 0.375\lambda$$

(Ref. **SMITH CHART. 9**)


0.1857
0.067
A
C
D
B
Circle A
Circle B
Stub 1
0.464λ
Stub 2
0.338λ
RADIALLY SCALED PARAMETERS
CENTER
SMITH CHART

SMITH CHART. 9

SUMMARY

- Properties Of Symmetrical Network:

$$Zin = Z_0 \left[\frac{Z_0 sinh\gamma + Z_R \, cosh\gamma}{Z_R sinh\gamma + Z_0 \, cosh\gamma}\right]$$

$$Z_0 = \sqrt{Z_{0c}.Z_{sc}}$$

$$\tanh\gamma = \sqrt{\frac{Z_{sc}}{Z_{0c}}}$$

- The transmission line is a structure which can transport electrical energy from one point to another. At low frequencies, a transmission line consists of two linear conductors separated by a distance.
  Co-axial cable
  Parallel wire transmission line
  Micro strip line
- The propagation constant, symbol γ, for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x, such that $\frac{I_1}{I_n} = e^{\gamma n}$. Since the propagation constant is a complex quantity can write $\gamma = \alpha + j\beta$
- The propagation constant symbol γ, for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x. $\gamma_1 + \gamma_2 + \cdots \ldots \ldots \ldots \ldots .. \gamma_n = ln\frac{\gamma_1}{\gamma_n}$
- Nodes are points of Zero voltage and current on the standing wave system antinodes are points of maximum voltage or current.
- A line terminated in R_0 has no standing waves and thus no nodes, antinodes are called smooth line. The ratio of the maximum to minimum magnitudes of current or voltage on a line having standing wave is called the standing wave ratio S.

$$S = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}}$$

- Smith chart is also known as circular chart
- A quarter wave line can transform a low impedance into a high impedance and vice versa, thus it can be considered as an impedance inverter.
- The quarter wave line may be used to provide mechanical support to the open wire line or centre conductor of a co-axial cable. Such a line as mechanical support is shorted at ground
- Two types of stub matching are,
  Single Stub Matching
  Double Stub Matching
- **Disadvantages of Single stub matching**:
  This is useful for a fixed frequency only because as the frequency changes, the location of the stub will have to be changed. For final adjustment, the stub has to be moved along the line

slightly. This is possible only in open wire and therefore on co axial line, single stub matching may become inaccurate in practice

Practically the two stubs must be separated by a distance $\frac{\lambda}{16}, \frac{\lambda}{8}, \frac{3\lambda}{16}, \frac{3\lambda}{8}$ etc. But the most commonly used separation between the two stubs.

PART A

1. **State the disadvantages of reflection**

   i) Reflected wave appears as echo at the sending end

   ii) The efficiency is reduced

   iii) Since some part of the energy is rejected by the load, the output reduces.

2. **What are the primary and secondary constants of the line?**

   The constants which do not vary with frequency like, R, L, G and C are called primary constants of the line. The constants which vary with frequency like $\propto, \beta, \gamma$ *and* Z_0 are called secondary constants of the line.

3. **What is a transmission line and its types?**

   The electrical lines which are used to transmit the electrical waves along them are called transmission lines. It is a conductive method of guiding electrical energy from one place to another. Various types are open-wire lines, cables, co-axial line, and waveguides.

4. **Define characteristic impedance**

   It is defined as the ratio of voltage to current at the input of an infinite line. It is independent of length of the line and the terminating load. It is dependent on the characteristics of the line per unit length,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

   R is the series resistance per unit length of the line

   L is the series inductance per unit length of the line

   G is the shunt conductance between conductors

   C is the shunt capacitance between conductors

5. **What are the significances of characteristic impedance?**

   a) When the line is terminated in its characteristic impedance, there is no reflection

   b) A line of finite length terminated in characteristic impedance acts as an infinite line.

   c) When a uniform transmission line is terminated in its characteristic impedance. Its Input impedance will be equal to the characteristic impedance.

   d) A network terminated in characteristic impedance at the input as well as at the output terminals is said to be correctly terminated network.

6. **Define *propagation constant***

   It is defined as the natural logarithm of ratio of current or voltage at any point to that at a point unit distance away from the first point. $\gamma = \left[\frac{I_1}{I_2}\right]$ *or* $\gamma = \left[\frac{V_1}{V_2}\right]$. Propagation constant is a