## DSP APPLICATIONS

Multirate signal processing: Decimation, Interpolation, Sampling rate conversion by a rational factor – Adaptive Filters: Introduction, Applications of adaptive filtering to equalization.

### 6.1.Introduction to Multistage signal processing:

Multirate digital signal processing is required in digital systems where more than one sampling rate is required. Different sampling rates can be obtained using an up sampler and down sampler. The basic operations in multirate processing to achieve this are decimation and Interpolation . Decimation is the process of reducing the sampling rate by a factor M (down sampling by M) and Interpolation is the process of increasing the sampling rate by an integer factor L (up sampling by L) is called interpolation.

In digital transmission system like teletype, facsimile, low bit rate speech where data has to be handled in different rates. Multirate signal processing is used. Multirate signal processing find its application in (i)Sub-band coding (ii)Implementation of narrow band filter (iii)digital filter banks (iv)Qudrature mirror filters.

There are various areas in which multirate signal processing is used. To list a few,
- (i) Communication System
- (ii) Speech and Audio processing systems
- (iii) Antenna systems, and
- (iv) Radar systems

The various advantages of multirate signal processing are
- (i) Computational requirements is less
- (ii) Storage for filter coefficients is less
- (iii) Finite arithmetic effects are less
- (iv) Filter order required in multirate application is low, and
- (v) Sensitivity to filter coefficient lengths is less

### 6.2.Sampling rate conversion :

The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Here the sequence x(n) which is got from sampling continuous time signal x(t) with a period T, to another sequence y(k) obtained from sampling x(t) with a period T'. Fig : shows the reconstruction of the signal with a D/A convertor , Low pass filter and resampler with sampling period T'.

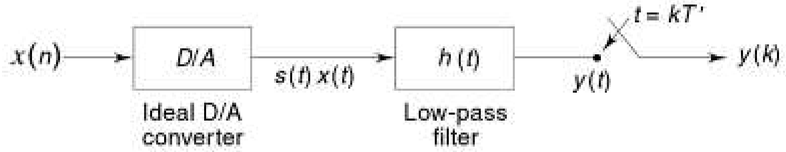



Fig:6.1 Conversion of a sequence x(n) to another sequence y(k)

The major advantage of sampling rate conversion in the digital domain is that the signal distortion introduced by the D/A convertor in the signal reconstruction and the quantization effects in the D/A convertor are avoided.

**6.3.Decimation : (Decimation by an Integer Factor)**

The process of reducing the sampling rate of a signal is called decimation (sampling rate compression) . let M be the Integer Sampling rate reduction factor for the signal x(n).

$$\frac{T'}{T} = M$$

The new sampling rate F' becomes

$$F' = \frac{1}{T'} = \frac{1}{MT}$$

$$F' = \frac{F}{M}$$

Let the signal x(n) be a full band signal,with non zero values in he frequency range –F/2 to F/2,where $\omega = 2\,\pi f T$

$$\left|X(e^{j\pi\omega})\right| \neq 0, |\omega| = |2\pi f T| \leq \frac{2\pi FT}{2}$$

To avoid aliasing in the decimation process x(n) is filtered with a digital low pass filter . The spectral response of LPF is,

$$H(e^{j\omega}) = \begin{cases} 1, |\omega| \leq \pi/M \\ 0, otherwise \end{cases}$$

The down sampler cab be represented as given below

y(n) = ↓M x(n)

If M=2 then,

y(n) = ↓2 x(n)

The usual notation is y(n) = (↓2)x(n)

The image signal is due to aliasing effect .In case of decimation by M there will be M-1 additional image of the input spectrum. Thus the output of the filter is the convolution of input signal x(n) and the filter impulse response h(n)

$$\omega(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

And W($e^{j\omega}$) is the spectrum of w(n). The output of filter ω(n) is down sampled by the factor M, The decimated signal y(m) is

$$y(m) = W(Mm) \qquad (1)$$

The entire process is represented in given figure.

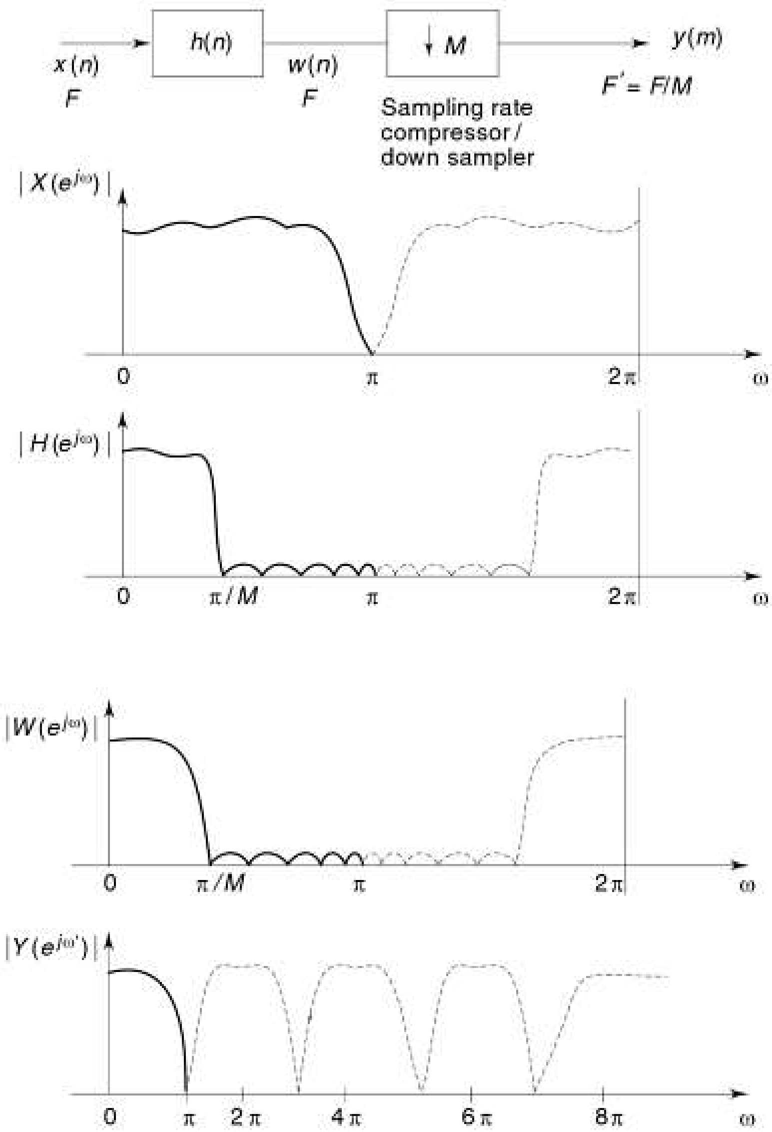



Fig 6.2: Decimation of x(n) by a factor M

$$y(m) = \sum_{k=-\infty}^{\infty} h(k)x(Mm-k)$$

or $$y(m) = \sum_{n=-\infty}^{\infty} h(Mm-n)x(n)$$

The decimator is also known as sub sampler, down sampler or under sampler. In the practical case where a non-ideal low pass filter is used, the output signal y(m) will not be a perfect one. The frequency domain characteristic of the output sequence y(m) can be obtained by relating input and output spectrum. It is convenient to define the intermediate term say (n).Consider the signal $\overline{w}$(n) is defined by

$$\varpi(n) = \begin{cases} \omega(n), n = 0, \pm M, \pm 2M, \dots \\ o \qquad , otherwise \end{cases} \quad (2)$$

Inverse Fourier transform of c(n) is $\varpi(n)$=$\omega(n)$ and in other cases it is zero.The general representation can be

$$\overline{\omega}= \omega(n)\left\{(\frac{1}{m}) \sum_{l=0}^{M-1} e^{\frac{j2\pi ln}{M}}\right\} ,-\infty < n < \infty$$

The term multiplied to $\omega(n)$ is the Fourier series representation of a periodic impulse train.Now ,the output signal becomes,

$$\text{y(m)}= \overline{\varpi}(Mm) = \omega(Mm)$$

The z-transform of the output signal is given by

$$y(z) = \sum_{m=-\infty}^{\infty} y(m)z^{-m}$$

$$y(z) = \sum_{m=-\infty}^{\infty} \varpi(mM)z^{-m}$$

Let mM=p , m=p/m

$$y(z) = \sum_{p=-\infty}^{\infty} \varpi(p)z^{-p/M}$$

Replace p⟶m

$$y(z) = \sum_{m=-\infty}^{\infty} \varpi(m)z^{-m/M}$$

Substituting the value for $\varpi(m), Y(z) becomes,$

$$y(z) = \sum_{m=-\infty}^{\infty} \omega(m)\left(\sum_{l=0}^{M-1} 1/Me^{j2\pi lm/M}\right)z^{-m/M}$$

$$\text{y(z)} = 1/\text{M}\sum_{l=0}^{M-1} w(e^{-j2\pi k/M}.z^{1/M})$$

But w(z)=H(z)X(z)

$$\text{Y(z)} = 1/\text{M} \sum_{l=0}^{M-1} H(e^{-j2\pi l/M}.z^{1/M})\, X(e^{-j2\pi k/M} z^{1/M})$$

When evaluated on a unit circle ,z=$e^{j\omega}$

$$Y(z) = 1/M \sum_{l=0}^{M-1} X(e^{-j2\pi k/M}.e^{j\omega/m})$$

$$= 1/M \sum_{l=0}^{M-1} X(e^{j(\omega-2\pi k)/M})$$

The relationship between the orginal and reduced sampling rate is $\omega_y$=M$\omega_y$

Ex M=2

$$Y(e^{j\omega}) = 1/M \sum_{l=0}^{M-1} X(e^{j(\omega-2\pi k)/2})$$

Hence second term is 2π times left shifted from first term , hence,

$$X(e^{j(\omega-2\pi)/2}) = X(e^{j\omega/2})$$

$$Y(e^{j\omega}) = 1/2[\ X(e^{j\omega/2}) + X(-e^{j\omega/2})\ ]$$

The spectrum for X($e^{jw'}$) and Y($e^{jw'}$) for a down sampler(factor two) with aliasing is shown in given below.

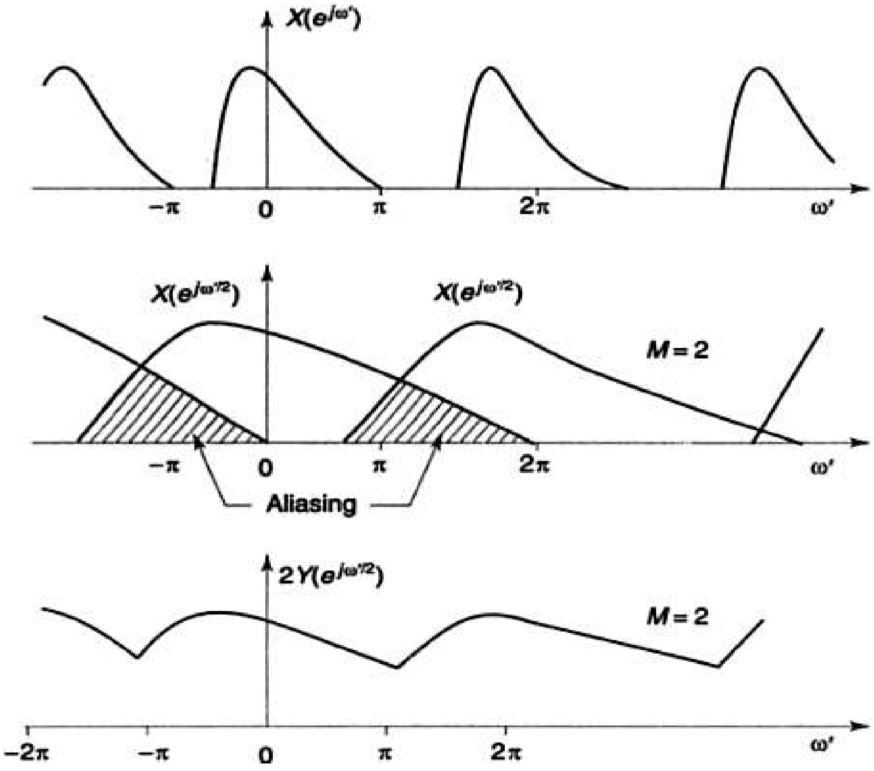



Fig 6.3: Aliasing Effect caused by down sampling

The original shape of X(ejw') is lost when x(n) is down sampled.This results in aliasing which is because of under sampling or down sampling.There will be no aliasing if

$$X(ejw')=0, \text{for} |w| \geq \frac{\pi}{2}$$

Decimation without aliasing is shown in given below.

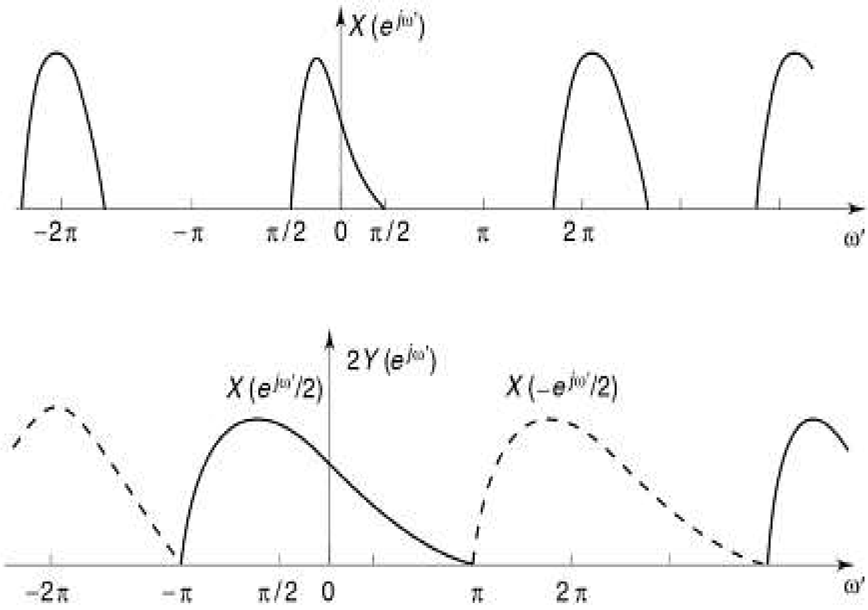



Fig 6.4: Down sampling without aliasing

1.Obtain the decimated signal y(n) by a factor three from the input signal x(n)

Solution: The decimated signal is given by

$$Y(n)=x(Mn), \text{where } M=3$$

The input and output signals are shown in figure.

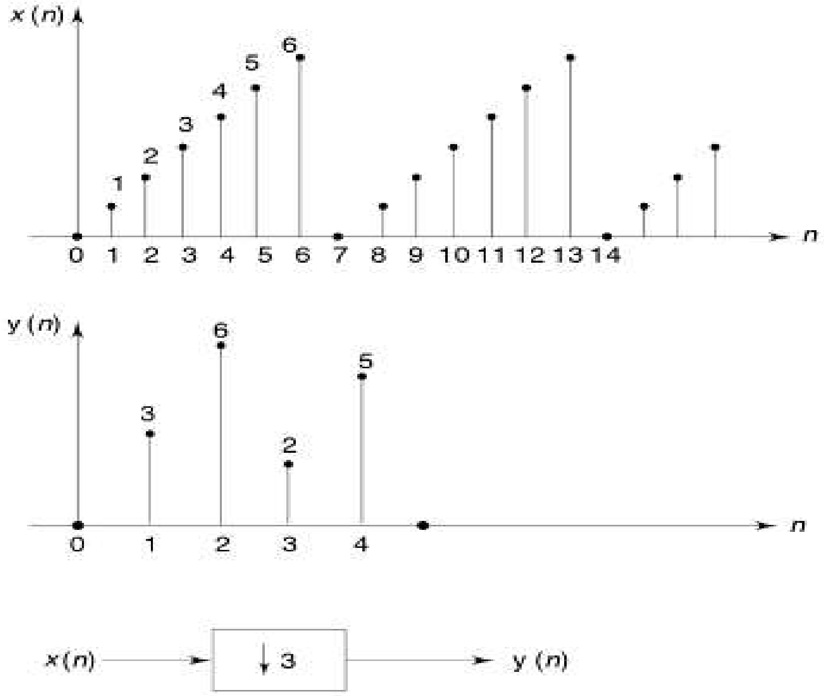



Fig 6.5 Decimation process with factor three

## 6.4.Interpolation : (up sampling by an integer factor L)

The process of increasing the sampling rate of a signal is interpolation (sampling rate expansion).Let L be an integer interpolating factor of the signal x(n),then

$$\frac{T'}{T} = \frac{1}{L}$$

The sampling rate is given by ,

$$F' = \frac{1}{L}$$

$$= \frac{1}{T/L}$$

$$= \frac{L}{T}$$

$$F' = LF$$

The up sampler is represented by the following representation,

x(n) = ↑L y(n)

If L=2 then,

x(n) = ↑2 y(n)

The usual notation is y(n) = (↑2) x(n)

Interpolation of a signal x(n) by a factor L refers to the process of interpolating L-1 samples between each pair of samples of x(n).

The signal w(m) is got by interpolating L-1 samples between each pair of the samles of x(n).

$$w(m) = \begin{cases} x(m/L), m = 0, \pm L, \pm 2l, \ldots. \\ o \quad , otherwise \end{cases}$$

The Z transform ω(n) is given by

$$W(z) = \sum_{n=-\infty}^{\infty} w(m) z^{-m}$$

$$W(z) = \sum_{n=-\infty}^{\infty} x(m/L) z^{-m}$$

Let m/L=p ; m=pL

$$W(z) = \sum_{n=-\infty}^{\infty} x(p) z^{-Lp}$$

Replace p=m

$$W(z) = \sum_{n=-\infty}^{\infty} x(m) z^{-Lm}$$

$$W(z) = X(z^L)$$

To remove the images an anti-imaging (low pass) filter is used. The ideal characteristic of the low pass filter is given by,

$$H(e^{j\omega}) = \begin{cases} G, |\omega'| \leq \pi/L \\ o, otherwise \end{cases}$$

Where G is the gain factor of the filter and it should be L in the passband. The frequency response of the output signal is given by,

$$Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega L})$$

$$Y(e^{j\omega}) = \begin{cases} GX(e^{j\omega L}), |\omega| \leq \pi/L \\ o, otherwise \end{cases}$$

When evaluated on a unit circle Z=$e^{j\omega}$

$$W(e^{j\omega}) = X(e^{j\omega} L)$$

The output signal y(m) is given by

$$y(m) = h(m) * w(m)$$

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k) \omega(k)$$

$$y(m) = \sum_{k=-\alpha}^{\alpha} h(m-k)x(k/L)$$

k/L is an integer

The given figure shows the entire process of interpolation by a factor L.The interpolation process is also known as up sampling.

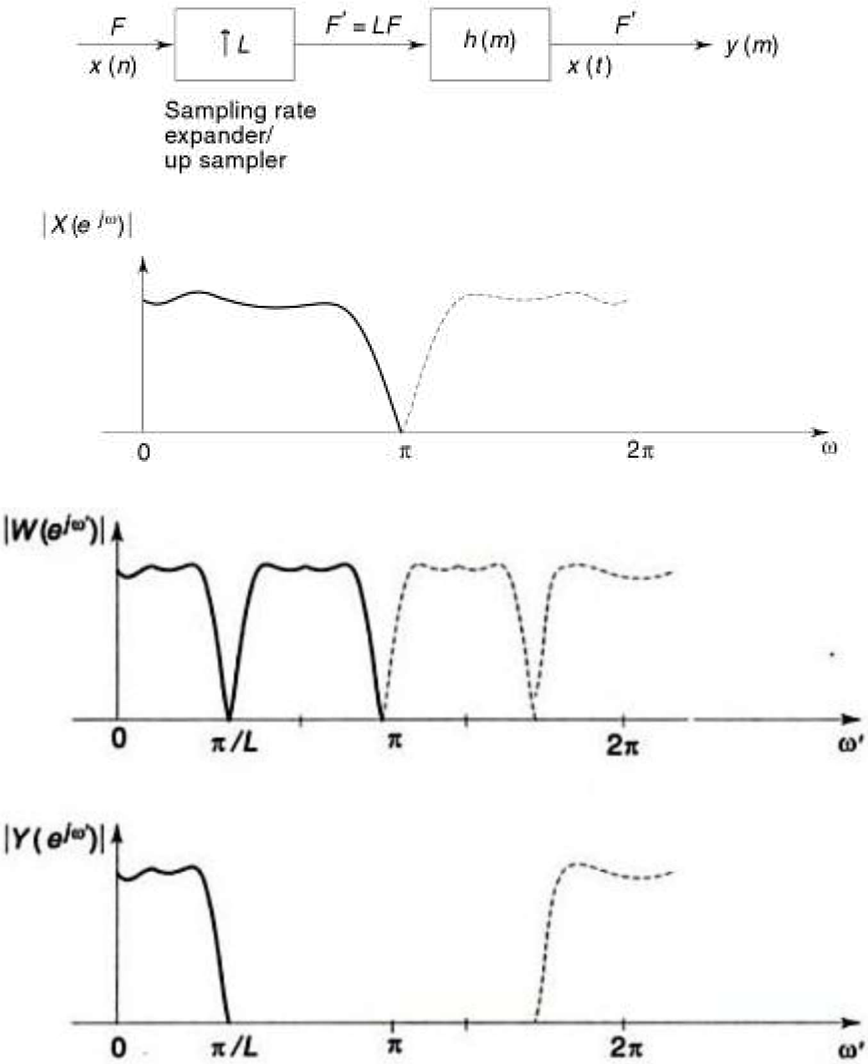



Fig 6.6: Interpolation of x(n) by a factor L

Remarks :

1. No information lost when a signal is up sampled.
2. The up sampler is a linear but not a time – invariant.

3. The up - sampler introduces spectral images.
4. Information is last when a signal is down sampled.
5. The down-sampler is a linear but not a time – invariant system.
6. The down - sampler causes aliasing.

2.Obtain the two-fold expanded signal y(n) of the input signal x(n)= $\begin{cases} n, n > 0 \\ 0, otherwise \end{cases}$

Solution:

The output signal y(n) is given by

$$y(n)=\begin{cases} x(\frac{n}{M}), n = samplesofM \\ 0, otherwise \end{cases}$$

where M=2.

x(n)=0,1,2,3,……….

y(n)=0,0,1,0,2,0,3,0,4………

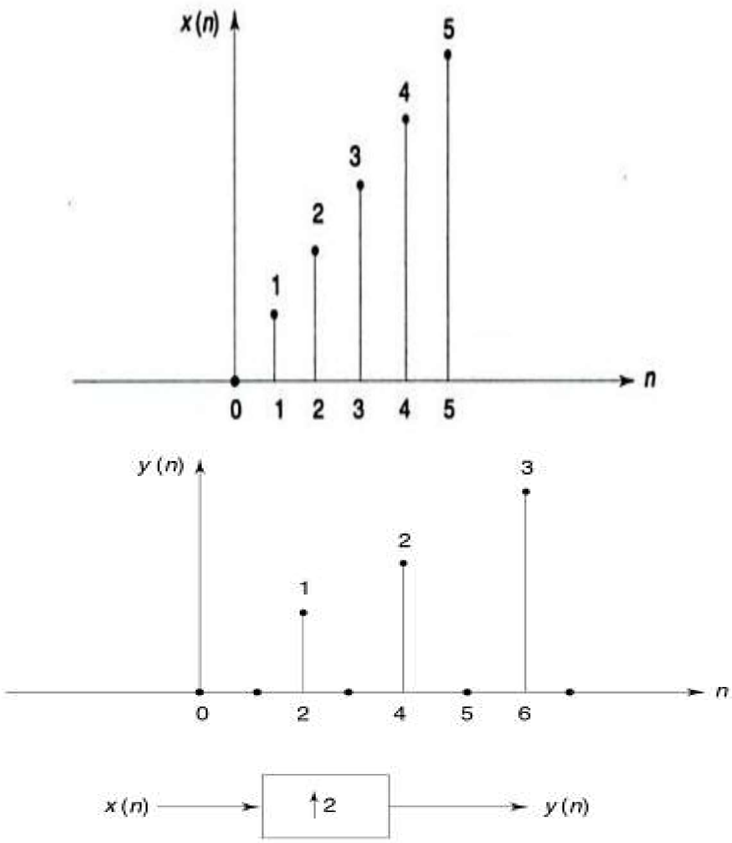



Fig 6.7: Interpolation process with a factor 2

## 6.5.Fractional Sampling Rate Alteration:

Consider the sampling rate conversion by a factor M/L

$$\frac{T'}{T} = \frac{M}{L}$$

The sampling rate F' is

$$F' = \frac{L}{M}F$$

The fractional conversion can be obtained by first increasing the sampling rate by L and then decreasing it by M. The interpolation process should be done before the decimation process. The given figure shows the process of fractional sampling rate conversion.

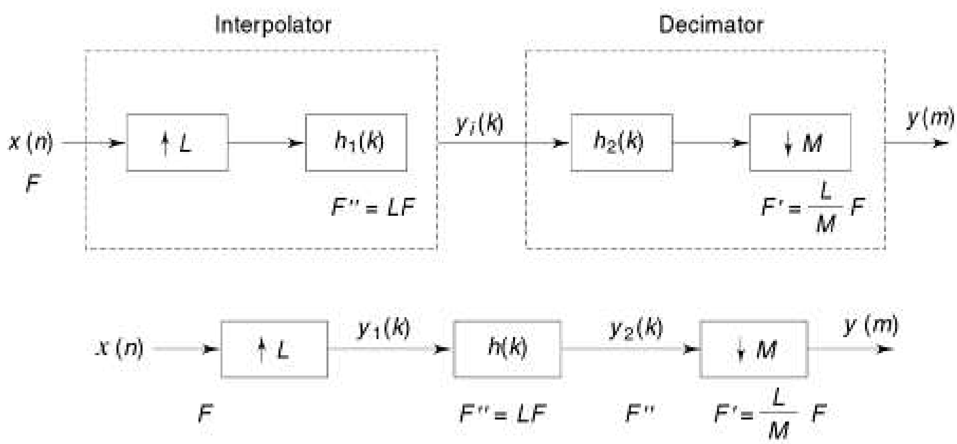



Fig 6.8: Efficient way of fractional sampling rate conversion

The filters h1(k) and h2(k) can be combined into one composite low-pass filter h(k),operating at the sampling rate LF. The characteristic of the h(k) filter should be

$$\overline{H}(e^{jw''}) = \begin{cases} L, |\omega''| \le \min(\frac{\pi}{L}, \frac{\pi}{M}) \\ 0, otherwise \end{cases}$$

Where, $\omega''$ =2 $\pi$ fT''=2 $\pi$ FT/L

Since T''=T/L

The sampling rate of the filters is F''=LF

Time domain relationships:

$$y_2(k) = \sum_{-\infty}^{\infty} h(k - rL)x(r)$$

$$y(M)=y_2(Mm)$$

Expressing y(m) in terms of x(r)

$$y(m)=\sum_{-\infty}^{\infty} h(Mm - rL)x(r)$$

Frequency –domain relationships:

The output spectrum is given by

$$Y(e^{jw'})=\frac{1}{M}\sum_{l}^{M-1} Y_2(e^{j(\omega'-2\pi l)/m})$$

Where , $Y_2(e^{jw''})=H(e^{jw''})X(e^{jw''L})$

$$Y_2(e^{jw''})=\frac{1}{M}\sum_{l=0}^{M-1} H(e^{j(w'-2\pi l/m)})X(e^{j(w'L-2\pi l/m)})$$

$$Y(e^{jw'})=\begin{cases} \frac{L}{M}X(e^{(jw'L/M), for |w'|} \le \min\left[\pi, \frac{\pi M}{L}\right] \\ 0, otherwise \end{cases}$$

**6.6.Polyphase FIR filter structure for Decimators and Interpolators:**

**6.6.1.Decimators:**

The decimator consisting of an anti – aliasing filter h(n) and a down sampler by a factor two.

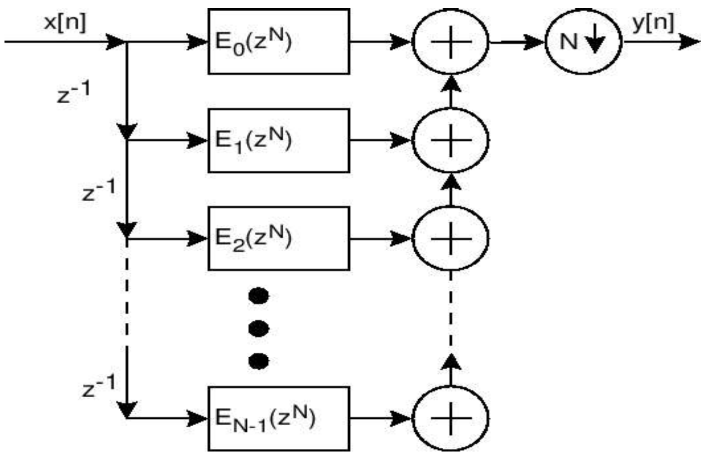



Fig 6.9: Decimation by a Factor two and its Frequency Response

Y(n) = x(n) * h(n)

$$y(n) = \sum_{k=0}^{N-1} x(k)h(n-k)$$

By down sampling y(m) = y(2n)

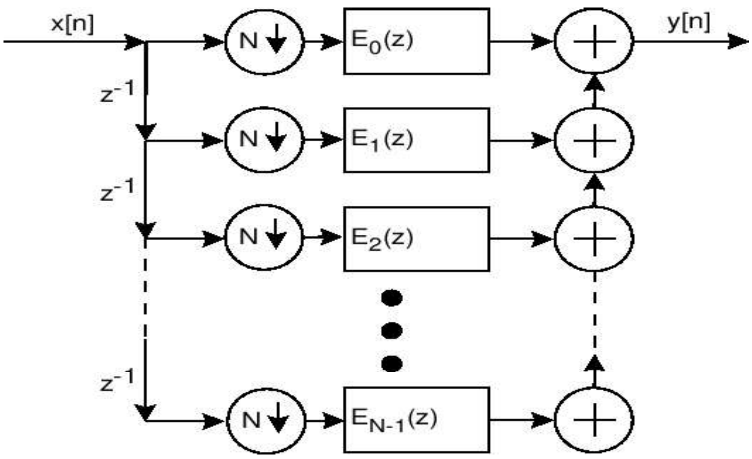



Fig 6.10: The Direct Implementation of the Decimator by a factor of Two

The above figure clearly represent the Direct implementation of FIR filter in transversal structure .This structure computes all the values of y(n).

As an alternative to the efficient transversal filter structure, polyphase filter structures can be used.The polyphase filter structures introduced here are of fundamental importance for analysis and synthesis of filter banks.

**6.6.2.Interpolators :**

Interpolators are the duals of decimators.Here down samplers and up ssamplers interchanged and input and outputs swapped. Because of this , the interpolator structures derived in this section are of similar form to the decimator structures.The interpolator consisting of an interpolation (anti - imaging) filter h(n) and up sampler by a factor two

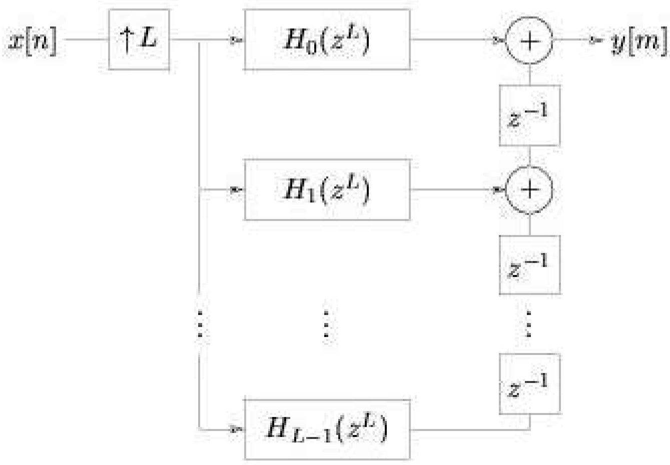



Fig 6.11: Interpoation by a Factor two and its Frequency Response

Assuming that the anti – imaging FIR filter has N coefficients , the filtering described by

y(n) = x'(n) * h(n)

y(n) $=\sum_{k=0}^{N-1} x'(k)h(n-k)$

The polyphase implementation using interpolator identities as shown below

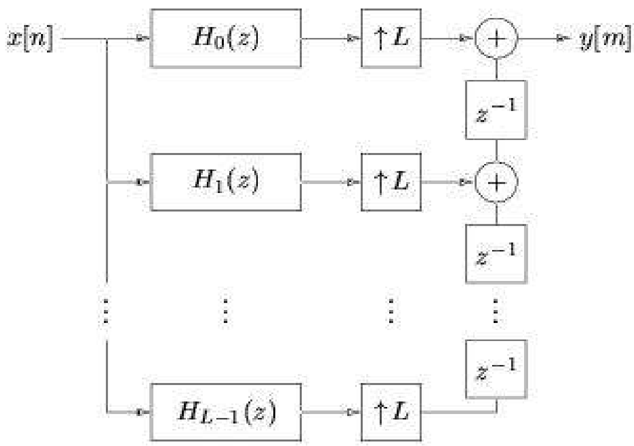



Fig 6.12: The Direct Implementation of the Interpolator by a factor of Two

**6.6.3.IIR structures for Decimators :**

The IIR filter is represented by the difference equations,

$$y(n) = \sum_{k=1}^{D} aky(n-k) + \sum_{k=1}^{D} bkx(n-k)$$

The system function for the above difference equation is given by

$$H(z) = \frac{\sum_{k=0}^{N-1} bkz^{-k}}{1 - \sum_{k=1}^{\alpha} akz^{-k}} = \frac{N(z)}{D(z)}$$

Let d = N-1 , so that the numerator and the denominator orders are same.

**6.6.4.Polyphase IIR Filter Strustures For Decimators :**

Consider the IIR transfer function H(z) = $\frac{P(z)}{D(z)}$

Express H(z) in the form $\frac{P'(z)}{D'(z^m)}$ by multiplying and dividing H(Z) with a properly chosen polynomial, apply polyphase decomposition to p'(z).

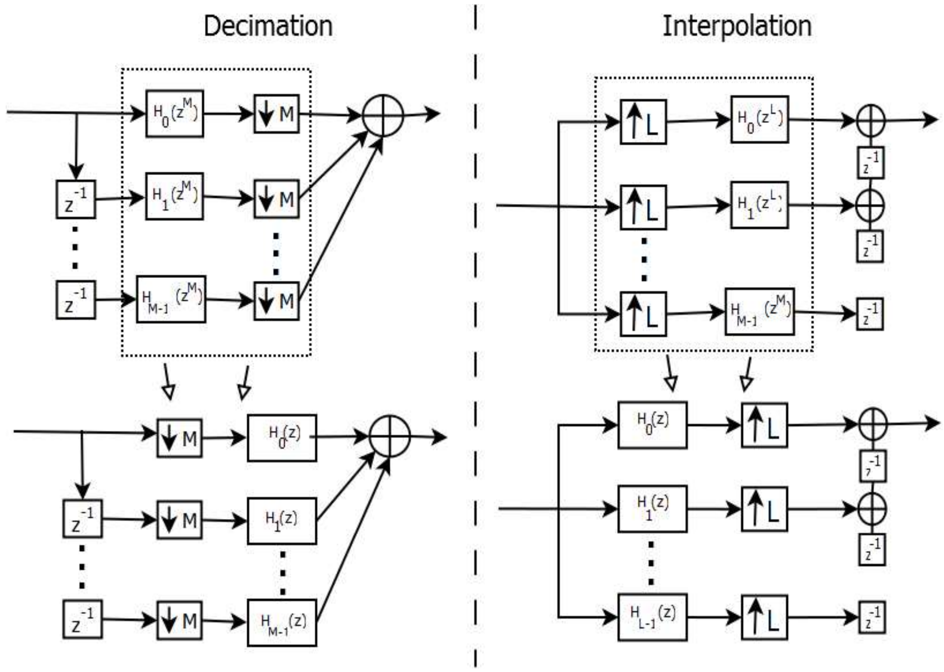



Fig 6.13: The Polyphase IIR Filter Structures for Decimators and interpolator

The above figure shows the polyphase IIR filter structure for decimators.

**6.7.Multistage implementation of sampling rate conversion:**

**(OR)**

**Multistage Decimators and Interpolators :**

In pratical applications mostly sampling rate conversion by a rational factor L/M is required. The figure represents the general structure of asystem where this conversion is used.\

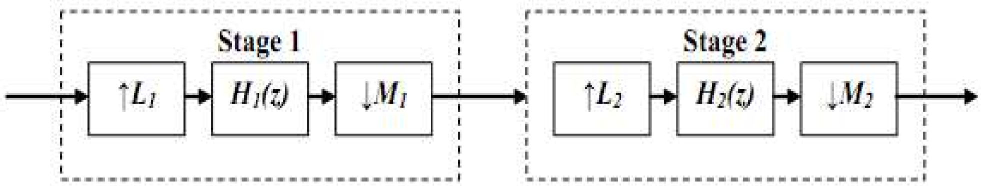



Fig 6.14: : Sampling Rate Conversion by a Rational Factor L/M

Consider a system for decimating a signal by an integer factor M. Let the input signal sampling frequency be $f_o$ then the decimated signal frequency will be $f_o/M$

The decimation factor can be factorised as.

$$M = \prod_{i=1}^{I} Mi$$

The resultant network is shown in fig

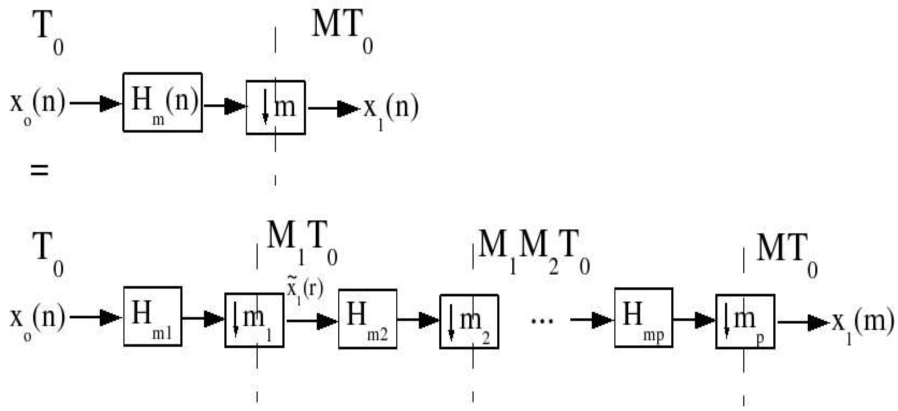



Fig 6.15:Multistage Decimator

Similarly ,the multistage interpolator is shown in fig.The 1 to L interpolator , has its interpolation factor represented by,

$$L = \prod_{i=1}^{I} Li$$

If the sampling rate alteration system is designed as a cascade system , the computational efficiency is improved significantly.

**Advantages :**

- Multistage systems requires reduced computation.
- Storage space required is less.
- Filter design problem is simple.
- Finite word length effects are less.

The demerits of the systems are that proper control structure is required in implementing the system and proper values of I should be chosen .

**6.8.Design of narrow Band Filter :**

**(or)**

**Filter Design For FIR Decimators and Interpolators :**

The various methods for the FIR filter design are given below.

1. Window method.
2. Optimal , equiripple linear phase method.
3. Half – band designs.
4. FIR interpolators design based on – time domain filter specifications.
5. Classical interpolation designs : linear and lagrangian.

Let us consider the equiripple FIR filter design . The equiripple response of the filter is shown below.

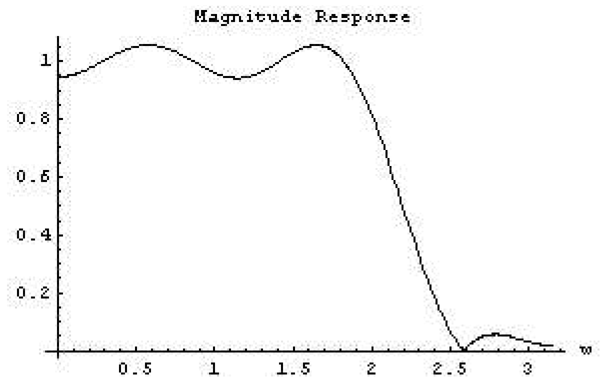



Fig : 6.16: The Equiripple Response of the Filter

A narrow band low pass filter is characterized by narrow pass band and a narrow transition band . It requires a very large number of coefficients . Due to high value of N it is susceptible to finite word length effects . In addition number of computations and memory locations required are very high . So multi rate approach of designing of low pass filter overcome this problem .

The design equation for calculating the stopband and passband frequency are described below . Let the highest frequency of the decimated signal or the total bandwidth of the interpolated signal be $\omega c \leq \pi$ , then the passband frequency is given by,

$$\omega p = \begin{cases} \dfrac{\omega c}{L} & 1\ to\ L\ interpolator \\ \dfrac{\omega c}{M} & M\ to\ 1\ decimator \\ \min\left(\dfrac{\omega c}{L}, \dfrac{\omega c}{M}\right) & conversion\ by\ L/M \end{cases}$$

The stop band frequency is given by ,

$$\omega s = \begin{cases} \dfrac{\pi}{L} & 1 \text{ to } L \text{ interpolator} \\ \dfrac{\pi}{M} & M \text{ to } 1 \text{ decimator} \\ \min\left(\dfrac{\pi}{L}, \dfrac{\pi}{M}\right) & \text{conversion by } L/M \end{cases}$$

The assumption is that there is no aliasing in the decimator or imaging in the interpolator. If aliasing is not allowed in the decimator or interpolator , then the stop band frequency is given by ,

$$\omega s = \begin{cases} \dfrac{(2\pi - \omega c)}{L} & 1 \text{ to } L \text{ interpolator} \\ \dfrac{(2\pi - \omega c)}{M} & M \text{ to } 1 \text{ decimator} \\ \min\left(\dfrac{(2\pi - \omega c)}{L}, \dfrac{(2\pi - \omega c)}{M}\right) & \text{conversion by } L/M \end{cases}$$

**6.9.Application of Multirate signal processing :**

**Two channel Quadrature Mirror Filter bank : (QMF)**

In application where sub – band filtering is is used , the following process are carried out.

1. The signal x(n) is first represented as sub – band signals Vc(n) by using an analysis filter bank.
2. The sub – band signals are processed.
3. The processed signals are compined using synthesis filter bank , to obtain the output signal y(n).

The sub – band signal are down sampled before processing .The signals are up sampled after processing .The structure used for this is known as quadrature mirror filter (QMF) bank .To list a few applications where QMF filters are used.

(i) Efficient coding of the signal x(n)
(ii) Analog voice privacy for secure telephone communication.

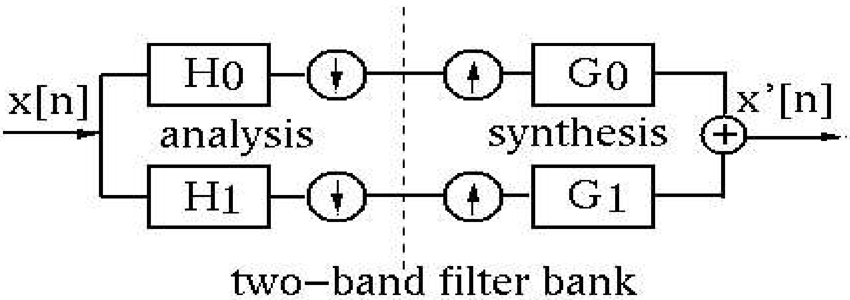



Fig 6.17: Two – channel Quadrature Mirror Filter Bank

The two channel QMF filter bank consists of two sections

1. Analysis section
2. Synthesis section

The analysis filter Ho(z) is a low pass filter and H1(z) is a high pass filter .The cut off frequency is taken as π/2 for those filters . The sub – band signals $v_k$(n) are down sampled. After down sampling these signals are processed (encoded) . In the receiving side the same signals are decoded , up – sampled and then passed through the synthesis filters Go(z) and G1(z) to get the output y(n).For perfect reconstruction the QMF filter banks should be properly selected.

**Analysis :**

The input – output relationship of the two channel QMF filter bank are given by ,

$$V_k(z) = H_k(z) + X_z$$

$$U_k(z) = [V_k(z^{1/2}) + V_k(-z^{1/2})]$$

$$\overline{V}_k(z) = U_k(z^2) \quad ,k = 0,1,$$

$$\overline{V}_k(z) = \frac{1}{2}[V_k(z) + V_k(-z)]$$

$$\overline{V}_k(z) = \frac{1}{2}[H_k(z)X(z) + H_k(-z)X(-z)]$$

The output is expressed as,

$$Y(z) = Go(z)\overline{V}_o(z)(z) + G1(z)\overline{V}_1(z)]$$

$$Y(z) = 1/2[Go(z)H_0(z)X(z) + H_0(-z)X(-z)Go(z)] + 1/2[G1(z)H_1(z)X(z) + H_1(-z)X(-z)]G1(z)]$$

$$Y(z) = 1/2X(z)[Go(z)H_0(z) + G1(z)H_1(z)] + 1/2X(-z)[H_0(-z)Go(z) + H_1(-z)G1(z)]$$

$$Y(z) = T(z)X(z) + A(z)X(-z)$$

Where T(z is the distribution transfer function given by

$$T(z) = Go(z)H_0(z) + G1(z)H_1(z)$$

And $$A(z) = H_0(-z)Go(z) + H_1(-z)G1(z)$$

In matrix form

$$Y(z) = ½ [x(z)x(-z)] \begin{bmatrix} Ho(z) & H1(z) \\ Ho(-z) & H1(-z) \end{bmatrix} \begin{bmatrix} Go(z) \\ G1(z) \end{bmatrix}$$

The matrix

$$H(z) = \begin{bmatrix} Ho(z) & H1(z) \\ Ho(-z) & H1(-z) \end{bmatrix}$$

Is referred as the aliasing component (Ac) matrix.

**Errors in QMF Filter Bank :**

The QMF filter structure is linear time varying system since the up sampler and down sampler are linear time varying. The analysis and synthesis filter banks should be properly selected to cancel the aliasing effect. To achieve this we have,

$$2A(z) = H_0(-z)Go(z) + H_1(-z)G1(z) = 0$$

One possible solution for this is,

$$Go(z) = H1(-z)$$

$$G1(z) = -Ho(-z)$$

$$Y(z) = T(z)\, X(z)$$

Where $T(z) = ½ [H_o(z)H_1(-z) + H_1(z)H_o(-z)$

Let analysed over a unit circle ,

$$Y(e^{j\omega}) = T(e^{j\omega})\, X(e^{j\omega})$$

$$T(e^{j\omega}) = |T(e^{j\omega})|\, e^{j\Phi\omega}$$

Hence 10 will be

$$Y(e^{j\omega}) = |T(e^{j\omega})|\, e^{j\Phi\omega}\, X(e^{j\omega})$$

$$|T(e^{j\omega})| = d \ddagger 0 \text{ where d is constant}$$

If $|T(e^{j\omega})|$ is constant for all ω , then we do not have any amplitude distortion . This condition is satisfied when $T(e^{j\omega})$ is an all pass filter . in the same way if $T(e^{j\omega})$ have linear phase there is no phase distortion . Therefore we need $T(e^{j\omega})$ to be a linear phase all pass filter in order to avoid any magnitude or phase distortion then it is called a perfect reconstruction (PR) QMF bank .

Then T(z) = d$z^{-no}$

Y(z) = d$z^{-no}$ X(z)

By using IZT

Y(n) = dx(n-no)

This is the reconstructed output of a PRQMF bank.

(i) **Alias Free Realisation :**

To achieve alias free QMF filter bank , we have

Go(z) = Ho(z) ;G1(z) = -H1(z) =-Ho(z)

From 7

T(z) =1/2[$H_o^2(z) + H_1^2(z)$]

=1/2[$H_o^2(z) + -H_o^2(-z)$]

**Polyphase Representation :**

**For analysis filters ,**

Ho(z) = Eo($z^2$) + $z^{-1}$E1($z^2$)

H1(z) = Eo($z^2$) - $z^{-1}$E1($z^2$)

In matrix form

$$\begin{bmatrix} Ho(z) \\ H1(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} Eo(z^2) \\ z^{-1}E1(z^2) \end{bmatrix}$$

**For synthesis filters ,**

$$[\ Go(z) \quad G1(z)\ ] = [z^{-1}E1(z^2) \quad Eo(z^2)\ ] \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The distortion transfer function in terms of polyphase components is given by,

T(z) = 2$z^{-1}$Eo($z^2$)E1($z^2$)

**(ii)** **Alias free QMF filter Bank :**

In pratical case , $|T(e^{j\omega})|$ is not a constant and hence the filter bank introduces amplitude distortion. The distortion transfer function becomes

$T(e^{j\omega}) = \frac{e^{-j(N-1)\omega}}{2}[|Ho(e^{j\omega})|^2 - (-1)^{N-1}|Ho(e^{j(\pi-\omega)})|^2$

If N is odd , then

$T(e^{j\omega}) = 0$

If N is even

$T(e^{j\omega}) = \frac{e^{-j(N-1)\omega}}{2}[|Ho(e^{j\omega})|^2 - (-1)^{N-1}|Ho(e^{j(\pi-\omega)})|^2$

Amplitude distortion occurs in the transition band which can be reduced by properly selecting the passband edge of Ho(z) Another method for reducing the amplitude distortion is by properly selecting the coefficients ho(n).

**IIR Filter Bank :**

The polyphase components are given by ,

Eo(z) = ½ Ao(z)

E1(z) = ½ A1(z)

Where Ao(z) and A1(z) represents all – pass filters

From 9 & 10

Ho(z) = ½[Ao($z^2$) + $z^{-1}$A1($z^2$)]

H1(z) = 1/2[Ao($z^2$) - $z^{-1}$A1($z^2$)]

In matrix form ,

$$\begin{bmatrix} Ho(z) \\ H1(z) \end{bmatrix} = 1/2 \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} Ao(z^2) \\ z^{-1}A1(z^2) \end{bmatrix}$$

## 6.10.Adaptive Filter

A filter will be optimal only if it is designed with same knowledge about input data. If this information is not known, then adaptive filters are used. The adjustable parameters in the filter are designed with values based on the estimated statistical native of the signals. The filters are adaptable to the changing environment. Adaptive filtering find its application in noise cancelling ,line enhancing, frequency tracking etc.,

Principles of Adaptive filtering:

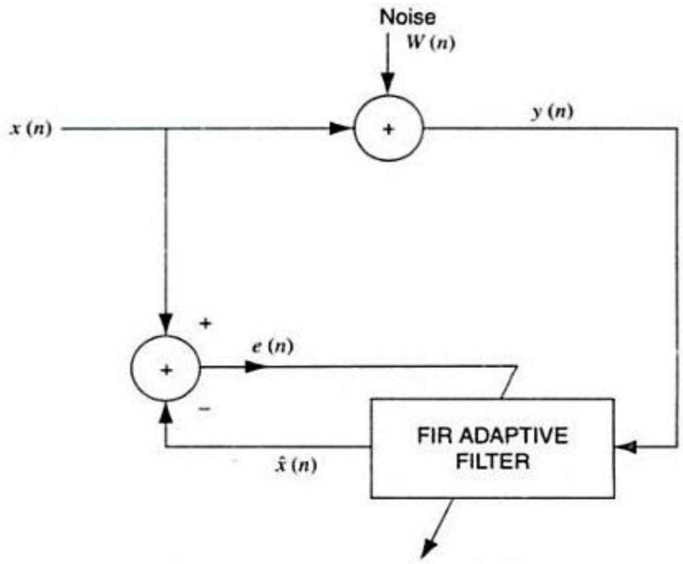



**Fig. 13.1(a)** *Principle of Adaptive FIR Filter*

(i).System modeling:

Consider an unknown system which has to be identified.The requirement is to develop an FIR filter.Let the unknown systems and the FIR filter be excited by the input signal x(n),the output from the dynamic systems be y(n) and$\bar{y}(n)$ be theoutput of FIR filter.

$$\bar{y}(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

Error signal,e(n)=y(n)- $\bar{y}(n)$

The error signal is to be minimizes using the criteria,minimum mean square error.

$$E_{min}=\sum_{n=0}^{\infty} |y(n) - \sum_{K=0}^{M-1} h(k)x(n-k)|^2$$

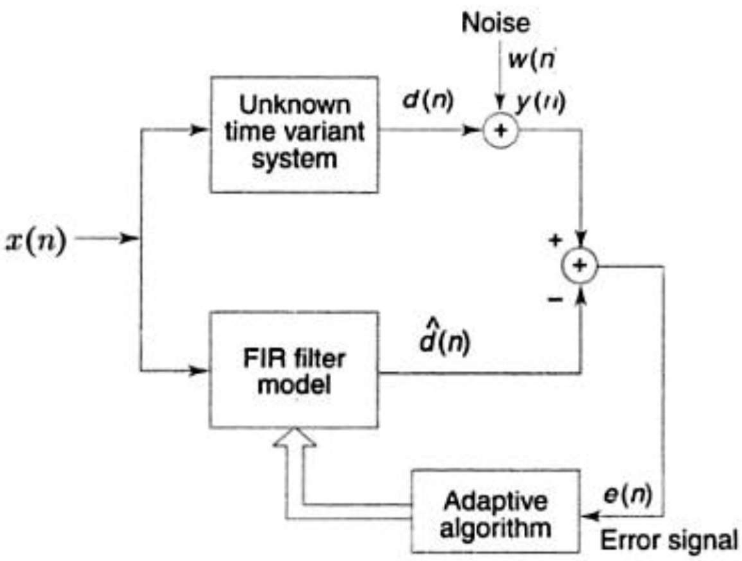



**Fig. 13.2** *System Identification*

(ii).Adaptive equalization:

Consider a digital communication system.The bandwidth of the channel has to be used efficiently.The factors that affect the data in the channel are inter-symbol interference (ISI) and thermal noise.The output of receiving filter is

$$s(t) = \sum_{k} a_k p(t - kTs)$$

$T_s$=duration of signaling interval

P(t)=impulse response of cascade connection of transmitting filter,channel and receiving filter.The sampled version of s(t) is

$$s(t) = \sum_{n} a_n p(k - n)$$

s(k)=$a_k$p(0)+ $\sum_n p(k-n)$

The first term corresponds to the desired symbol and the remaining terms represent ISI.To avod ISI problem,adaptive equalizers can be used.The purpose of using adaptive equalizer is for compensating the channel distortion so that the detected signal will be reliable.

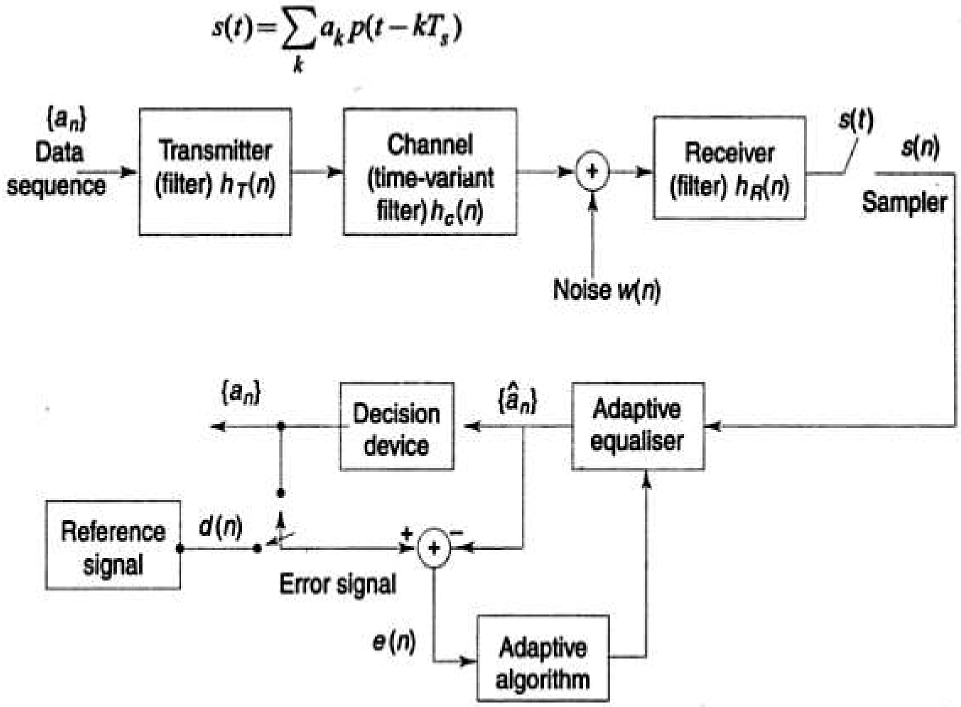



**Fig. 13.3** *Adaptive Equalisation*

The adaptive equalization process is done in two steps,ie.,(i).Training mode and (ii).tracking mode.In training mode, a known test signal is transmitted.Using a synchronized version of the test signal at the receiver side and comparing this signal with the received signal the resultant error signal gives the information about the channel.This error signal is used to adjust the coefficients of equalizer.After this training process is done,the adaptive equalizer can be continuously adjusted in the decision directed mode.

(iii).Adaptive line enhancer

If the sine wave is corrupted by additive noise,the adaptive line enhancer can be used to detect the low level sine wave.

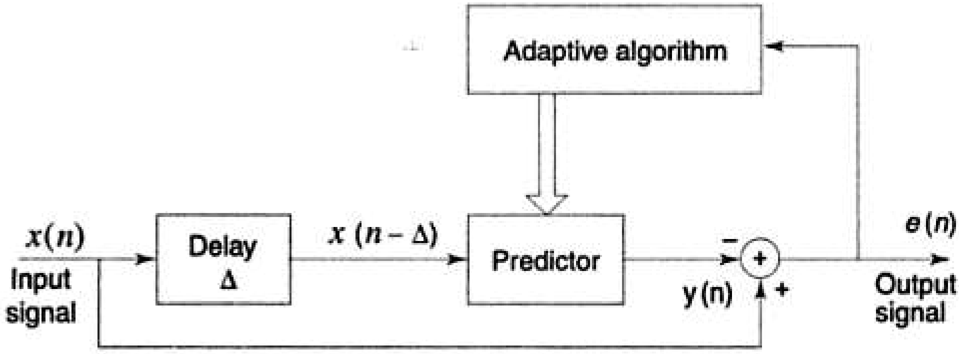



**Fig. 13.4** *Adaptive Line Enhancer*

Let the input signal be x(n).The signal is passed through a delay element with a delay S,and its output is x(n-s).This is passed through a predictor.The predicted output y(n) is subtracted from the input signal resulting in the error signal e(n)=x(n)-y(n).The error signal controls the predictor coefficients.The delay Δ is used for removing the correlation between x(n) and x(n-Δ).Δ is also known as decorrelation parameter.The adaptive line enhancer is used to suppress broadband noise components and passed only the narrowband signals with less attenuation.

(iv).Adaptive noise cancelling:

Let x(n) be the desired signal.This signal is computed by an additive noise signal $w_1(n)$ and an additive interference signal $w_2(n)$.The output of the unknown linear system $z_2(n)$ has the filtered additive interference noise component.Another noise signal $w_3(n)$ is added to the filtered noise component $z_2(n)$ and the observed signal z(n) is $z(n)-z_2(n)+w_3(n)$

The estimate of interference signal $w_2(n)$ is obtained from an FIR filter.The estimate of the desire signal x(n) is the error signal given by $e(n)=y(n)-\overline{w_2}(n)$.

Where $\overline{w_2}(n)$ is the estimate of $w_2(n)$.The error signal is used for adjusting the FIR filter coefficients.

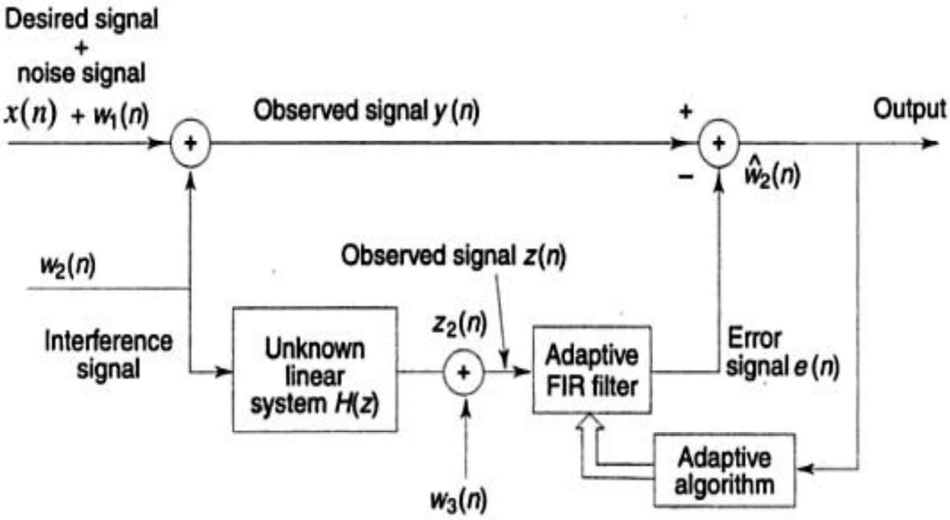



**Fig. 13.5** *Adaptive Noise Cancelling*

(v).Echo cancellation:

Consider the four-wire and two wire transmissions in the telephone connections.At the hybrid (connect 4 to 2 wire transmission),an echo is generated.Assume that a call is made over a long distance using satellites.There is a delay of 270ms in the satellite communication.When A speaks to B ,the speech takes the upper transmission path and a part of the signal is retained through the lower transmission path.The retained echo signal has the delay of 540ms.

The echo cancellation is done by finding an estimate of the echo and subtracting it from the return signal.The return signal is given by,y(n)=$\sum_{k=0}^{\infty} h(k)x(n-k)+v(n)$

Where,x(n) is the speech of speaker A

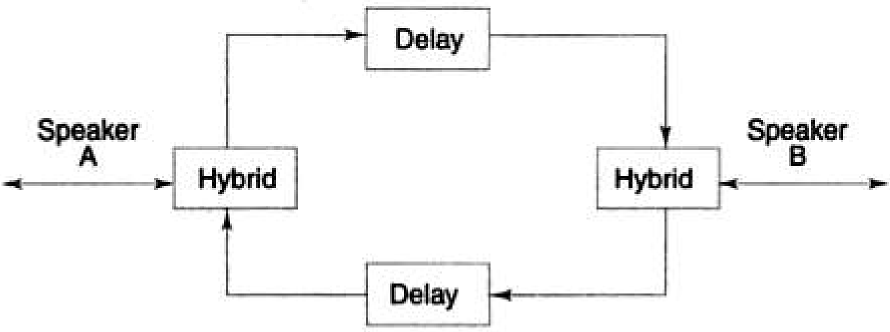
V(n) is the speech of speaker B+noise

H(k) is the impulse response of echo path.

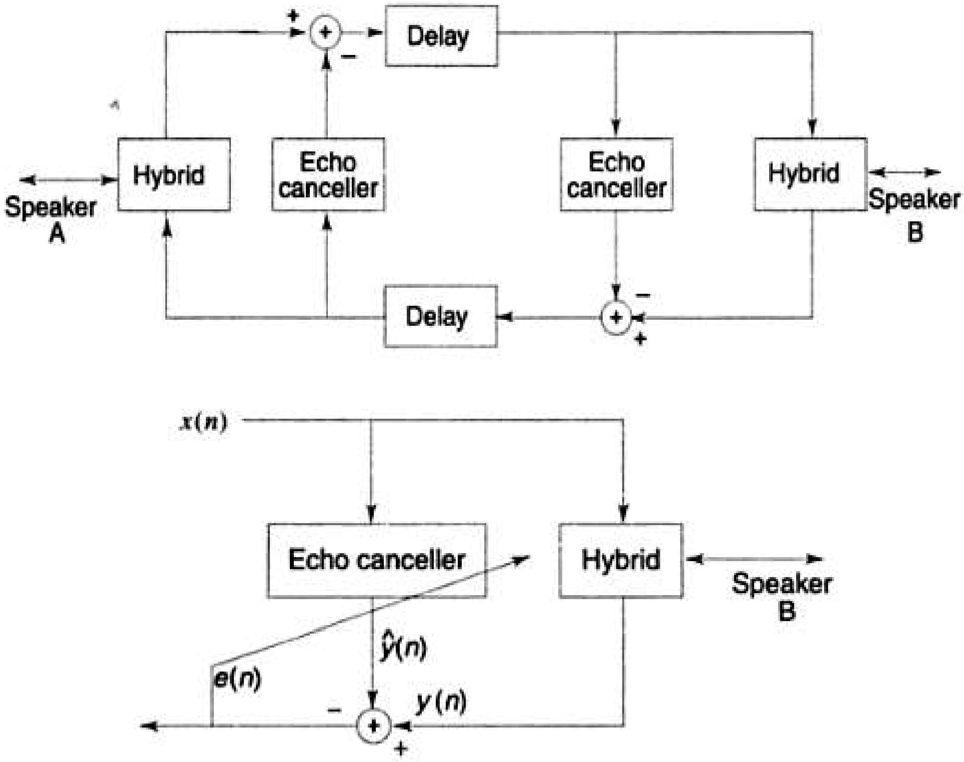
The estimate of echo is $\bar{y}$(n)=$\sum_{k=0}^{\infty} \bar{h}(k)x(n-k)$

Where $\bar{h}(k)$ is the estimate of impulse response of echo path.The error signal is e(n)=y(n)- $\bar{y}$(n)

By adaptively controlling h(k),after some iterations,the echo effect can be minimized.




(a) Without echo protection




(b) With echo suppressors

**Fig. 13.6** *Echo Cancellation*