UNIT IV

PASSIVE FILTERS

Characteristic impedance of Symmetrical Networks – Filter fundamentals Design of filters: Constant K Filters - Low pass, High pass, Band pass, Band elimination filters - m - derived sections – Low pass, High pass composite filters.

4. INTRODUCTION

The neper (Symbol: Np) is a logarithmic unit of ratio. In filter circuit and other electric networks it is frequently convenient to appraise the performance of a circuit in terms of the ratio of input current to output currents magnitudes. If the input and output image impedance or the ratios of voltage to current at input and output of network are equal then the ratios of the input and output currents or input and output voltages may equally will be,

$$\left|\frac{I_1}{I_2}\right| = \left|\frac{V_1}{V_2}\right|$$

In common,

$$\left|\frac{I_1}{I_2}\right| = \left|\frac{V_1}{V_2}\right| = e^N$$

Under conditions of equal impedance associated with input and output circuits. The unit of N has been given the name neper and defined as,

$$N\ neper = ln\left|\frac{V_1}{V_2}\right| = ln\left|\frac{I_1}{I_2}\right|$$

Two voltage or currents differ by one neper when of them is e times as large as the other $\frac{P_1}{P_2} = e^{2N}$. The unit based on logarithms to the base 10 naming the unit bel. The bel is defined as the logarithms of the power ratios.

$$Number\ of\ bels = log\frac{P_1}{P_2}.$$

A decibel is one-tenth of a bel, i.e. 1B=10 dB. The bel (B) is the logarithm of the ratio of two power quantities of 10:1, and for two field quantities in the ratio √10:1[8].

$$db = 10log\left|\frac{P_1}{P_2}\right|.$$

For the case of equal impedance in input and output circuits

$$db = 20log\left|\frac{I_1}{I_2}\right| = 20log\left|\frac{V_1}{V_2}\right|.$$

Equating the values for the power ratios $e^{2N}=10^{db/10}$ and taking log on both sides

$$1\,NP = \frac{20}{ln10}\,dB = 20log10e\,dB = 8.685889638\,dB \qquad and$$

$$1\,dB = \frac{ln\,10}{20}\,Np = \frac{1}{20\,log_{10}\,e}\,Np = 0.115129254\,Np$$

4.1. CHARACTERISTIC IMPEDANCE OF SYMMETRICAL NETWORKS

A two-port network makes possible the isolation of either a complete circuit or part of it and replacing it by its characteristic parameters. When $Z_1=Z_2$ or the two series arms of a T network are equal or $Z_a = Z_c$ and the shunt arms of a π network and equal the network is said to be symmetrical.

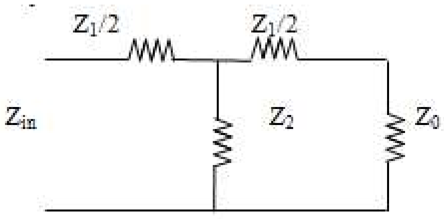


Fig.4.1: Z_0 for T section

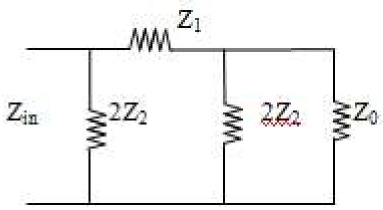


Fig. 4.2: Z_0 for π section

For a symmetrical network when the input impedance is equal to Z_0. The input impedance is called characteristic impedance (or) iterative impedance (or) its impedance transformation ratio is unity.

The value of Z_0 for symmetrical network can be easily determined. For T network terminated in an impedance Z_0 the input impedance is,

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2(\frac{Z_1}{2}+Z0\ )}{\frac{Z_1}{2}+Z_2+Z0}$$

It can be assumed that if Z_0 is properly chosen in terms of network arms it should be possible to make Z_{in} equal to Z_0 ie, $Z_{in} = Z_0$

$$Z_0 = \frac{Z_1^2/4 + Z_1Z_2 + Z_2Z_0 + Z_1\frac{Z_0}{2}}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$= \frac{\frac{Z_1^2}{4} + Z_1Z_2 + Z_0(Z_2 + \frac{Z_1}{2})}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$Z0 - \frac{Z_0(Z_2 + \frac{Z_1}{2})}{\frac{Z_1}{2} + Z_2 + Z_0} = \frac{\frac{Z_1^2}{4} + Z_1Z_2}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$Z0\ (1 - \frac{\left(Z_2 + \frac{Z_1}{2}\right)}{\frac{Z_1}{2} + Z_2 + Z_0}) = \frac{\frac{Z_1^2}{4} + Z_1Z_2}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$Z0\ (\frac{\frac{Z_1}{2} + Z_2 + Z_0 - Z_2 - \frac{Z_1}{2}}{\frac{Z_1}{2} + Z_2 + Z_0}) = \frac{\frac{Z_1^2}{4} + Z_1Z_2}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$Z02 = \frac{Z_1^2}{4} + Z_1Z_2$$

For symmetrical T section $Z_{oT} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2} = \sqrt{Z_1Z_2(1 + \frac{Z_1}{4Z_2})}$

It becomes the characteristic impedance. Similarly for the π section network the impedance is ,

$$Zin = \frac{\left[Z_1 + \left(\frac{2Z_2Z_0}{2Z_2 + Z_0}\right)\right] 2Z_2}{Z_1 + \frac{2Z_2Z_0}{2Z_2 + Z_0} + 2Z_2}$$

For symmetrical network $Z_{in} = Z_0$

$$Z_{0\pi} = \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}}$$

Which is the characteristic impedance for a symmetrical π section. If the open circuit impedance is Z_{oc} and short circuit impedance is Z_{sc} for a T network,

$$Zoc = \frac{Z_1}{2} + Z_2$$

$$Zsc = \frac{Z_1}{2} + \frac{Z_1Z_2/2}{\frac{Z_1}{2} + Z_2}$$

$$Zoc.Zsc = \frac{Z_1^2}{4} + Z_1Z_2 = ZoT2$$

Similarly for the π section network

$$Zoc.Zsc = \frac{4Z_2^2Z_1}{4Z_2 + Z_1} = Z_{0\pi}^2$$

Therefore for a symmetrical network, $Z_0 = \sqrt{Zoc.Zsc}$

4.2. FILTER FUNDAMENTALS DESIGN OF FILTERS:

The ideal filter, whether it is a low pass, high pass, or band pass filter will exhibit no loss within the pass band, i.e. the frequencies below the cut off frequency. Then above this frequency in what is termed the stop band the filter will reject all signals. In reality it is not possible to achieve the perfect pass filter and there is always some loss within the pass band, and it is not possible to achieve infinite rejection in the stop band. Also there is a transition between the pass band and the stop band, where the response curve falls away, with the level of rejection rises as the frequency moves from the pass band to the stop band.

4.2.1. Basic types of RF filter

There are four types of filter that can be defined. Each different type rejects or accepts signals in a different way, and by using the correct type of RF filter it is possible to accept the required signals and reject those that are not wanted. The four basic types of RF filter are:

i. Low pass filter
ii. High pass filter
iii. Band pass filter
iv. Band reject filter

A low pass filter only allows frequencies below what is termed the cut off frequency through. This can also be thought of as a high reject filter as it rejects high frequencies. Similarly a high pass filter only allows signals through above the cut off frequency and rejects those below the cut off frequency. A band pass filter allows frequencies through within a given pass band. Finally the band reject filter rejects signals within a certain band. It can be particularly useful for rejecting a particular unwanted signal or set of signals falling within a given bandwidth. Ideally it is desired that a filter network transmit or pass a desired frequency band without loss.

The propagation constant γ = α+jβ. If α=0 or $I_1=I_2$ then there is no attenuation only phase shift in transmitting the signal through filter and the operation is pass band of frequency. When α has positive value then I_2 is smaller in magnitude than I_1, attenuation has occurred and operation is in an attenuation or stop band frequencies.

$$sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Assume the network contain only pure reactance and thus $\frac{Z_1}{4Z_2}$ will be real, it will be positive or negative depending on the type of reactance used for Z_1 and Z_2.

$$sinh\gamma/2 = sinh(\frac{\alpha}{2} + j\frac{\beta}{2})$$

$$= sinh\frac{\alpha}{2}cos\frac{\beta}{2} + j\, cosh\frac{\alpha}{2}sin\frac{\beta}{2}$$

If Z_1 and Z_2 are same type of reactance then $\left|\frac{Z_1}{4Z_2}\right| > 0$ or the ratio is positive and real.

$$cosh\frac{\alpha}{2}sin\frac{\beta}{2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$sinh\frac{\alpha}{2}cos\, j\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2)$$

$$(1) \rightarrow sin\frac{\beta}{2} = 0;\ \beta = n\pi \quad where\ n = 0,2,4,\ldots\ldots..$$

$$(2) \rightarrow cos\frac{\beta}{2} = 1;\ then,$$

$$sinh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\alpha = 2sinh - 1\sqrt{\frac{Z_1}{4Z_2}}$$

Thus the condition that $\left|\frac{Z_1}{4Z_2}\right| > 0$ implies a stop or attenuation band of frequencies. If Z_1 and Z_2 are opposite type of reactance then $\left|\frac{Z_1}{4Z_2}\right| < 0$ or the ratio is negative and imaginary.

$$sinh\frac{\alpha}{2}cos\frac{\beta}{2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (3)$$

$$cosh\frac{\alpha}{2}sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (4)$$

$$sinh\frac{\alpha}{2} = 0\,;\ \alpha = 0, \beta \neq 0, sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Condition\ I$$

$$If\ \ cos\frac{\beta}{2} = 0\ ;\ sin\frac{\beta}{2} = \pm 1\ and\ \ \alpha \neq 0\ , \beta = (2n-1)\pi, cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}\ ...Condition\ II$$

Condition I leads to a pass band, or region of zero attenuation which limited by the upper limits on the sine (or)

$$sin\frac{\beta}{2} = 1\ (or)\ \ -1 < \left|\frac{Z_1}{4Z_2}\right| < 0$$

The phase angle in this pass band will be

$$\beta = 2sin - 1\sqrt{\frac{Z_1}{4Z_2}}$$

Condition II leads to stop band or attenuation band α≠ 0 the phase angle is π the attenuation is

$$\alpha = 2cosh - 1\sqrt{\frac{Z_1}{4Z_2}}$$

The $cosh^{-1}$ has no value below unity. $\left|\frac{Z_1}{4Z_2}\right| < -1$.The value of $\frac{Z_1}{4Z_2}$ can be classified into three region with corresponding value of α and β.

$\frac{Z_1}{4Z_2}$	+∞ to 0	0 to -1	-1 to -∞
Reactance Type Band α β	Same Stop 2sinh-1 $\sqrt{\frac{Z_1}{4Z_2}}$ π	Opposite Pass 0 2sin-1 $\sqrt{\frac{Z_1}{4Z_2}}$	Opposite Stop 2cosh-1 $\sqrt{\frac{Z_1}{4Z_2}}$ π

Table4.1. $\frac{Z_1}{4Z_2}$ **values of 0,-1,1,∞,-∞**

The frequency at which the network changes from a pass network to stop network or vice versa are called cut off frequencies. These frequencies occur when,

$$\frac{Z_1}{4Z_2} = 0\ \ (or)\ Z_1\ = 0$$

$$\frac{Z_1}{4Z_2} = -1\ (or)\ Z_1 = -4Z_2$$

Where Z_1 and Z_2 are opposite type of reactance. Since Z_1 and Z_2 have a number of configurations as L and C elements as parallel and series combinations.

4.2.2. ***Behaviour Of The Characteristic Impedance***

For symmetrical T network the characteristic impedance $Z_{oT} = \sqrt{Z_1Z_2(1+\frac{Z_1}{4Z_2})}$. If the network is purely reactance the expression for the characteristic impedance is

$Z_{oT}=\sqrt{-X_1X_2(1+\frac{X_1}{4X_2})}$. The negative sign under the radical due to j^2. The stop band exists where X_1 and X_2 are the same type of reactance. The ratio $\frac{X_1}{4X_2}$ will be real and positive and the characteristic impedance will be pure reactance. The pass band was shown to exist where X_1 and X_2 were of opposite reactance type and $-1<\frac{X_1}{4X_2}<0$ Z_0 is real. The stop band exists with X_1 and X_2 of opposite types but $\frac{X_1}{4X_2}<-1$. Therefore Z_0 will be pure reactance. In pass band Z_0 is real and positive if the reactive network is terminated with a resistive $Z_0= R_0$ then the input impedance is R_0. So the network can accept power and will transmit it to the resistive load without loss or attenuation. In a stop band Z_0 has been shown to be reactive. If the network is terminated in its reactance Z_0 it will appear as totally reactive. It cannot accept or transmit the power. Since there is no resistive element in which the power may be dissipated. The network may transmit voltage or current, but with 90° phase angle between the two with attenuation. Similarly for π network $Z_{0\pi}=\sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}}$ Z_1Z_2 is always real for Z_1 & Z_2 as pure reactance. Thus the conditions for pass band and stop band for T section likewise apply for π section.

4.3. CONSTANT K FILTERS

Constant k filters, also k-type filters, are a type of electronic filter designed using the image method. They are the original and simplest filters produced by this methodology and consist of a ladder network of identical sections of passive components. Historically, they are the first filters that could approach the ideal filter frequency response to within any prescribed limit with the addition of a sufficient number of sections. However, they are rarely considered for a modern design, the principles behind them having been superseded by other methodologies which are more accurate in their prediction of filter response.

4.4. CONSTANT K LOW PASS FILTERS

If Z_1 and Z_2 of a reactance network are $Z_1Z_2= K^2$ Where K is a constant independent of frequency network or filter section for which this relation holds are called constant K filters. As the special case at

$$Z_1 = j\omega L \text{ and } Z_2 = \frac{-j}{\omega C}$$

$$\therefore Z_1Z_2 = j\omega L(\frac{-j}{\omega C}) = \frac{L}{C} = Rk^2$$

If Z_1 and Z_2 are opposite type R_k is used K is real. Fig Shows the LPF section and the reactance of Z_1 and $4Z_2$ will vary with frequency. We know the pass band starts at frequency at which $Z_1 = 0$ and runs to the frequency at which $Z_1 = -4Z_2$. Thus the reactance curve shoes that a pass band starts at f=0 and continue to some higher frequency. All the frequency above f_c lie in a stop or attenuation band. Thus the network is called low pass filter.

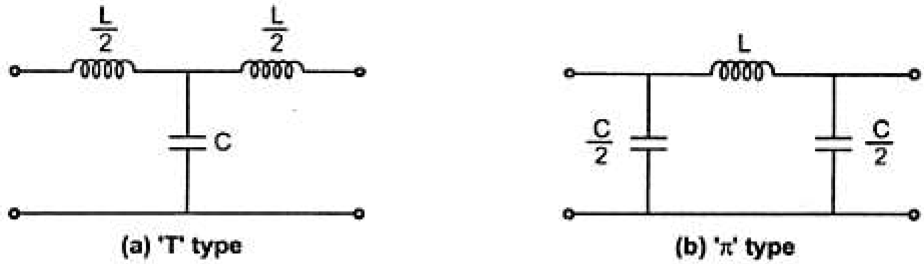


Fig. 4.1 Prototype T and π low pass filter sections

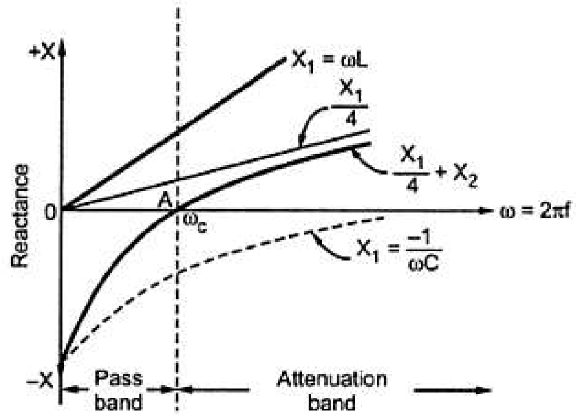


Fig 4.2. Low pass filter and its reactance curve

At the cut off frequency $Z_1 = -4Z_2$

$$j\omega L = \left(\frac{-j}{\omega C}\right)[\omega c = 2\pi fc]$$

$$2\pi fcL = \frac{4}{2\pi fcC}$$

$$fc^2 = \frac{4}{4\pi^2 LC}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$Then\ sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{j\omega L\ \omega C}{-4j}}$$

$$= \sqrt{\frac{-\omega^2 LC}{4}}$$

$$= \frac{j\omega}{2\pi f_c} = \frac{j2\pi f}{2\pi f_c} = j\frac{f}{f_c}$$

If the frequency f is in pass band (or)

$$\frac{f}{fc} < 1, -1 < \frac{Z_1}{4Z_2} < 0\ then$$

$$\frac{f}{f_c} < 1, \alpha = 0, \beta = 2\sin^{-1}\frac{f}{f_c}$$

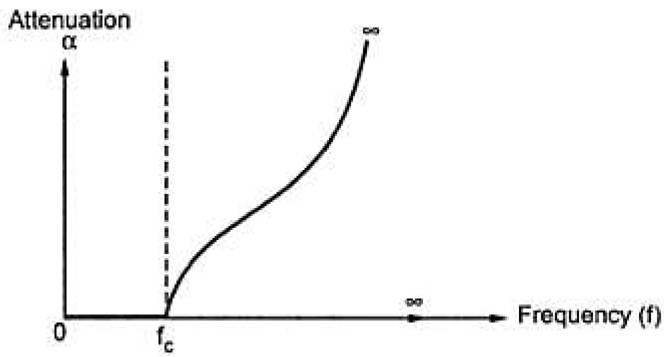


Fig 4.4 Variation of α and β with frequency of Low pass filter

If the frequency f is in attenuation band $\frac{f}{f_c}>1$ $\frac{Z_1}{4Z_2}<-1$ then $\frac{f}{f_c}>1$, $\alpha= 2\cosh^{-1}(\frac{f}{f_c})$, $\beta= \pi$. Therefore the attenuation α is zero throughout the pass band but raises gradually from the cut off frequency at $\frac{f}{f_c}= 1.0$ to ∞ at infinite frequency. The phase shift β is zero frequency and increases gradually though the pass band reaching π at f_c and remaining at π for all higher frequencies. The phase shift β is zero at zero frequency and increases gradually though the pass band reaching π at for all higher frequencies. The characteristic impedance for a T sections

$$Z_{oT} = \sqrt{Z_1 Z_2 (1 + \frac{Z_1}{4Z_2})}$$

$$= \sqrt{\frac{L}{C}(1 - \frac{\omega^2 LC}{4})}$$

$$= \sqrt{R_k^2 \left[1 - \left(\frac{f}{f_c}\right)^2\right]}$$

$$Z_{oT} = R_k \sqrt{1 - (\frac{f}{f_c})^2}$$

A low pass filter may be designed from desired cut off frequency and load resistance. The design of Low pass filter is,

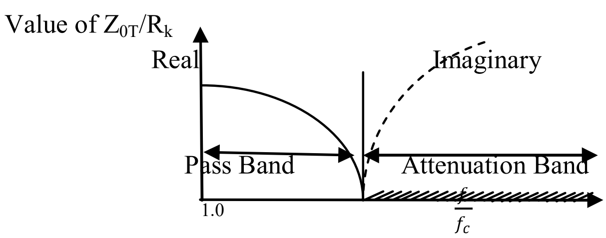


Fig 4.5 Variation of Z_{0T}/R_k for frequency of Low pass filter

$$At\ cut\ of\ frequency, \qquad Z1 = -4Z2$$

$$j\omega cL = 4(\frac{j}{\omega_c C})[\omega c = 2\pi f c]$$

$$2\pi f cL = \frac{4}{2\pi f cC}$$

$$\pi^2 LC\, fc2 = 1$$

$$We\ know\ the\ relation\ R = \sqrt{\frac{L}{C}} \qquad \therefore C = \frac{1}{\pi^2 L\, fc^2}$$

$$= \frac{1}{\pi^2\, fc^2 R^2 C}$$

$$C_2 = \frac{1}{\pi^2\, fc^2 R^2}$$

$$C = \frac{1}{\pi f cR} \quad Similarly, \qquad L = \frac{R}{\pi f_c}$$

4.5. CONSTANT K HIGH PASS FILTERS

If the position of inductance and capacitance are interchanged ie,

$$Z_1 = j\omega L\ and\ Z_2 = \frac{-j}{\omega C}$$

$$\therefore Z_1 Z_2 = j\omega L(\frac{-j}{\omega C}) = \frac{L}{C} = Rk^2$$

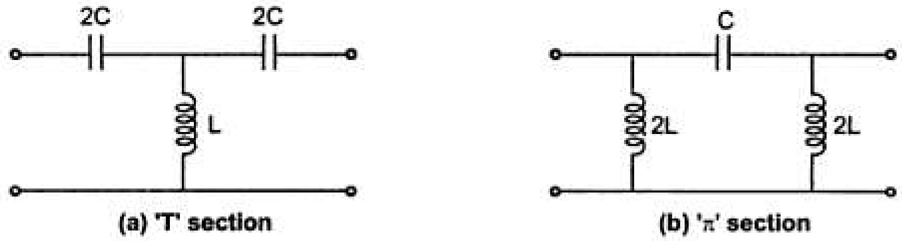


Fig. 4.4 Prototype T and π high pass filter sections

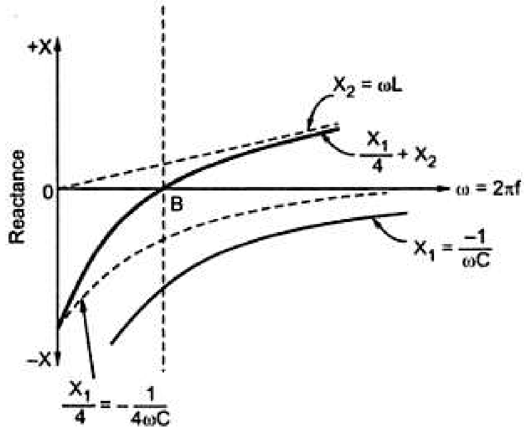


Fig. 4.5 Reactance-frequency sketch for prototype high pass filter

For high pass filter the cut off frequency at the point which Z_1 equals $-4Z_2$ with a pass band from frequency to infinity where $Z_1 = 0$. The cut off frequency is determined as the frequency at which

$$Z1 = -4Z2 \quad or \quad (\frac{-j}{\omega_c C}) = -j4\omega_0 L$$

$$4\omega 2LC = 1$$

$$fc = \frac{1}{4\pi\sqrt{LC}}$$

Using the above expression $$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega L\ \omega C}}$$

$$= \frac{-j}{2\omega\sqrt{LC}} = j\frac{f_c}{f}$$

The region in which $\frac{f_c}{f}<1$ is a pass band, so that the variation of γ inside and outside the pass band will be identical with the value for the low pass filter. Here the phase angle β will be negative changing from 0 at infinite frequency or $\frac{f_c}{f}=1$. The high pass filter may be designed by

again choosing the resistive load R equal to R_k, $$R = R_k = \sqrt{\frac{L}{C}}$$

At cut of frequency, $Z_1 = -4Z_2$

$$4\omega c2LC = 1\ and\ \frac{L}{C} = R2$$

$$C = \frac{1}{4\pi f_c R} \quad and\ L = \frac{R}{4\pi f_c}$$

The characteristic impedance of the high pass filter is,

$$ZoT = Rk\sqrt{1 - (\frac{f_c}{f})^2}$$

4.6. CONSTANT K BAND PASS FILTERS

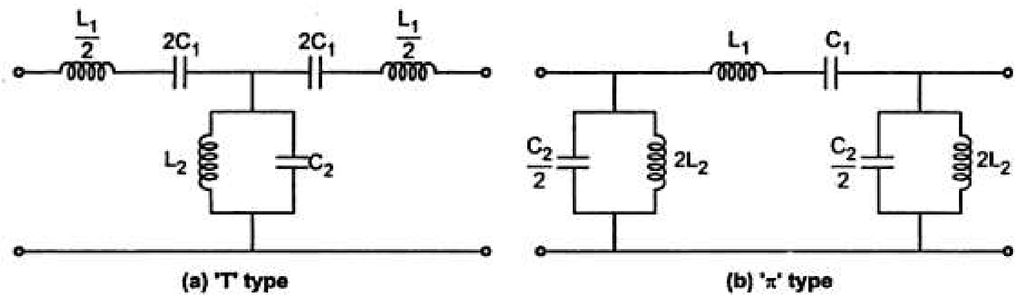


Fig. 4.6 Prototype T and π band pass filter sections

It is desirable to pass band of frequency and to attenuate frequencies on both sides of the pass and. The operation of band pass filter is same as low pass filter and high pass filter in series. The cut off frequency of low pass filter is above the cut off frequency of high pass filter, the overlapping thus allowing only a band of frequencies to pass. i.e, Band pass filter. Consider the above circuit with series resonant series arm and an anti resonant shunt arm. The reactance curve is,

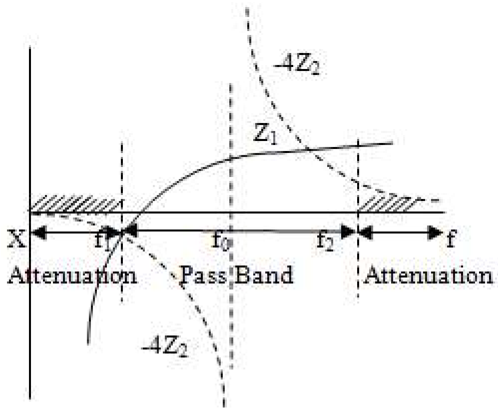


Fig 4.8. Reactance curve Band pass with resonant and anti resonant frequency are properly adjusted

At Resonant frequency,

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2 \quad and \qquad L_1 C_1 = L_2 C_2$$

The impedance of the arms are,

$$Z_1 = j(\omega L_1 - \frac{1}{\omega C_1}) = j(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1})$$

$$Z_2 = \frac{j\omega L_2(-\frac{j}{\omega C_2})}{j(\omega L_2 - \frac{1}{\omega C_2})} = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$For\ a\ constant\ K\ filter, \quad Rk^2 = Z_1 Z_2 = j(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1})(\frac{j\omega L_2}{1-\omega^2 L_2 C_2})$$

$$= \frac{-L_2(\omega^2 L_1 C_1 - 1)}{C_1(1-\omega^2 L_2 C_2)}$$

$$If \quad L_1 C_1 = L_2 C_2, \qquad Z_1 Z_2 = \frac{L_2(1-\omega^2 L_1 C_1)}{C_1(1-\omega^2 L_2 C_2)}$$

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = Rk^2$$

$$At\ cut\ of\ frequency, \qquad Z_1 = -4Z_2$$

$$Multiplying\ Z_1, \qquad Z_1 Z_2 = -4\, Z_1 Z_2 = -4\, Rk^2$$

$$Z_1 = \pm 2j\, Rk$$

So Z_1 at lower cut off f_1 = - Z_1 at upper cut off f_2

i.e,

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = \omega_2 L_1 - \frac{1}{\omega_2 C_1}$$

$$\frac{1-\omega_1{}^2 L_1 C_1}{\omega_1 C_1} = \frac{\omega_2{}^2 L_1 C_1 - 1}{\omega_2 C_1}$$

$$1-\omega_1{}^2 L_1 C_1 = \frac{\omega_1}{\omega_2}(\omega_2{}^2 L_1 C_1 - 1)$$

We know that At Resonant frequency, $\omega_0{}^2 L_1C_1 = 1 = \omega_0{}^2 L_2C_2$

$1-\omega_0{}^2 L_1C_1 = 0$ so $\omega_0{}^2 L_1C_1 = 1$

$$L_1C_1 = \frac{1}{\omega_0^2}$$

$$1-\omega_1{}^2 L_1 C_1 = \frac{\omega_1}{\omega_2}(\omega_2{}^2 L_1 C_1 - 1)$$

$$1-\frac{\omega_1^2}{\omega_0^2} = \frac{\omega_1}{\omega_2}(\frac{\omega_2^2}{\omega_0^2} - 1)$$

$$1-\frac{f_1^2}{f_0^2} = \frac{f_1}{f_2}(\frac{f_2^2}{f_0^2} - 1)$$

$$\frac{f_0^2 - f_1^2}{f_0^2} = \frac{f_1}{f_2}(\frac{f_2^2 - f_0^2}{f_0^2})$$

$$f_0^2 f_2 - f_1^2 f_2 = f_2^2 f_1 - f_0^2 f_1$$

$$f_0^2(f_2 + f_1) = f_2 f_1 (f_2 + f_1)$$

$$f_0 = \sqrt{f_1 f_2}$$

If the filter terminate by load R = R_k determine the circuit components in terms of R and the cut off frequencies f_1 and f_2. At lower cut off frequency

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2R$$

$$\frac{1-\omega_1{}^2L_1C_1}{\omega_1C_1}=2R$$

$$1-\omega_1{}^2L_1C_1=2R\omega_1C_1$$

$$1-\frac{(2\pi f_1)^2}{\omega_0^2}=4\pi Rf_1C_1$$

$$1-\frac{(2\pi f_1)^2}{(2\pi f_0)^2}=4\pi Rf_1C_1$$

$$1-\frac{f_1{}^2}{f_0^2}=4\pi Rf_1C_1$$

$$C_1=\frac{1}{4\pi Rf_1}[\frac{f_1f_2-f_1^2}{f_1f_2}]$$

$$=\frac{1}{4\pi R}[\frac{f_2-f_1}{f_1f_2}]$$

$$Sub\ L1C1=\frac{1}{\omega_0^2} \qquad \frac{L_1}{4\pi R}\left[\frac{f_2-f_1}{f_1f_2}\right]=\frac{1}{\omega_0^2}$$

$$Sub\quad f_0^2=f_1f_2 \qquad \frac{L_1}{4\pi R}=\frac{1}{(2\pi f_0)^2}\left[\frac{f_1f_2}{f_2-f_1}\right]$$

$$L1=\frac{R}{\pi(f_2-f_1)}$$

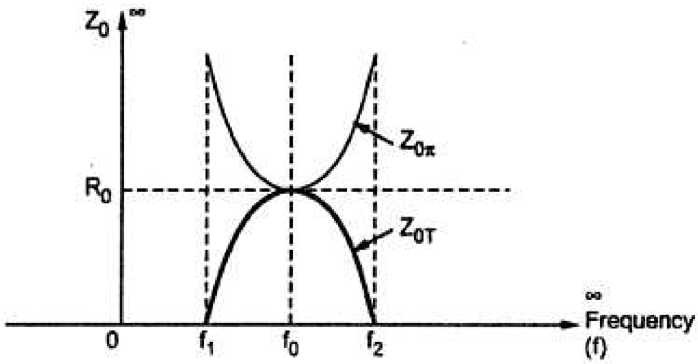
Similarly for the Shunt arm,

$$L_2=C_1R_2=\frac{R^2}{4\pi R}\left[\frac{f_2-f_1}{f_1f_2}\right]$$

$$=\frac{R}{4\pi}\left[\frac{f_2-f_1}{f_1f_2}\right]$$

$$C_2=\frac{L_1}{R}=\frac{R}{R^2\pi(f_2-f_1)}$$

$$C_2=\frac{1}{R\pi(f_2-f_1)}f$$



(a) Variation of Z_{0T} and $Z_{0\pi}$ with frequency

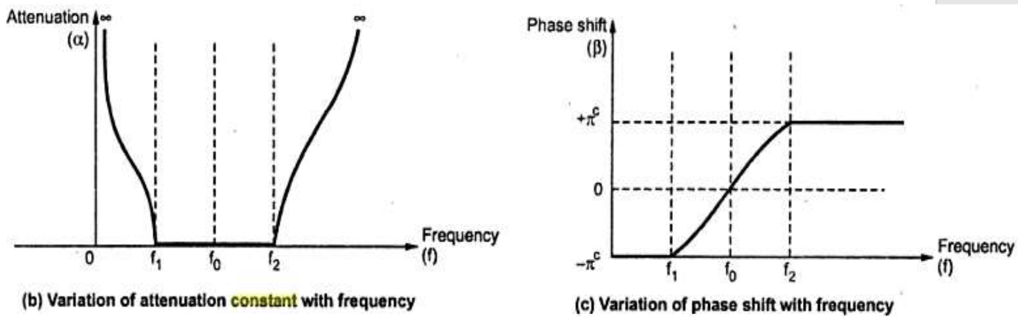


Fig 4.9. Variation of Z_{0T}, $Z_{0\pi}$ and attenuation constant ,phase constant with frequency

4.7. CONSTANT K BAND ELIMINATION FILTERS

If the series and parallel tuned arms of the band pass filter are interchanged the resultant band elimination filter is

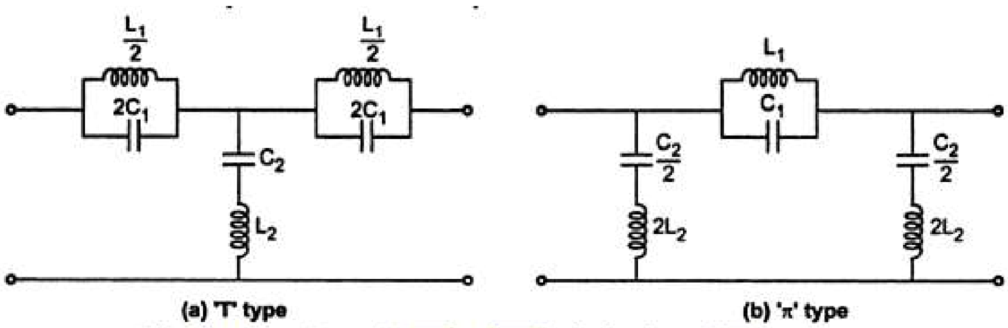


Fig. 4.10 Prototype T and π band elimination filter sections

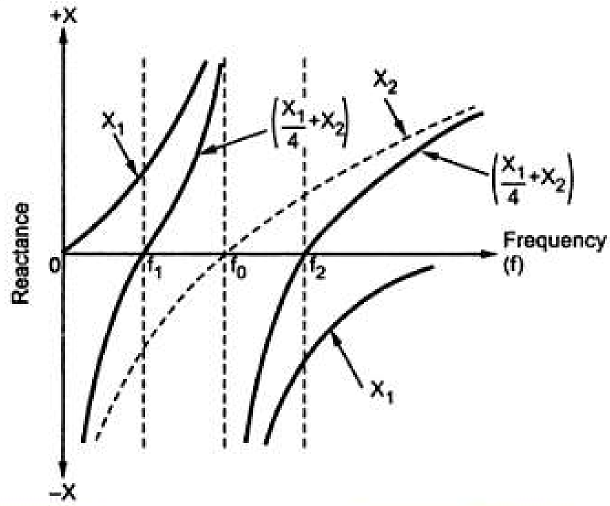


Fig. 4.11 Reactance frequency sketch for constant K band elimination filter

The operation of band elimination filter is the LPF is parallel with HPF section in which the cut off frequency of low pass filter is below the high pass filter. As for the band pass filter the series arms and shunt arms are made anti resonant and resonant at the same frequency.

$$Z1 = \frac{j\omega L_2(-\frac{j}{\omega C_2})}{j(\omega L_2 - \frac{1}{\omega C_2})} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z2 = j(\omega L1 - \frac{1}{\omega C_1}) = j(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1})$$

For a constant K filter,

$$Rk^2 = Z_1 Z_2 = (\frac{j\omega L_2}{1 - \omega^2 L_2 C_2})\ j(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1})$$

$$= \frac{-L_2(\omega^2 L_1 C_1 - 1)}{C_1(1 - \omega^2 L_2 C_2)}$$

$$If \quad L_1 C_1 = L_2 C_2, \qquad Z_1 Z_2 = \frac{L_2(1 - \omega^2 L_1 C_1)}{C_1(1 - \omega^2 L_2 C_2)}$$

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = Rk^2$$

We know that At Resonant frequency,

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$\therefore \quad 1 - \omega_0^2 L_2 C_2 = 0$$

$$\omega_0^2 L_2 C_2 = 1$$

$$L_1 C_1 = \frac{1}{\omega_0^2}$$

$$1-{\omega_1}^2L_1C_1=\frac{\omega_1}{\omega_2}({\omega_2}^2L_1C_1-1)$$

$$1-\frac{\omega_1^2}{\omega_0^2}=\frac{\omega_1}{\omega_2}(\frac{\omega_2^2}{\omega_0^2}-1)$$

$$1-\frac{f_1^2}{f_0^2}=\frac{f_1}{f_2}(\frac{f_2^2}{f_0^2}-1)$$

$$\frac{f_0^2-f_1^2}{f_0^2}=\frac{f_1}{f_2}(\frac{f_2^2-f_0^2}{f_0^2})$$

$$f_0^2f_2-f_1^2f_2=f_2^2f_1-f_0^2f_1$$

$$f_0^2(f_2+f_1)=f_2f_1(f_2+f_1)$$

$$f_0=\sqrt{f_1f_2}$$

At the cut off frequency, $Z_1=-4Z_2$

$$Z_1Z_2=-4{Z_2}^2=Rk^2$$

$$Z2=\pm\frac{j\,Rk}{2}$$

If the filter is terminated in a load $R=Rk$ at lower cut off frequency

$$j(\frac{1}{\omega_1C_2}-\omega2L2)=\frac{j\,Rk}{2}$$

Since $L_2C_2=\frac{1}{\omega_0^2}$

$$\frac{1-{\omega_1}^2L_2C_2}{\omega_1C_2}=\frac{R}{2}$$

$$1-\frac{\omega_1^2}{\omega_0^2}=\frac{R\omega_1C_2}{2}$$

$$1-\frac{f_1^2}{f_0^2}=\pi Rf_1C_2$$

$$C_2=\frac{1}{\pi Rf_1}[1-\frac{f_1^2}{f_0^2}]$$

$$=\frac{1}{\pi Rf_1}[\frac{f_0^2-f_1^2}{f_0^2})] \qquad [f_0=\sqrt{f_1f_2}]$$

$$C_2=\frac{1}{\pi Rf_1}[\frac{f_1f_2-f_1^2}{f_1f_2}]$$

$$C_2=\frac{1}{\pi R}\left[\frac{f_2-f_1}{f_1f_2}\right]$$

Then,

$$f_0=\sqrt{f_1f_2}=\frac{1}{2\pi\sqrt{L_2C_2}}$$

$$\sqrt{f_1f_2}=\frac{1}{2\pi\sqrt{L_2(\frac{1}{\pi R}\left[\frac{f_2-f_1}{f_1f_2}\right])}}$$

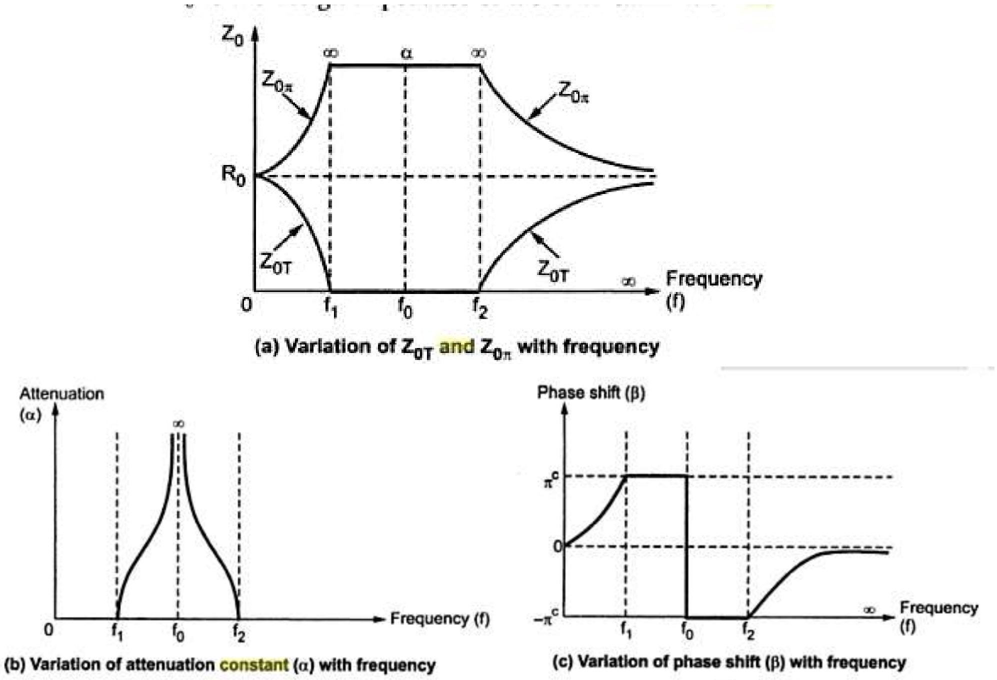
$$\sqrt{L_2} = \frac{1}{2\pi\sqrt{f_1 f_2}\sqrt{\frac{1}{\pi R}\left[\frac{f_2 - f_1}{f_1 f_2}\right]}}$$

$$L_2 = \frac{R}{4\pi(f_2 - f_1)}$$

Similarly, $L_1 = C_2 R_2$ $\qquad = R^2 \frac{1}{\pi R}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$

$$= R\,\frac{1}{\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$$

And $C_1 = \frac{L_2}{R^2}$ $\qquad C_1 = \frac{1}{4\pi R(f_2 - f_1)}$



(a) Variation of Z_{0T} and $Z_{0\pi}$ with frequency

(b) Variation of attenuation constant (α) with frequency

(c) Variation of phase shift (β) with frequency

Fig 4.10. Variation of Z_{0T} , $Z_{0\pi}$ and attenuation constant ,phase constant with frequency

DISADVANTAGES OF PROTOTYPE FILTER SECTION

The constant K prototype filter section through simple has two major disadvantages.

1. The attenuation does not raises vary rapidly at cut off so that frequencies just outside the pass band are not attenuated with respect to frequencies just inside the pass band. And the characteristic impedance varies widely over the pass band so the impedance matching is not possible.

2. At the resonant frequency the circuit is short circuited or the attenuation becomes infinity. Below f_c the shunt circuit appear as a capacitance from the m-derived circuit.

4.8. M - DERIVED SECTIONS

The first disadvantages of prototype filter section can be overcome by connecting two or more prototype section of same type. m-derived filters or m-type filters type was originally intended for use with telephone multiplexing and was an improvement on the existing constant k type filter.

M- derived T section

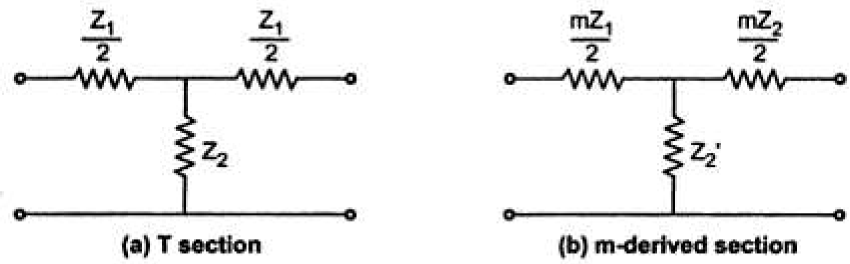


Fig 4.10. m-derived T section

$Z_1^1 = mZ_1$ then $Z_0^1 = Z_0$ and We know that

$$Z02 = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$\left(\frac{mZ_1}{4}\right)^2 + mZ_1 Z_2^1 = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$mZ_1 Z_2^1 = \frac{Z_1^2}{4} + Z_1 Z_2 - \left(\frac{mZ_1}{4}\right)^2$$

$$= \frac{Z_1^2}{4}(1 - m^2) + Z_1 Z_2$$

$$= \frac{Z_2}{m} + \frac{Z_1}{4m}(1 - m^2)$$

It appear that the shunt arm consists of two impedance in series.

Here $\frac{1-m^2}{4m}$ must be positive forcing the terms 1-m^2 and m always to be positive. Thus m must always 0<m<1.

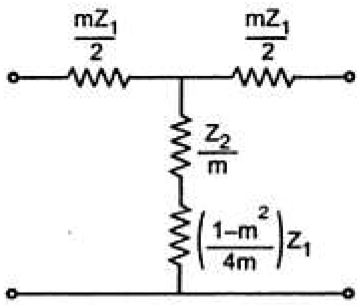


Fig 4.11. m-derived T section

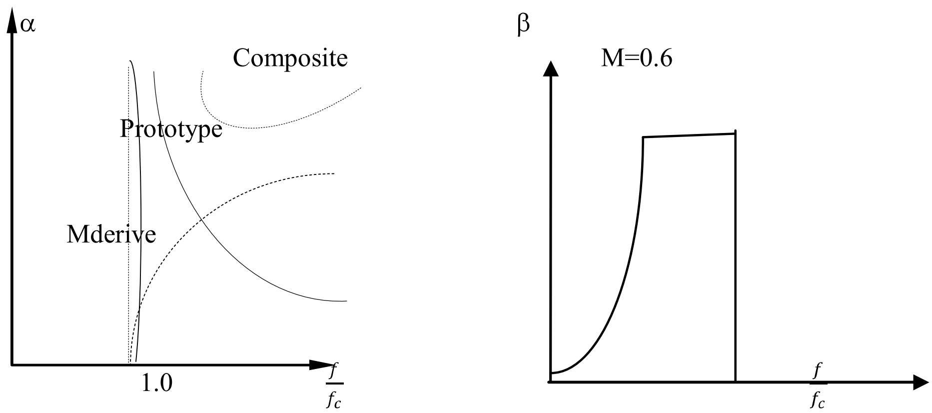


Fig 4.12. Variation of attenuation constant and phase shift for m- derived filter

M- derived π section

The characteristic impedance of section is,

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 (1+\frac{Z_1}{4Z_2})}}.$$

The characteristic impedance of the prototype and m derived sections are equal so that may without mismatch. By the use of transformation for the shunt arm,

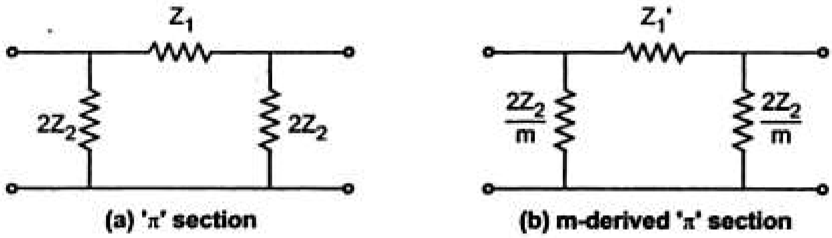


Fig 4.13. m-derived π section

The characteristic impedance,

$$\frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 (1+\frac{Z_1}{4Z_2})}} = \frac{Z_1^1 Z_2/m}{\sqrt{\frac{Z_1{}^1 Z_2}{m}(1+\frac{mZ_1{}^1}{4Z_2})}}$$

$$\frac{\left(\frac{Z_1^1 Z_2}{m}\right)^2}{\frac{Z_1{}^1 Z_2}{m}(1+\frac{mZ_1{}^1}{4Z_2})} = \frac{(Z_1 Z_2)^2}{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$= \frac{1}{\frac{1}{mZ_1} + \frac{1}{\frac{4m}{1-m^2} Z_2}}$$

It is apparent that the series arm Z_1^1 in represented by two impedance in parallel. One in mZ_1 and other is $\frac{4m}{(1-m^2)Z_2}$ in value.

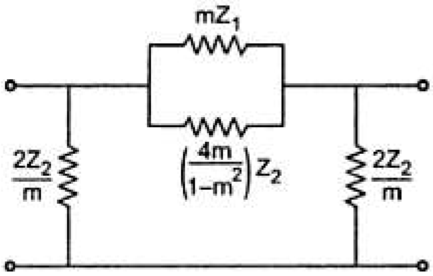


Fig 4.14. m-derived π section

4.9. M- DERIVED LOW PASS FILTER

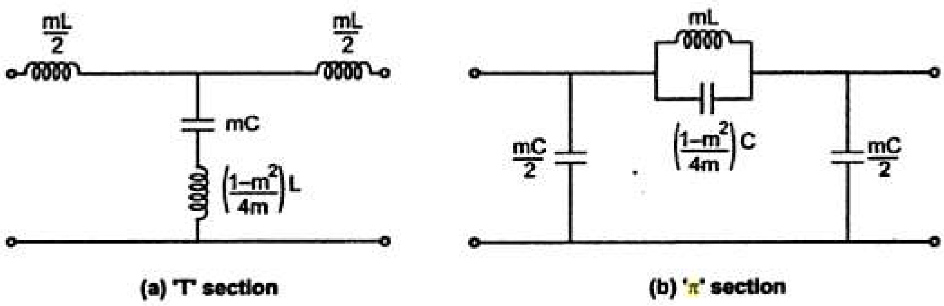


Fig 4.15. m- derived Low Pass Filter Section

Consider the shunt arm T section resonate at frequency of infinite attenuation f_∞ which is selected just above cut off frequency f_c.

$$f\infty = \frac{1}{2\pi\sqrt{\left(\frac{1-m^2}{4m}\right) LmC}}$$

$$f_\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}}$$

But Low Pass Filter cut off frequency is given by

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$\therefore \quad f_\infty = \frac{fc}{\sqrt{(1-m^2)}}$$

$$\sqrt{(1-m^2)} = \frac{fc}{f\infty}$$

$$(1 - m^2) = \left(\frac{\text{fc}}{\text{f}\infty}\right)^2$$

$$m^2 = 1 - \left(\frac{\text{fc}}{\text{f}\infty}\right)^2$$

$$m = \sqrt{1 - \left(\frac{\text{fc}}{\text{f}\infty}\right)^2}$$

Variations of Characteristic impedance, attenuation constant, phase constant

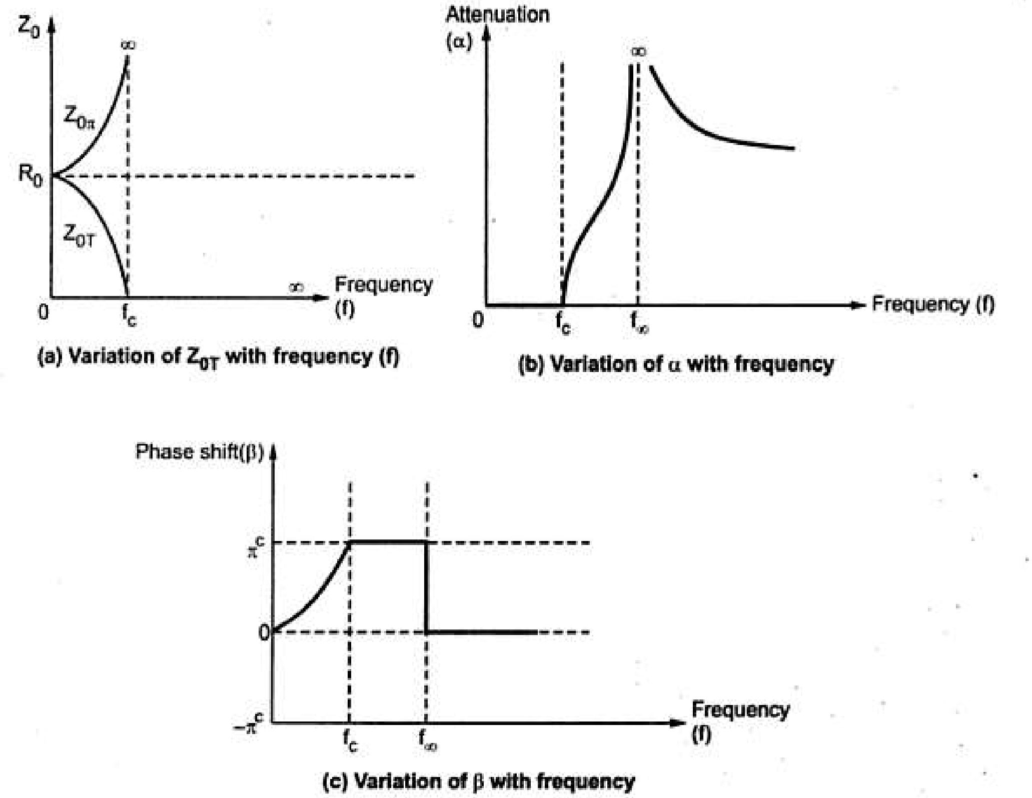


Fig 4.16. Variation of Z_0, α, β with frequency

Variation of attenuation constant α,

$$\alpha = 2cosh - 1\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2cosh - 1\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{1 - \left(\frac{f}{f_\infty}\right)^2}}\right] \; for\, fc < f < f\infty \; and$$

$$\alpha = 2sinh-1\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2sinh-1\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{\left(\frac{f}{f_\infty}\right)^2-1}}\right] for\, f > f\infty$$

The phase shift β in pass band,

$$\beta = 2sin-1\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2sin-1\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{1-(1-m^2)\left(\frac{f}{f_c}\right)^2}}\right]$$

4.10. M – DERIVED HIGH PASS FILTER

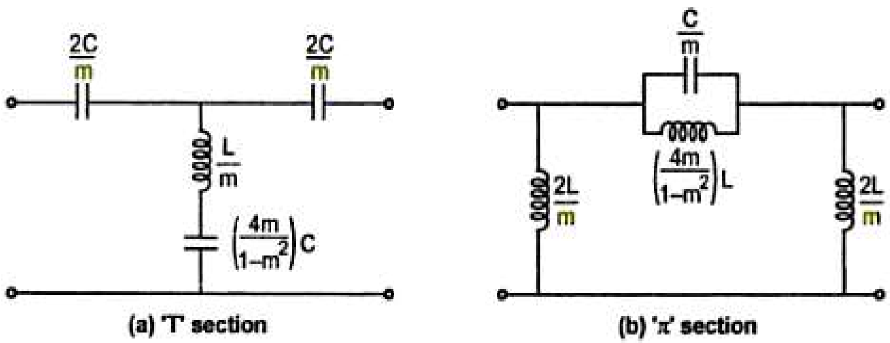


Fig 4.17. m- derived High Pass Filter Section

Consider the shunt arm of T section resonate at frequency of infinite attenuation ie f∞. which is selected below cut off frequency fc

$$f\infty = \frac{1}{2\pi\sqrt{\left(\frac{4m}{1-m^2}\right)\left(\frac{L}{m}\right)C}}$$

$$= \frac{1}{2\pi\sqrt{\left(\frac{4LC}{1-m^2}\right)}}$$

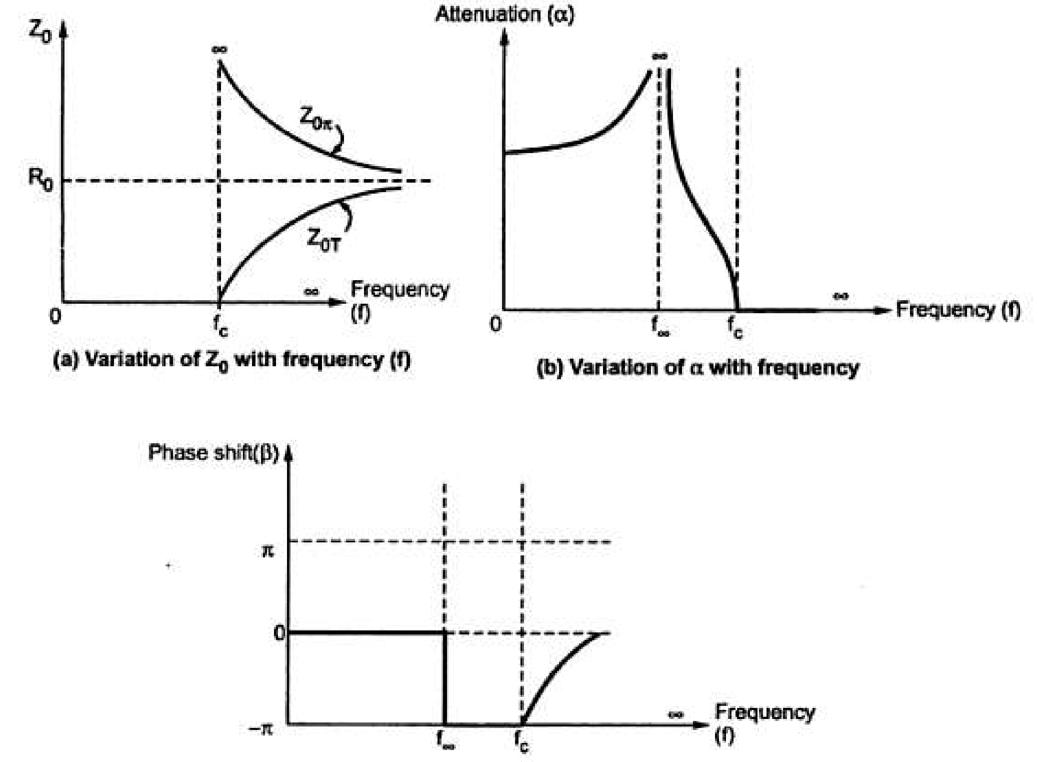
$$= \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} \quad for\ HPF\ fc = \frac{1}{4\pi\sqrt{LC}}$$

∴ $f_\infty = fc\sqrt{(1-m^2)}$

$$\frac{f\infty}{fc} = \sqrt{(1-m^2)}$$

$$m = \sqrt{\left(1-\left(\frac{f\infty}{fc}\right)^2\right)}$$

Variations of Characteristic impedance, attenuation constant, phase constant



(a) Variation of Z_0 with frequency (f)

(b) Variation of α with frequency

(c) Variation of β with frequency

Fig 4.18. Variation of Z_0, α, β with frequency

Variation of attenuation constant α,

$$\alpha = 2cosh - 1\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2cosh - 1\left[\frac{m\left(\frac{fc}{f}\right)}{\sqrt{1-\left(\frac{f\infty}{f}\right)^2}}\right] \; for\ f\infty < f < fc\ \ and$$

$$\alpha = 2sinh - 1\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2sinh - 1\left[\frac{m\left(\frac{fc}{f}\right)}{\sqrt{\left(\frac{f\infty}{f}\right)^2 - 1}}\right] \; for\ f < f\infty$$

The phase shift β in pass band,

$$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\sin^{-1}\left[\frac{m\left(\frac{fc}{f}\right)}{\sqrt{1-(1-m^2)\left(\frac{fc}{f}\right)^2}}\right]$$

1.12.3. m- derived Band Pass Filter Section

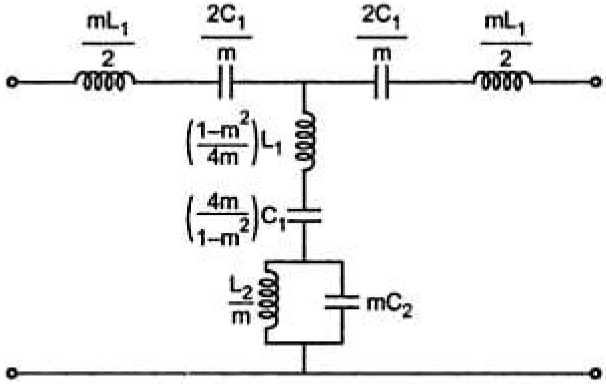


Fig. 4.19. m- derived Band Pass Filter

The T section in each case will have a shunt impedance, $\frac{Z_2}{m} + \left[\frac{1-m^2}{4m}\right] Z_1$. Where Z_1 and Z_2 are the impedance of the prototype sections. If substitute value of Z_1 and Z_2 of prototype bandpass filter section,

$$\frac{Z_2}{m} + \left[\frac{1-m^2}{4m}\right] Z_1 = 0$$

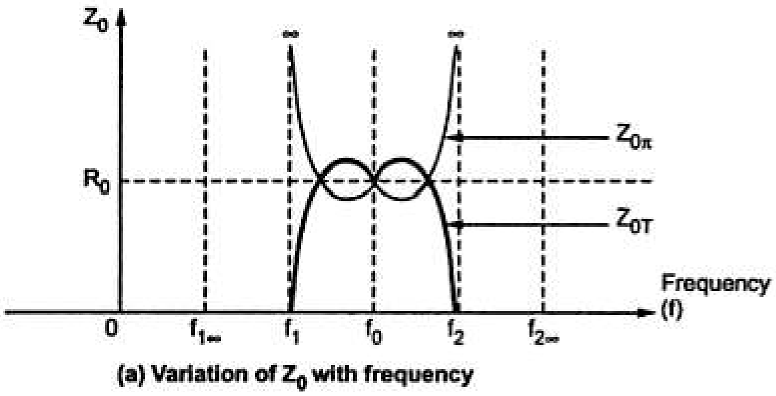
The resonant frequency $f_0 = \sqrt{f_{1\infty} f_{2\infty}} = \sqrt{f_1 f_2}$

$$f_{2\infty} - f_{1\infty} = \frac{f_2 - f_1}{\sqrt{(1-m^2)}}$$

$$\sqrt{(1-m^2)} = \frac{f_2 - f_1}{f_{2\infty} - f_{1\infty}}$$

$$m = \sqrt{\left(1 - \left(\frac{f_2 - f_1}{f_{2\infty} - f_{1\infty}}\right)^2\right)}$$

Variations of Characteristic impedance, attenuation constant, phase constant



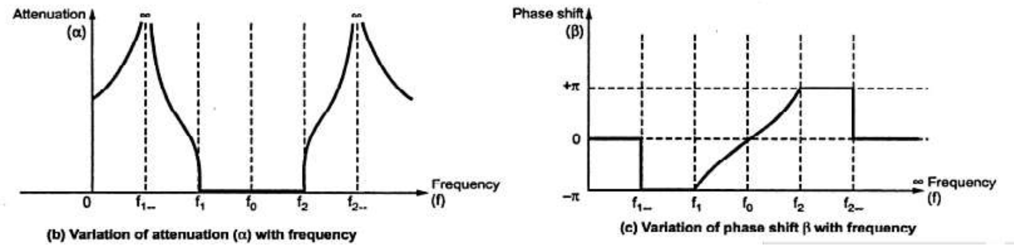


Fig 4.20. Variation of Z_0, α, β with frequency

1.12.4. m- derived Band Elimination Filter Section

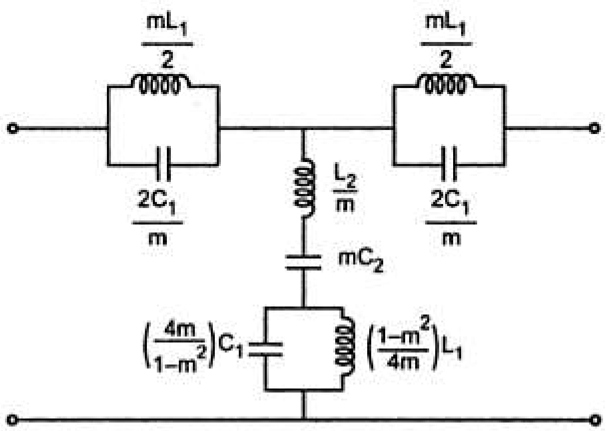


Fig 4.21. m- derived Band Elimination Filter Section

The m derived band elimination filter can be derived from the prototype band elimination filter section. The relationship between frequency of infinite attenuation ($f_{1\infty}$,$f_{2\infty}$) and cut off frequencies

(f_1,f_2) is given by, $f_{2\infty} - f_{1\infty} = (f_2 - f_1)\sqrt{(1 - m^2)}$

If the frequency of resonance is f_0 then it is the geometric mean of two cut off frequencies as well as of two frequencies of infinite attenuation.

$$f_0 = \sqrt{f_{1\infty} f_{2\infty}} = \sqrt{f_1 f_2}$$

$$\sqrt{(1-m^2)} = \frac{f_{2\infty} - f_{1\infty}}{f_2 - f_1}$$

$$m = \sqrt{1 - \left(\frac{f_{2\infty} - f_{1\infty}}{f_2 - f_1}\right)^2}$$

Variations of Characteristic impedance, attenuation constant, phase constant

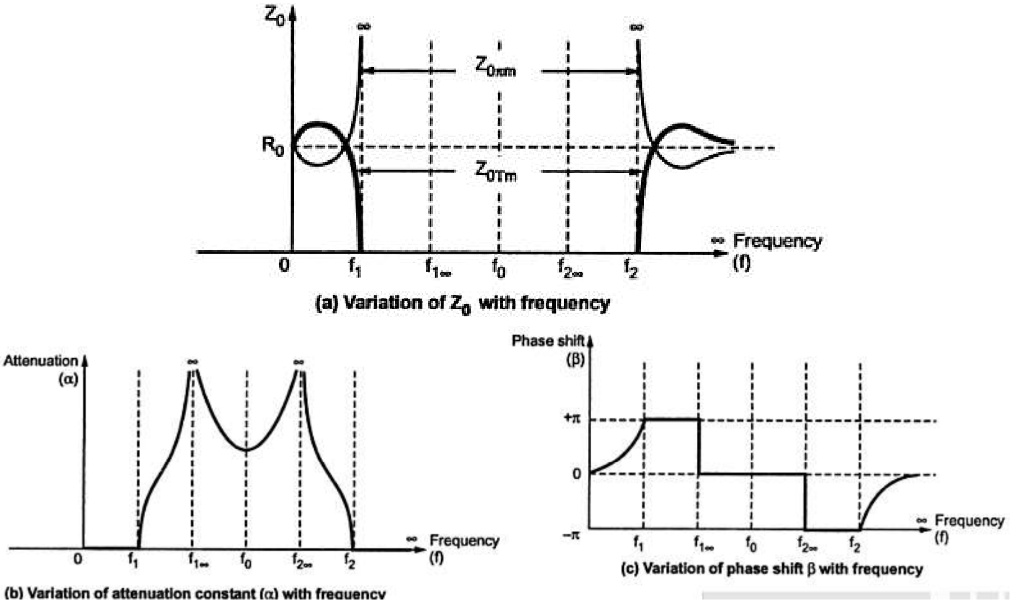
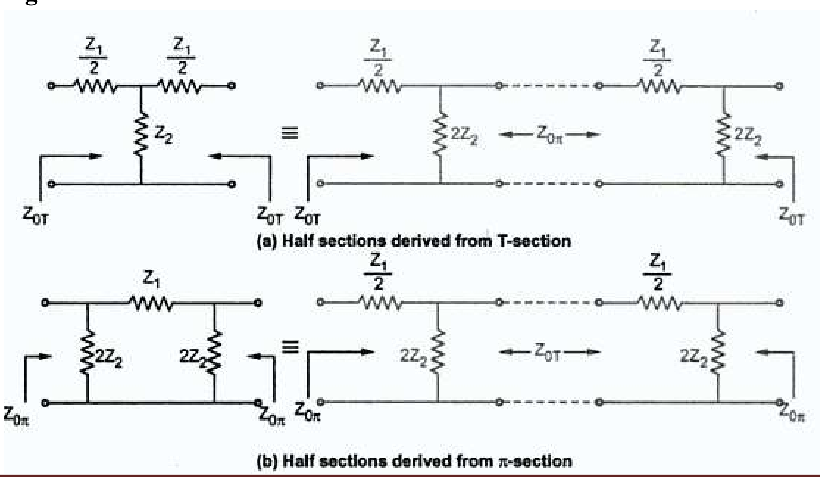


Fig 4.22. Variation of Z_0, α, β with frequency

Terminating Half section



4.11.Terminating m-derived Half Sections

The m-derived half sections can be obtained by splitting m-derived T or π-section centrally. When m-derived T-section is splitted centrally, the image impedances of the resulting m-derived half sections are Z_{0T} and $Z_{0\pi m}$ as shown in the Fig.

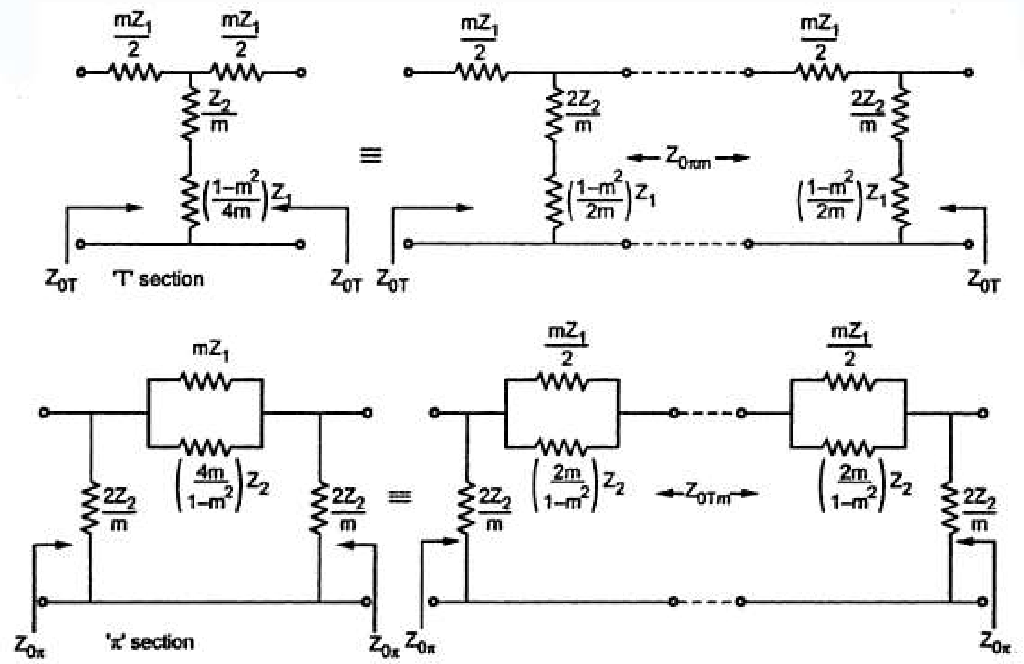


Fig. 4.23. m-derived half section

Consider one of the m-derived half sections as shown in the Fig. 6.44.

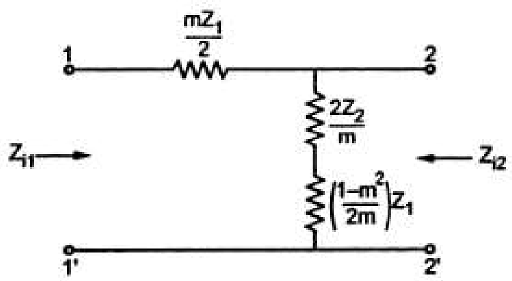


Fig. 4.24 m-derived half section

4.11.COMPOSITE FILTERS.

In prototype filter sections, the attenuation characteristic is not very sharp in the attenuation band as it is expected. This drawback can be overcome by using m-derived filter sections which are derived from respective prototype filter sections. But it is observed that in the stop band attenuation drastically reduces after f_∞ in low pass section and before f_∞ in high pass section. This drawback of m-derived filter can be overcome by connecting number of sections including prototype sections and m-derived sections with terminating half sections. Such a combination of different sections is called **composite filter.**

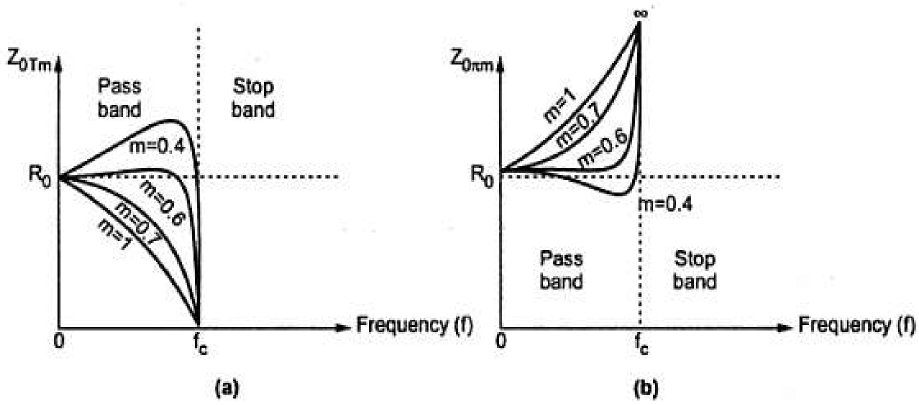


Fig.4.25 Variation of Z_{0Tm} and $Z_{0\pi m}$ in pass band of low pass filter

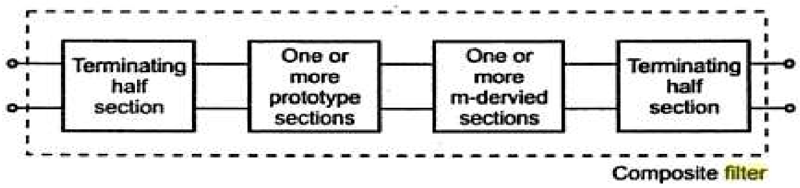


Fig. 4.25. Block schematic of composite filter

In composite filter, cut off frequency and the design impedance are the two important design specifications. The number of various sections in the composite filter totally depends on the attenuation characteristics required. If it is required that the attenuation should rise sharply in the attenuation band, we must select at least one m-derived section with low value of m. In general, for lower values of m, attenuation at cut off rises rapidly. The typical value of m for such attenuation at cut off is m = 0.3 to 0.35. If it is required to maintain this attenuation at a high value in attenuation band, we must connect either a prototype section or another m-derived section with comparitively larger value of m. If required both the sections can be used in the composite filter. To have proper impedance matching and constant characteristic impedance throughout pass band, we must connect the terminating sections with m = 0.6.

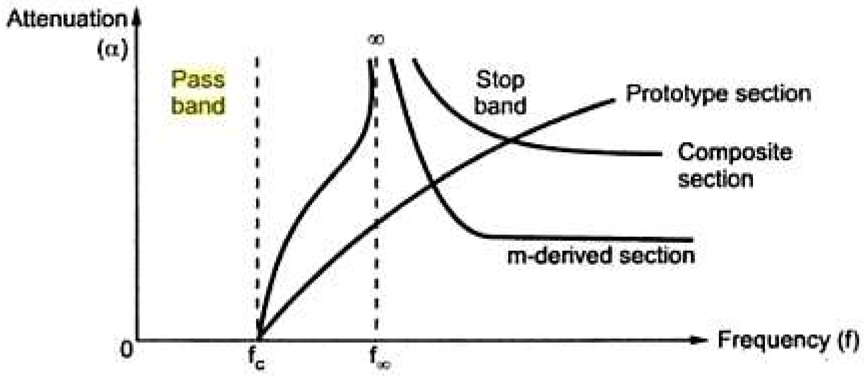


Fig 4.26. Variation of attenuation constant α in prototype, m-derived and composite sections

PROBLEMS

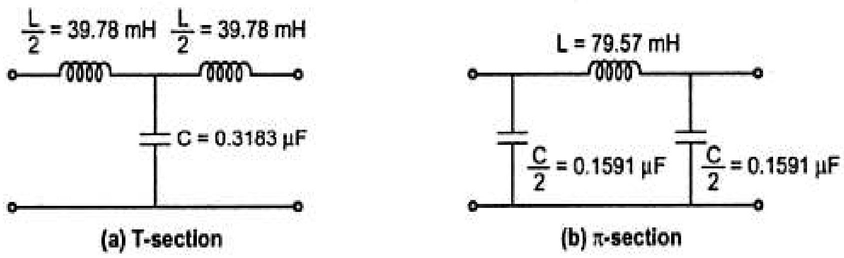
Example : *Design a prototype low pass filter sections if design impedance $R_0 = 500\ \Omega$ and cut-off frequency $f_c = 2000$ Hz.*

Solution : Using design equations for prototype low pass filter, we get,

$$L = \frac{R_0}{(\pi f_c)} = \frac{500}{(3.142 \times 2000)} = 79.57 \text{ mH}$$

$$C = \frac{1}{(\pi f_c)(R_0)} = \frac{1}{(3.142 \times 2000)\ (500)} = 0.3183\ \mu F$$

Prototype low pass filter T and π-sections are as shown in the Fig.



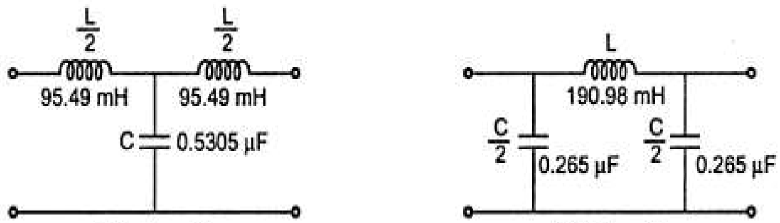
Example 4.2 : *A prototype LPF is to be designed which must have R_0 = 600 Ω; f_c = 1 kHz. Find filter elements (L and C).*

Solution : For prototype LPF, using design equations, we can write

$$L = \frac{R_0}{(\pi f_c)} = \frac{600}{(\pi \times 1 \times 10^3)} = 190.98 \text{ mH}$$

$$C = \frac{1}{(\pi f_c) R_0} = \frac{1}{(\pi \times 1 \times 10^3)(600)} = 0.5305 \ \mu\text{F}$$

With the above values of LPF elements, we can draw T and π sections as shown in the Fig.



(a) 'T' section (b) 'π' section

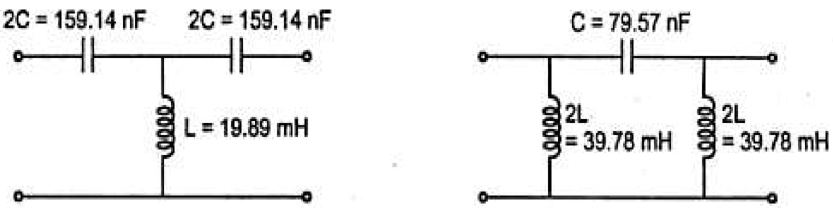
Example 4.3 : *Design a prototype high pass filter sections if design impedance R_0 = 500 Ω and cut-off frequency f_c = 2000 Hz.*

Solution : Using design equations for prototype high pass filter; we get

$$L = \frac{R_0}{(4\pi f_c)} = \frac{500}{(4 \times \pi \times 2000)} = 19.89 \text{ mH}$$

$$C = \frac{1}{(4\pi f_c) R_0} = \frac{1}{(4 \times \pi \times 2000)(500)} = 79.57 \text{ nF}$$

Prototype high pass filter T and π-sections are as shown in the Fig. 5.20.



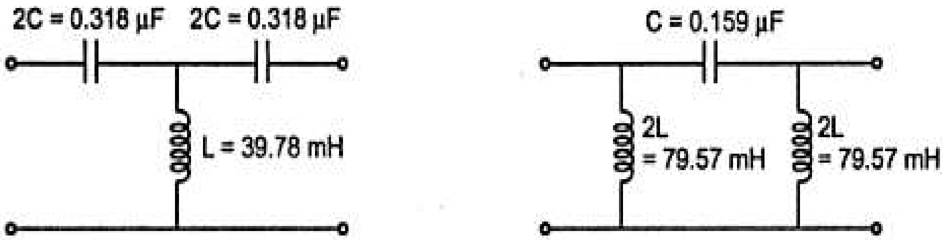
(a) T-section (b) π-section

Example 4.4 : *Design a HPF, T and π section to work into impedance 500 Ω and have cut-off frequency of 1 kHz. For this filter calculate phase angle 'β' at frequency 1.5 kHz and attenuation 'α' in neper at frequency of 0.9 kHz.*

Solution : For prototype HPF, using design equations, we can write,

$$L = \frac{R_0}{(4\pi f_c)} = \frac{500}{(4\times\pi\times1\times10^3)} = \mathbf{39.788\ mH}$$

$$C = \frac{1}{(4\pi f_c)R_0} = \frac{1}{(4\times\pi\times1\times10^3)(500)} = \mathbf{0.159\ \mu F}$$



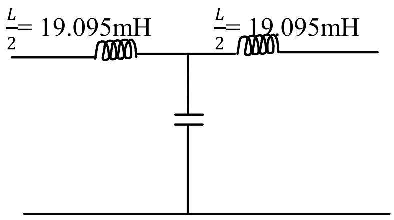
(a) 'T' section **(b) 'π' section**

Problem 4.5: Design m derived T section LPF having a cut off frequency of 5000 Hz and a design impedance of 600 ohms. The frequency of infinite attenuation is 1.25fc.

Solution:

Given $R_0 = 600\Omega$ fc = 5000Hz f∞ = 1.25fc = 1.25 X 5000 = 6250Hz

i. Design prototype T section



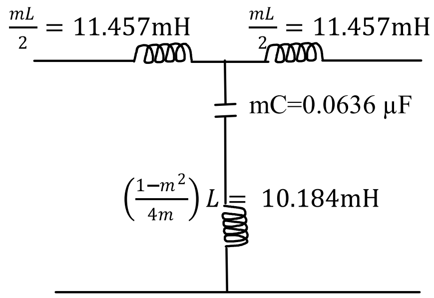
$L = \frac{R}{\pi f_c}$

C= 0.106 µF

$= \frac{600}{\pi\ X\ 5000} = 38.19$ mH

$C = \frac{1}{\pi f c R}$

$= \frac{1}{\pi\ X\ 5000\ X\ 600} = 0.106\ \mu F$



ii. Design of m derived T section

$$m = \sqrt{1 - \left(\frac{fc}{f\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{5000}{6250}\right)^2}$$

= 0.6

$$\frac{\text{mL}}{2} = \frac{0.6 \text{ X } 38.19\text{x}10^{-3}}{2} = 11.457\text{mH}$$

$$\text{mC} = 0.6 \text{ X } 38.19\text{x}10^{-3} = 0.0636\ \mu\text{F}$$

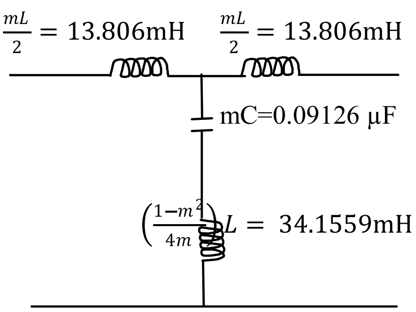
$$\left(\frac{1 - m^2}{4m}\right) L = \left(\frac{1 - 0.6^2}{4 X 0.6}\right) 38.19\text{x}10^{-3}$$

= 10.184m H

Problem 4.6: Design a LPF with cut off frequency of 2600 Hz to match 550 ohms. Use m derived section with infinite attenuation at 2850 Hz.

Solution:Given R_0 = 550Ω fc = 2600Hz f∞ =2850Hz

i. Design prototype T section



$$\text{L} = \frac{R}{\pi f_c}$$

$$= \frac{550}{\pi \ X\ 2600} = 67.33 \text{ mH}$$

$$\text{C} = \frac{1}{\pi \text{fcR}}$$

$$= \frac{1}{\pi \text{ X } 2600 \text{ X } 550} = 0.2226\ \mu\text{F}$$

ii. Design of m derived T section

$$m = \sqrt{1 - \left(\frac{\text{fc}}{\text{f}\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{2600}{2850}\right)^2}$$

= 0.41

$$\frac{\text{mL}}{2} = \frac{0.41 \text{ X } 67.334\text{x}10^{-3}}{2} = 13.8036\text{mH}$$

$$\text{mC} = 0.41 \text{ X } 0.2226\text{x}10^{-6} = 0.091266\ \mu\text{F}$$

$$\left(\frac{1 - m^2}{4m}\right) L = \left(\frac{1 - 0.41^2}{4 X 0.41}\right) 67.334\text{x}10^{-3}$$

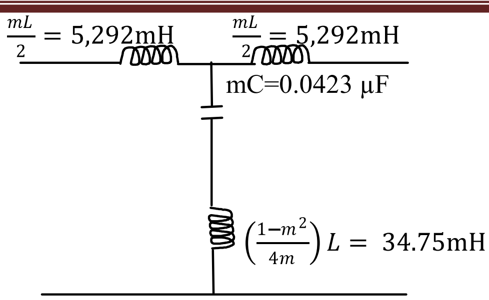
= 34.1559m H

Problem 4.7: Design m derived T type LPF to work into load of 500Ω with cut off frequency of 4KHz and peak attenuation at 4.15KHz.

Solution:

Given R_0 = 500Ω fc = 4KHz f∞ = 4.15KHz

i. Design prototype T section



$L = \frac{R}{\pi f_c}$

$= \frac{500}{\pi \ X\ 4000} = 39.788$ mH

$C = \frac{1}{\pi fcR}$

$= \frac{1}{\pi\ X\ 4000\ X\ 500} = 0.1591$ µF

Design of m derived T section

$$m = \sqrt{1 - \left(\frac{fc}{f\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{4000}{4150}\right)^2}$$

= 0.266

$$\frac{mL}{2} = \frac{0.266\ X\ 39.788x10^{-3}}{2} = 5.292mH$$

$$mC = 0.266\ X\ 0.1591x10^{-6} = 0.0423\ \mu F$$

$$\left(\frac{1-m^2}{4m}\right)L = \left(\frac{1-0.266^2}{4\ X\ 0.266}\right) 39.78x10^{-3}$$

= 34.75m H

Example 4.8 : *Design a prototype band pass filter if design impedance $R_0 = 4\,k\Omega$ and pass band between 1.25 kHz and 2 kHz.*

Solution : Given $R_0 = 4\,k\Omega$, f_1 = 1.25 kHz, f_2 = 2 kHz.

Using design equations for prototype band pass filter, we get

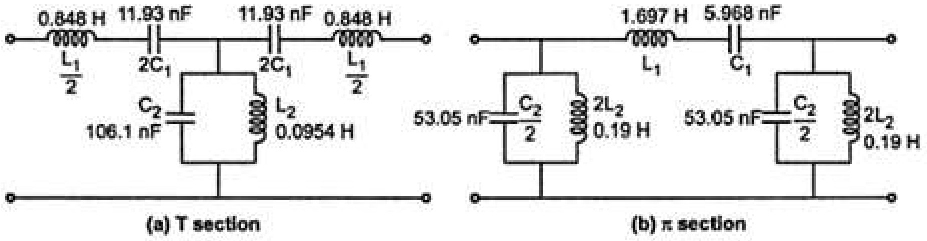
$$L_1 = \frac{R_0}{\pi(f_2-f_1)} = \frac{4000}{\pi\,(2000-1250)} = 1.697\ H$$

$$C_1 = \frac{(f_2-f_1)}{4\pi R_0\ (f_1\,f_2)} = \frac{(2000-1250)}{4\times\pi\times4000\times(1250\times2000)} = 5.968\ nF$$

$$L_2 = \frac{(f_2-f_1)\,R_0}{4\pi f_1\,f_2} = \frac{(2000-1250)\ (4000)}{4\times\pi\times1250\times2000} = 0.0954\ H$$

$$C_2 = \frac{1}{\pi R_0\ (f_2-f_1)} = \frac{1}{\pi\times4000\times(2000-1250)} = 106.1\ nF$$

The prototype band pass filter 'T' and 'π' sections are as shown in the Fig.



(a) T section (b) π section

Example 4.9 : *Design a constant k band pass filter section to be terminated in 600 Ω resistance having cut-off frequencies of 2 kHz and 5 kHz.*

Solution : Given $R_0 = 600\ \Omega$, $f_1 = 2$ kHz, $f_2 = 5$ kHz

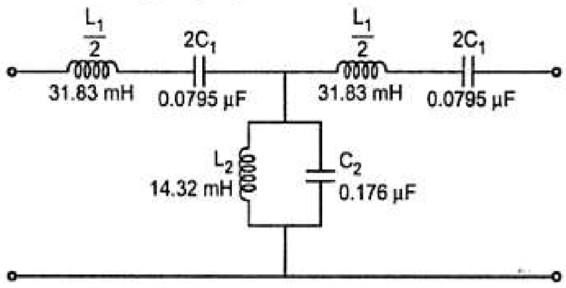
For band pass filter, using design equations, we can get values of elements in the series and shunt arm as follows.

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{600}{\pi(5000-2000)} = 63.66 \text{ mH}$$

$$C_1 = \frac{(f_2 - f_1)}{(4\pi f_1 f_2) R_0} = \frac{(5000-2000)}{(4\times\pi\times 2000\times 5000)\,600} = 0.0398\ \mu\text{F}$$

$$L_2 = \frac{(f_2 - f_1) R_0}{(4\pi f_1 f_2)} = \frac{(5000-2000)\,600}{(4\times\pi\times 2000\times 5000)} = 14.32 \text{ mH}$$

$$C_2 = \frac{1}{\pi(f_2 - f_1) R_0} = \frac{1}{\pi(5000-2000)\,600} = 0.176\ \mu\text{F}$$



Example 4.10 : *Design a band pass filter to operate into input and output resistance of 100 Ω and have a pass band between 4.8 kHz and 5.2 kHz.*

Solution : Given $R_0 = 100\ \Omega$, $f_1 = 4.8$ kHz, $f_2 = 5.2$ kHz

For band pass filter, using design equations we can write,

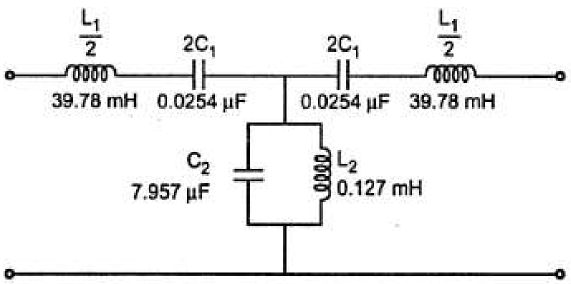
$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{100}{\pi(5200-4800)} = 79.57 \text{ mH}$$

$$C_1 = \frac{(f_2 - f_1)}{(4\pi f_1 f_2) R_0} = \frac{(5200-4800)}{(4\times\pi\times 4800\times 5200)(100)} = 0.0127\ \mu\text{F}$$

$$L_2 = \frac{R_0 (f_2 - f_1)}{4\pi f_1 f_2} = \frac{100(5200-4800)}{4\times\pi\times 4800\times 5200} = 0.127 \text{ mH}$$

$$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)} = \frac{1}{\pi\times 100\times(5200-4800)} = 7.957\ \mu\text{F}$$

The prototype band pass filter T section is as shown in the



Example 4.12 : *Design a prototype band elimination filter sections if design impedance is 400 Ω and cut-off frequencies are 1250 Hz and 2000 Hz.*

Solution : Given $R_0 = 400\ \Omega$, $f_1 = 1.25$ kHz, $f_2 = 2$ kHz

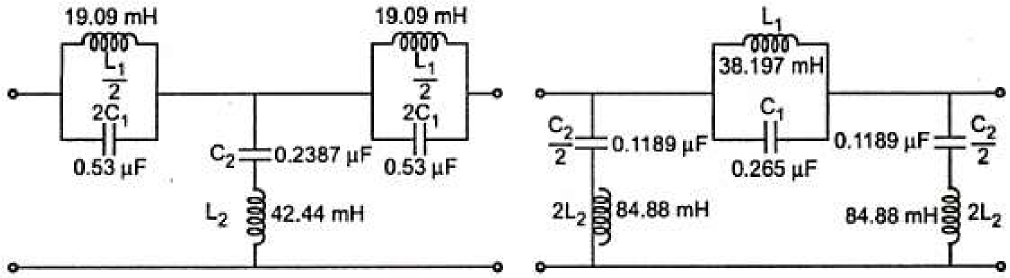
Using design equations for prototype band elimination filter, we can write,

$$L_1 = \frac{R_0 (f_2 - f_1)}{\pi f_1 f_2} = \frac{400(2000-1250)}{\pi(1250)(2000)} = 38.197 \text{ mH}$$

$$C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)} = \frac{1}{4\times\pi\times 400\times(2000-1250)} = 0.265\ \mu\text{F}$$

$$L_2 = \frac{R_0}{4\pi(f_2 - f_1)} = \frac{400}{4\times\pi\times(2000-1250)} = 42.44 \text{ mH}$$

$$C_2 = \frac{(f_2 - f_1)}{\pi R_0 (f_1 f_2)} = \frac{(2000-1250)}{\pi\times 400\times(1250)(2000)} = 0.2387\ \mu\text{F}$$



(a) 'T' type section (b) 'π' type section

Example 4.13 : *Design m-derived T type low pass filter to work into load of 500 Ω and cut-off frequency at 4 kHz and peak attenuation at 4.5 kHz.*

Solution : Given $R_0 = 500\ \Omega$, $f_c = 4$ kHz, $f_\infty = 4.5$ kHz

Before designing m-derived section, we have to design prototype section first.

Using design equations, values of L and C are given by

$$L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 4000} = 39.78 \text{ mH}$$

$$C = \frac{1}{(\pi f_c) R_0} = \frac{1}{\pi \times 4000 \times 500} = 0.1591\ \mu\text{F}$$

In m-derived low pass filter, value of m is given by,

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

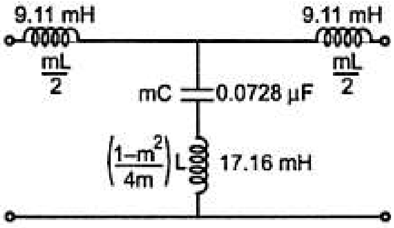
$$\therefore \quad m = \sqrt{1 - \left(\frac{4000}{4500}\right)^2}$$

$$\therefore \quad m = 0.458$$

$$\frac{mL}{2} = \frac{(0.458)(39.78 \times 10^{-3})}{2} = 9.109 \text{ mH} \approx \mathbf{9.11 \text{ mH}}$$

$$m\,C = (0.458)(0.1591 \times 10^{-6}) = \mathbf{0.0728\ \mu F}$$

$$\left(\frac{1-m^2}{4m}\right) L = \left[\frac{1-(0.458)^2}{4(0.458)}\right](39.78 \times 10^{-3}) = \mathbf{17.16 \text{ mH}}$$



Example 4.14 : *Design m-derived LPF having cutoff frequency of 5 kHz and impedance of 600 Ω. The frequency of infinite attenuation is 1.25 times the cutoff frequency.*

Solution : Given : R_0 = 600 Ω, f_c = 5 kHz, f_∞ = (1.25 × 5) = 6.25 kHz

$$L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 5 \times 10^3} = 38.197 \text{ mH.}$$

$$C = \frac{1}{(\pi f_c) R_0} = \frac{1}{\pi \times 5 \times 10^3 \times 600} = 0.106 \ \mu\text{f}$$

For m derived LPF m is given by,

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{5\times 10^3}{6.25\times 10^3}\right)^2}$$

$$= 0.6$$

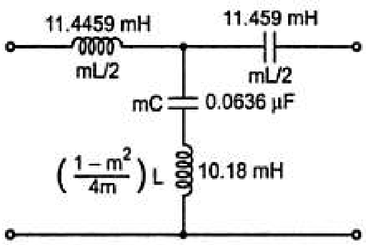
The actual values of components in series & shunt arms of m-derived filter are

$$\frac{mL}{2} = \frac{0.6\times 38.197\times 10^{-3}}{2} = 11.459 \text{ mH}$$

$$mC = 0.6\ (0.106\times 10^{-6}) = 0.0636\mu\text{F}$$

$$\left(\frac{1-m^2}{4m}\right) L = \left(\frac{1-(0.6)^2}{4(0.6)}\right)(38.197\times 10^{-3}) = 10.18 \text{ mH}$$

∴ The low pass filter is as shown below.



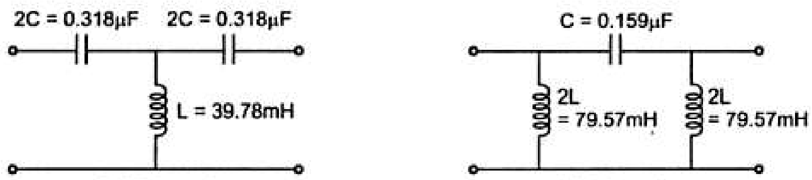
Example 4.4 : *Design a HPF, T and π section to work into impedance 500 Ω and have cut-off frequency of 1 kHz. For this filter calculate phase angle 'β' at frequency 1.5 kHz and attenuation 'α' in neper at frequency of 0.9 kHz.*

Solution : For prototype HPF, using design equations, we can write,

$$L = \frac{R_0}{(4\pi f_c)} = \frac{500}{(4\times\pi\times 1\times 10^3)} = \mathbf{39.788\ mH}$$

$$C = \frac{1}{(4\pi f_c) R_0} = \frac{1}{(4\times\pi\times 1\times 10^3)(500)} = \mathbf{0.159\ \mu F}$$

With the help of above values of L and C, we can construct T and π sections of prototype HPF as shown in the Fig. 9.60.



(a) 'T' section (b) 'π' section

To calculate β at f = 1.5 kHz

Frequency f = 1.5 kHz lies in pass band of high pass filter. In pass band, phase constant β is given by

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

$$\therefore \beta = 2\sin^{-1}\left(\frac{1\times 10^3}{1.5\times 10^3}\right)$$

$$\therefore \beta = 2\,(41.81°)$$

$$\therefore \beta = \mathbf{-83.62°}$$

∴ β is – ve because in pass band it varies from – π^c to 0.

To calculate α at f = 0.9 kHz

Frequency f = 0.9 kHz lies in stop band of high pass filter. In stop band, attenuation constant α is given by

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)$$

$$\therefore \alpha = 2\cos^{-1}\left(\frac{1\times 10^3}{0.9\times 10^3}\right)$$

$$\therefore \alpha = 2\,(0.4671)$$

$$\therefore \alpha = \mathbf{0.9342\ N}$$

Example 4.5. : *Design a composite high pass filter to operate into a load of 600 Ω and have a cut-off frequency of 1.2 kHz. The filter is to have one constant k section, one m-derived section with f_∞ = 1.1 kHz and suitable termination half section.*

Solution : Given $R_0 = 600\ \Omega$, $f_c = 1.2$ kHz, $f_\infty = 1.1$ kHz

Consider high pass filter using 'T' section throughout.

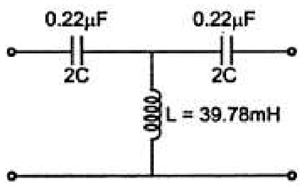
(1) Design of prototype (constant K section) :

Using design equation, values of L and C are given by,

$$L = \frac{R_0}{(4\pi f_c)} = \frac{600}{(4\times\pi\times1.2\times10^3)} = 39.78 \text{ mH}$$

$$C = \frac{1}{(4\pi f_c)R_0} = \frac{1}{(4\times\pi\times1.2\times10^3\times600)} = 0.11\ \mu\text{F}$$

Hence a prototype high pass filter T type is as shown in the Fig. 9.61.



(2) Design of m-derived high pass T section filter :

The value of m in high pass filter is given by

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

$$\therefore m = \sqrt{1-\left(\frac{1.1\times10^3}{1.2\times10^3}\right)^2}$$

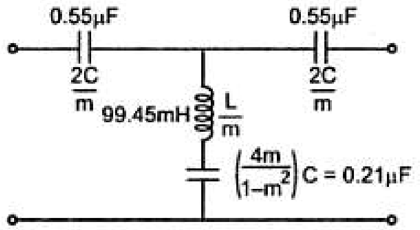
$$\therefore m = 0.399 \approx 0.4$$

The elements in the series and shunt arms of m-derived section are given as follows,

$$\frac{2C}{m} = \frac{2(0.11\times10^{-6})}{0.4} = 0.55\ \mu\text{F}$$

$$\frac{L}{m} = \frac{(39.78\times10^{-3})}{0.4} = 99.45 \text{ mH}$$

$$\left(\frac{4m}{1-m^2}\right)C = \left[\frac{4(0.4)}{1-(0.4)^2}\right]\left[0.11\times10^{-6}\right] = 0.21\ \mu\text{F}$$



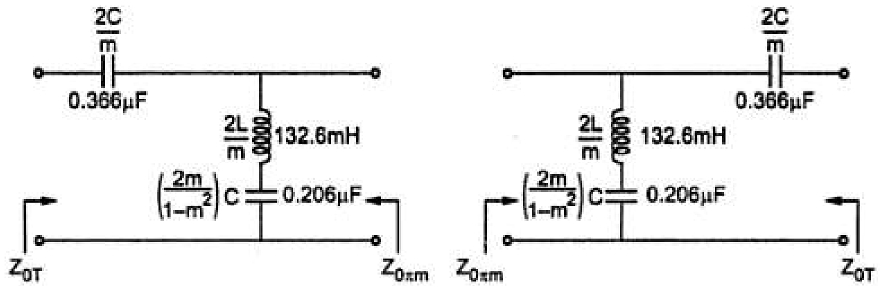
(3) Design of terminating section with m = 0.6

The elements in series and shunt arms of terminating sections are given as follows

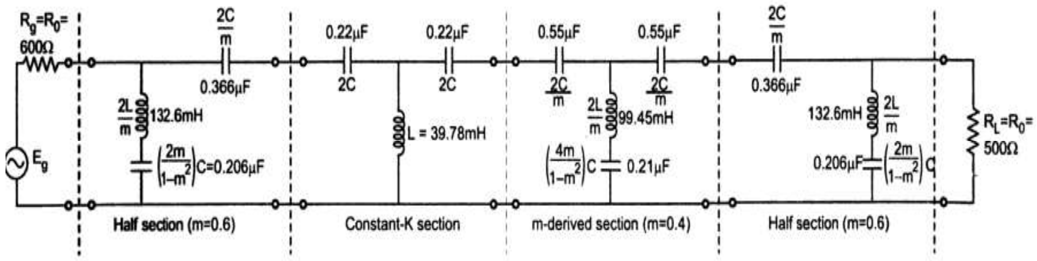
$$\frac{2C}{m} = \frac{2\,(0.11\times10^{-6})}{0.6} = 0.366\,\mu\text{ F}$$

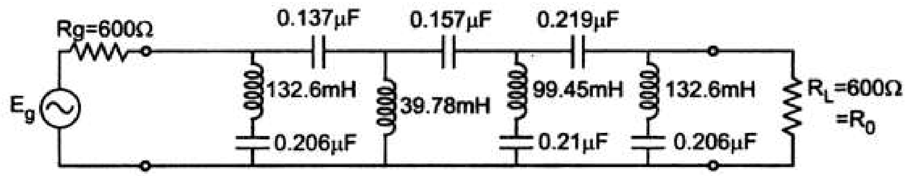
$$\frac{2L}{m} = \frac{2\,(39.78\times10^{-3})}{0.6} = 132.6\text{ mH}$$

$$\left(\frac{2m}{1-m^2}\right)C = \left[\frac{2\,(0.6)}{1-(0.6)^2}\right](0.11\times10^{-6}) = 0.206\,\mu\text{F}$$



m-derived terminating half sections





Combined Circuit

Example 4.16 : *Design an m-derived lowpass T-section filter to have termination of 600 Ω resistance. The cut-off frequency is 1.8 kHz and infinite attenuation occurs at 2 kHz.*

Solution : Given $R_0 = 600\ \Omega$, $f_c = 1.8$ kHz, $f_\infty = 2$ kHz

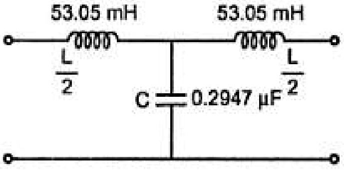
(1) Design of prototype lowpass filter section -

Using design equations,

$$L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 1.8 \times 10^3} = 106.1 \text{ mH}$$

$$C = \frac{1}{(\pi f_c) R_0} = \frac{1}{(\pi \times 1.8 \times 10^3 \times 600)} = 0.2947\ \mu\text{F}$$

Hence prototype T section of low pass filter is as shown in the Fig. .



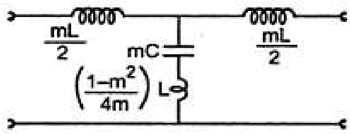
(2) Design of m-derived lowpass filter :

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1 - \left(\frac{1800}{2000}\right)^2} = 0.4358$$

Thus the elements in series and shunt arms of m-derived sections are as follows.

$$\frac{mL}{2} = \frac{(0.4358)(106.1 \times 10^{-3})}{2} = \mathbf{23.12\ mH}$$

$$mC = (0.4358)\,(0.2947 \times 10^{-6}) = \mathbf{0.1284\ \mu F}$$



$$\left(\frac{1-m^2}{4m}\right)L = \left[\frac{1-(0.4358)^2}{4(0.4358)}\right] \times (106.1 \times 10^{-3}) = \mathbf{49.305\ mH}$$

Example 4.17 : *Design a composite low pass filter consisting a prototype T-section and two terminating half sections. The cut-off frequency is to be 7 kHz and load is 500 Ω.*

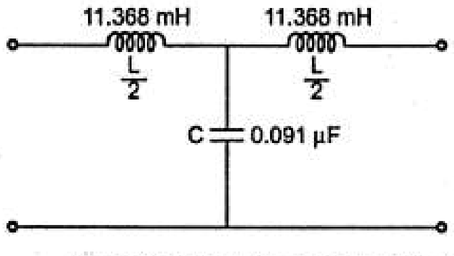
Solution : Given $R_0 = 500\ \Omega$, $f_c = 7$ kHz

1) Design of prototype (constant-k) section :

Using design equations, values of L and C are given by,

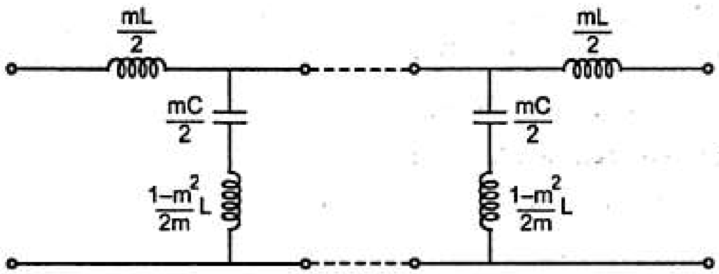
$$L = \frac{R_0}{(\pi f_c)} = \frac{500}{(\pi \times 7 \times 10^3)} = 22.736 \text{ mH}$$

$$C = \frac{1}{(\pi f_c) R_0} = \frac{1}{(\pi \times 7 \times 10^3)(500)} = 0.091\ \mu\text{F}$$



Constant-k T type low pass filter

2) Design of terminating half sections with m = 0.6 :



m-derived half sections

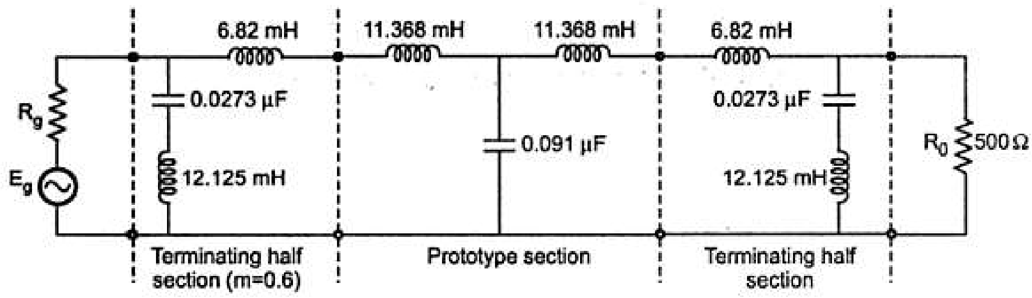
The elements in the terminating half section are given by

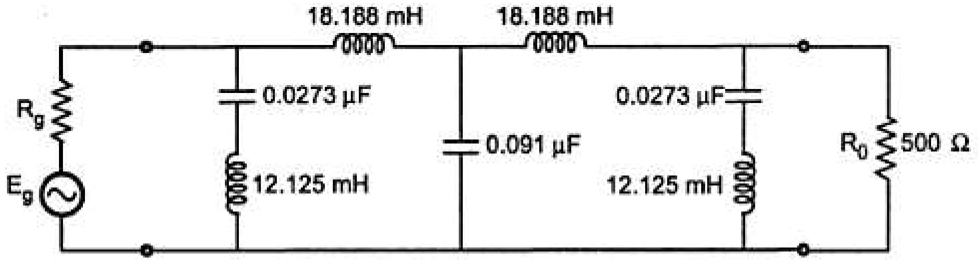
$$\frac{m\,L}{2} = \frac{(0.6)(22.736\times10^{-3})}{2} = 6.82 \text{ mH}$$

$$\frac{mC}{2} = \frac{(0.6)(0.091\times10^{-6})}{2} = 0.0273\ \mu\text{F}$$

$$\frac{1-m^2}{2m}\,L = \frac{1-(0.6)^2}{2(0.6)}\,(22.736\times10^{-3}) = 12.125 \text{ mH}$$

Hence the filter is as shown in the Fig.





SUMMARY

THE NEPER : THE DECIBEL

- The neper (Symbol: Np) is a logarithmic unit of ratio. It is not an SI unit but is accepted for use alongside the SI.
- A decibel is one-tenth of a bel, i.e. 1B=10 dB. The bel (B) is the logarithm of the ratio of two power quantities
- 1 NP = 20log10e dB= 8.685889638 dB and 1 dB = 0.115129254 Np

Neper ,N = $\ln\left|\frac{V_1}{V_2}\right| = \ln\left|\frac{I_1}{I_2}\right|$ Decibel db = 20 log $\left|\frac{P_1}{P_2}\right|$

CHARACTERISTIC IMPEDANCE:

- A two-port network makes possible the isolation of either a complete circuit or part of it and replacing it by its characteristic parameters.

Symmetrical T network : $Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$

Symmetrical π network : $Z_0\pi = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_2}{4Z_2}}}$

CURRENT AND VOLTAGE RATIOS

- $\left|\frac{v_1}{v_N}\right| = \left|\frac{I_1}{I_N}\right| = e^{\gamma N}$

PROPAGATION CONSTANT :

- The propagation constant symbol γ, for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x. $\gamma_1 + \gamma_2 + \cdots \ldots \ldots \ldots \ldots \gamma_N = \ln\frac{I_1}{I_2}$

PROPERTIES OF SYMMETRICAL NETWORK:

$$Z_{in} = Z_0 \left[\frac{Z_0 \sinh\gamma + Z_R \cosh\gamma}{Z_R \sinh\gamma + Z_0 \cosh\gamma}\right]$$

$$Z_0 = \sqrt{Z_{0c}.Z_{sc}}$$

$$\tanh\gamma = \sqrt{\frac{Z_{sc}}{Z_{0c}}}$$

FILTER FUNDAMENTALS:

$Z_1 / 4Z_2$	$-\infty\ to\ -1$	-1 to 0	0 to∞
Band	Stop	Pass	Stop
α	$2\ \sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	0	$2\ \cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$
β	$2n\pi$	$2\ \sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	$(2n-1)\pi$

CONSTANT K FILTER: $Z_1Z_2 = K^2$

Filter	f_c	$\sinh{}^{\gamma}/_{2}$	Z_0	Capacitance, C	Inductance, L
LPF	$\frac{1}{\pi\sqrt{LC}}$	$j\,{}^{f}/_{f_c}$	$RK\sqrt{\left({}^{f}/_{f_c}\right)^2}$	$\frac{1}{\pi f_c R}$	$\frac{R}{\pi f_c}$
HPF	$\frac{1}{4\pi\sqrt{LC}}$	$j\,{}^{f_c}/_{F}$	$RK\sqrt{1-\left({}^{f}/_{f_c}\right)^2}$	$\frac{1}{4\pi R f_c}$	$\frac{R}{4\pi f_c}$

Filter	F_0	L_1	C_1	L_2	C_2
BPF	F_1F_2	$\frac{R}{\pi\ (F_1F_2)}$	$\frac{1}{4\pi R}\left(\frac{F_2-F_1}{F_1F_2}\right)$	$\frac{R}{4\pi}\left(\frac{F_2-F_1}{F_1F_2}\right)$	$\frac{1}{\pi R(F_1 - F_2)}$
BEF	$\sqrt{F_1F_2}$	$\frac{R}{4\pi}\left(\frac{F_2-F_1}{F_1F_2}\right)$	$\frac{1}{4\pi R(F_1 - F_2)}$	$\frac{1}{4\pi R(F_2 - F_1)}$	$\frac{1}{\pi R}\left(\frac{F_2-F_1}{F_1F_2}\right)$

M DERIVED T SECTION : $Z_1^{'} = mZ_1$

$$Z_2^{'} = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$$

For LPF f∞ $= \frac{fc}{\sqrt{1-m^2}}$ For HPF

f∞$=f_c\sqrt{1-m^2}$

m = $\sqrt{1-\frac{fc^2}{f\infty^2}}$ m = $\sqrt{1-\frac{f\infty^2}{fc^2}}$

m derived π section: $Z_2^{'} = \frac{Z_2}{m}$

$$Z_1^{'} = \frac{1}{\frac{1}{mZ_1}} + \frac{1-m^2}{4mZ_2}$$

CONSTANT K FILTERS

- Constant k filters, also k-type filters, are a type of electronic filter designed using the image method. They are the original and simplest filters produced by this methodology and consist of a ladder network of identical sections of passive components

FILTER PERFORMANCE

- The laboratory filters are assembled in accordance with the design of the circuit. The inductor used were toriods on compressed molybdenum permally

PART A

1. Define neper & bel

Neper :- It is defined as N nepers = ln $\left|\frac{V_1}{V_2}\right|$ = ln $\left|\frac{I_1}{I_2}\right|$. Also It is defined as the natural algorithm of input voltage or current to the output voltage or current.

Bel :- The bel is defined as the logarithm of a power ratio. Number of bels = log $\frac{P_1}{P_2}$

2. Define decibel.

Decibel: It is the 10 times of common logarithm of ratio of input power to output power.

$$D = 10 \log \left|\frac{P_1}{P_2}\right|$$

Where P1 = input Power P2 = output power

1 NP = 20log10e dB= 8.685889638 dB and 1 dB = 0.115129254 Np

3. What is filter?

Filter: - It is the electronics device which is designed to separate and pass or suppress a group of signal through a mixer of signals. And it also passes freely a desired band of frequency. While almost suppressed other band of frequency.

4. What are the types of filter?

Types of filter

a. Active filters: They contains transistor, inductors and op-amp.

b. Passive filters: They contains resistor, capacitor.

5. What is symmetrical networks?

When $Z_1 = Z_2$ or the two series arms of a T network are equal, or Za = Z0 and the shunt arms of a π network are equal the network works are said to be symmetrical.

6. Write the equivalent value of neper to decibel?

1 Neper = 8.686 db or N = 0.115D

7. What are the parameter of filter?

The Parameter of filter are

i. Characteristics impedance (Z_0). ii. Passband. iii. Stopband. iv. Cut off Frequency. v. Attenuation.

8. How will construct band pass filter by using LPF & HPF?