UNIT 1

SAMPLING & QUANTIZATION

Low pass sampling – Aliasing- Signal Reconstruction-Quantization - Uniform & non-uniform quantization - quantization noise - Logarithmic Companding of speech signal- PCM - TDM

1.1 DIGITAL COMMUNICATION SYSTEM: AN OVERVIEW

Figure 1.1 illustrates the basic elements of a digital communication system. In most of the practical systems, the information sources are analog in nature. Voice, video, TV sources are common examples of analog sources. However, computer data are in discrete form.

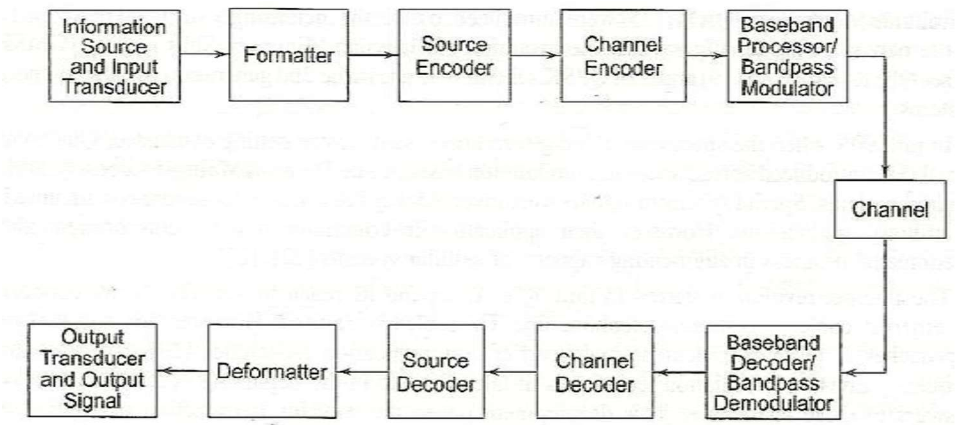

Fig. 1.1 A Typical Digital Communication System

In a digital communication system, the input signal should be in digital form so that digital signal processing techniques can be employed on these signals. The input transducer converts these signals into analog electric signals. Then electrical signals are converted into a sequence of digital signals. The block performing this is known as **Formatter**. If the output of the information source is digital, then formatter is not used. Here the source of information is keyboard or typewriter connected to the computer.

To represent these digital signals by as few digits as possible, a coding system can be employed which minimizes the requirement of number of digits. This process is called source encoding and the block performing this task is known as **source encoder**. This source encoder block compresses the total digits of message signal for transmission.

To reduce noise in the communication channel, some redundancy is introduced in the message. This is done by the **channel encoder** block.

In low-speed wired transmission, the channel encoded signal is generally not modulated. The transmission takes place in baseband. However, for proper detection in the receiver and to reduce noise and interference, line coding is used. Some pulse shaping is also done to reduce interference. Some special filters are also employed in the receiver to reduce noise. All these are collectively called as **baseband processor**.eg. Fixed telephony and data storage systems.

However for transmission of high speed digital data (eg. computer communication systems), the digital signal needs to be modulated i.e. frequency translated. The primary purpose of the **bandpass modulator** is to map the digital signal to high frequency analog signal waveforms. A performance measure of the modulator is spectral efficiency which is the number of bits sent per second for every Hz of channel bandwidth. The purpose of modulation is to increase the spectral efficiency as much as possible.

In the **communication channel**, the transmitted signal gets corrupted by random noise. The noise is from various sources: either from **electronic devices** (thermal noise, shot noise), or from **man-made** disturbances (automobile noise, electromagnetic interference from other electronic equipments etc.), or from **natural sources** (atmospheric noise, electrical lightning discharges during thunderstorm, radiation from space falling in the electromagnetic spectrum).

At the receiver, **the bandpass demodulator** block processes the channel corrupted transmitted waveform and converts them back to a sequence of number that represents the estimate of transmitted data sequence. In case of baseband, the task of converting back the line coded pulse waveform to transmitted data sequence is carried out by the **baseband decoder** block.

This sequence of numbers representing the data sequence is passed to the **channel decoder**, which reconstructs the original information sequence (source encoded) by using channel encoding algorithm.

Performance measure of demodulator and decoder is the frequency of bit error (bit error rate(BER in the decoded sequence. BER depends on channel coding characteristic, types of analog signal used in transmission at modulator, transmitter power, channel characteristic (i.e. amount of noise, nature of interference) and the method of demodulation and decoding.

Source decoder estimates the digital signal from the information sequence. The difference of this estimate and the original digital signal is the distortion introduced by the digital communication system.

If the original information source was not in digital data form and the output of the receiver needs to be in the original form of information, a **deformatter** block is used to convert back the digital data to either discrete form (like keyboard characters) or analog form (eg. speech signal).

Output transducer converts the estimate of digital signal to analog non-electrical signal. However in data communication systems, e.g. computer communication, the input signal and reconstructed signal both are in digital form. So, an output transducer may not be always present in digital data communication systems

1.2. SAMPLING

1.2.1 Representation of CT Signals by Its Samples

The CT signals must be represented in samples because,

- A CT signal cannot be processed in digital signal processor or computer
- To enable digital transmission of CT signals

Fig. 1.2 shows the CT signal and its sampled DT signal. In this figure observe that the CT signal is sampled at t = 0, T., 2T., 3T, ... and so on.

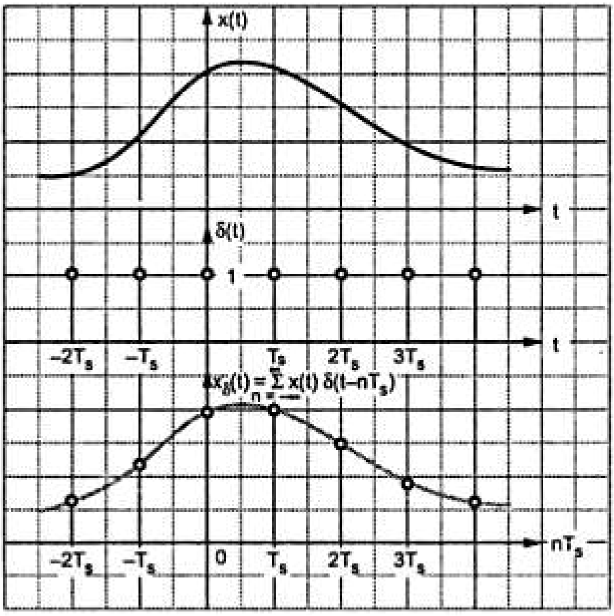

Fig. 1.2 CT and its DT signal

- Here sampling theorem gives the criteria for spacing T_s between two successive samples.
- The samples $x_\delta(t)$must represent all the information contained in $x(t)$
- The sampled signal $x_\delta(t)$is called discrete time (DT) signal. It is analyzed with the help of DTFT and z-transform.

1.2.2. Sampling Theorem for Lowpass (LP) Signals

A lowpass or LP signal contains frequencies from 1 Hz to some higher value

Statement of sampling theorem

1) A band limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$seconds and

2) A band limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

The first part of above statement tells about sampling of the signal and second part tells about reconstruction of the signal. Above statement can be combined and stated alternately as follows:

A continuous time *signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal. i.e.,*

$$f_s \geq 2W$$

Here f_s is the sampling frequency and

W is the higher frequency content

Proof of sampling theorem

There are two parts: I) Representation of $x(t)$ in terms of its samples

II) Reconstruction of $x(t)$ from its samples.

Part I: Representation of $x(t)$ in terms of its samples

Step1: Define $x_\delta(t)$

Step 2: Fourier transform of $x_\delta(t)$ i.e. $X_\delta(f)$

Step. 3: Relation between *X(f)* and $X_\delta(f)$

Step 4: Relation between *x(t)* and $x(nT_s)$

Step 1: Define $x_\delta(t)$

Refer Fig. 1.2. The sampled signal $x_\delta(t)$ is given as,

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s) \qquad --- (1)$$

Here observe that $x_\delta(t)$ is the product of x_δ and impulse train $\delta(t)$ as shown in Fig. 1.2. In the above equation $\delta(t - nT_s)$ indicates the samples placed at $\pm T_s, \pm 2T_s, \pm 3T_s \ldots$ *and so on.*

Step 2: Fourier transform of $x_\delta(t)$ i.e. $X_\delta(f)$

Taking Fr of eqn (1)

$$X_\delta(f) = FT\left\{\sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s)\right\}$$

$$= FT\{Product\ of\ x(t)\ and\ impulse\ train\}$$

We know that FT of product in time domain becomes convolution in frequency domain. i.e.,

$$X_\delta(f) = FT\{x(t)\} * FT\{\delta(t - nT_s)\} \qquad \text{---(2)}$$

By definitions, $x(t) \overset{FT}{\longleftrightarrow} X(f)$ and

$$\delta(t - nT_s) \overset{FT}{\longleftrightarrow} f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Hence equation (2) becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Since convolution is linear

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad \text{By shifting property of impulse function}$$

$$= \cdots f_s X(f - 2f_s) + f_s X(f - f_s) + f_s X(f) + f_s X(f - f_s) + f_s X(f - 2f_s) \ldots$$

(i) The RHS of above equation shows that $X(f)$ is placedat $\pm f_s, \pm 2f_s, \pm 3f_s$ ...

(ii) This means $X(f)$ is periodic in f_s

(iii) If sampling frequency is$f_s = 2W$, then the spectrums $X(f)$ just touch each other.

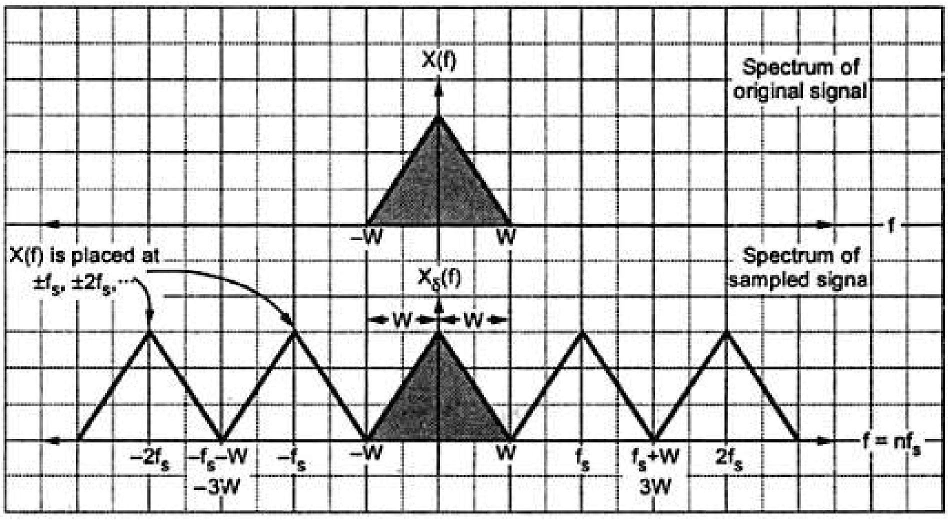

Fig. 1.3 Spectrum of original signal and samples signal

Step 3: Relation between *X(f)* and $X_\delta(f)$

Assume that $f_s = 2W$, then as per above diagram

$$X_\delta(f) = f_s X(f) \quad for - W \le f \le W \; and \; f_s = 2W$$

$$or \quad X(f) = \frac{1}{f_s} X_\delta(f) \qquad --(3)$$

Step 4 : Relation between $x(t)$and $x(nT_s)$

$$DTFT \; is, \qquad X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$$\therefore \; X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi fn} \qquad --(4)$$

In above equation of is the frequency of DT signal. If we replace *X(f)* and $X_\delta(f)$,then *'f'* becomes frequency of CT signal. i.e.,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi \frac{f}{f_s} n}$$

In above equation 'f' is frequency of CT signal. And $\frac{f}{f_s}$ = Frequency of DT signalin equation (4)

Since $x(n)$=$x(nT_s)$ i.e. samples of $x(t)$then we have,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi fnT_s} \qquad since \frac{1}{f_s} = T_s$$

Putting above expression in equation (3),

$$X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi fnT_s}$$

Inverse Fourier Transform (IFT) of above equation gives $x(t)$i.e.,

$$x(t) = IFT \left\{ \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi fnT_s} \right\} \qquad --(5)$$

i) Here $x(t)$is represented completely interms of $x(nT_s)$

il) Above equation holds for $f_s = 2W$. This means if the samples are taken at therate of 2W or higher, $x(t)$is completely represented by its samples.

Part II: Reconstruction of $x(t)$from its samples

Step 1:Take inverse Fourier transform of *X(f)* in terms of $X_\delta(f)$

Step, 2: Show that *x(t)*is obtained back with the help of interpolation function

Step 1: Take inverse Fourier transform of *X(f)* in terms of $X_\delta(f)$

The IFT of equation (5) becomes,

$$x(t) = \int_{-\infty}^{\infty} \left\{ \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} \right\} e^{j2\pi ft}\, df$$

Here the integration can be taken from$-W \le f \le W$. Since $X(f) = \frac{1}{f_s} X_\delta(f)for-W \le f \le W$.

$$x(t) = \int_{-W}^{W} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} . e^{j2\pi ft}\, df$$

Interchanging the order of summation and integration,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-W}^{W} e^{j2\pi f(t-nT_s)}\, df$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)} \right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)} \right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)} \right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{\sin 2\pi W(t-nT_s)}{j2\pi(t-nT_s)} \right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \left[\frac{\sin \pi(2Wt - 2WnT_s)}{\pi(f_s t - f_s nT_s)} \right]$$

Here $f_s = 2W$, hence $T_s = \frac{1}{f_s} = \frac{1}{2W}$.Simplifying above equation.

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\,\text{sinc}(2Wt - n) \quad since\ \frac{\sin \pi\theta}{\pi\theta} = sinc\ \theta \ --- (6)$$

Step 2: Let us interpret the above equation. Expanding we get,

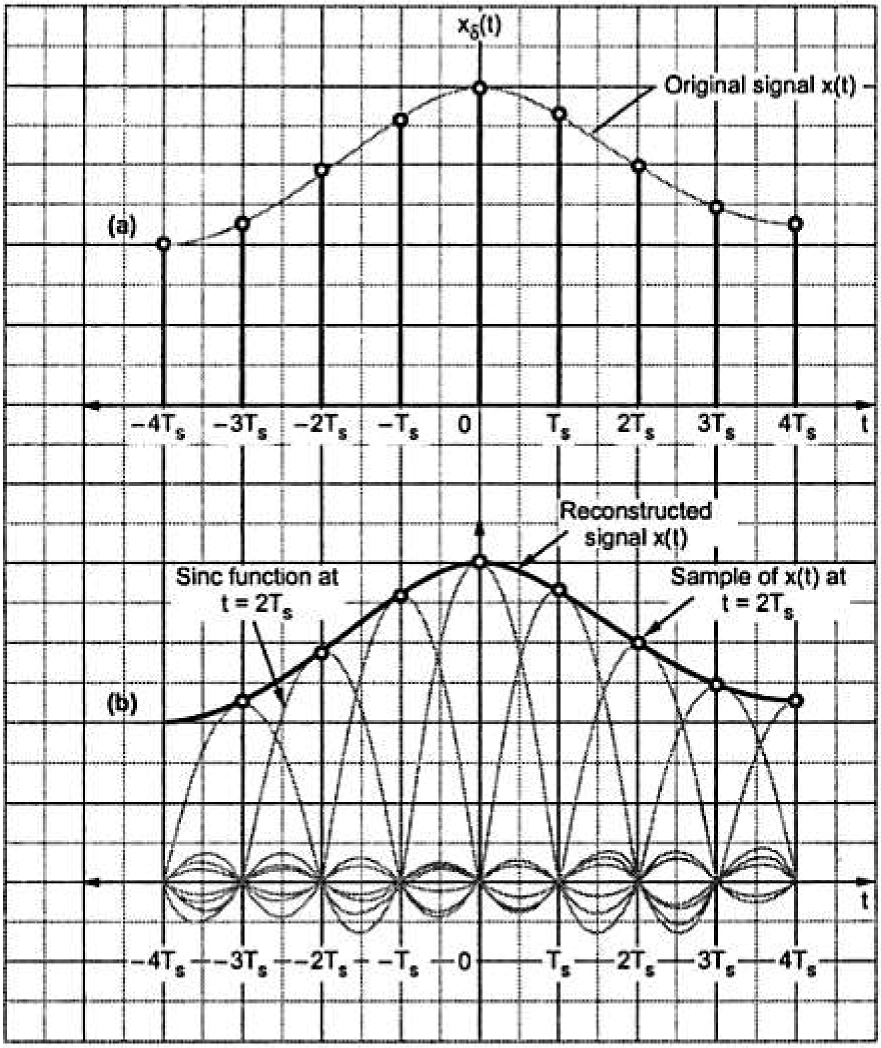

Fig. 1.4 (a) Sampled version of signal $x(t)$ (b) Reconstruction of $x(t)$ from its samples

i) The samples $x(nT_s)$are weighted by sinc functions.

ii) The sinc function is the interpolating function. Fig. 1.4 shows, how *x(t)* is interpolated.

Step 3 : Reconstruction of *x(t)* by lowpass filter

When the interpolated signal of equation (6) is passed through the low pass filter of bandwidth $-W \leq f \leq W$,then the reconstructed waveform shown in aboveFig. 1.4 (b) is obtained. The individual sinc functions are interpolated to get smooth $x(t)$.

1.2.3 Sampler Implementation

The implementation of a sampler is done with a SAMPLE AND HOLD circuit. In this operation, a switch and storage mechanism (for example, a transistor and a capacitor) is used to form a sequence of samples of the analog input waveform. These samples look like a PAM (Pulse Amplitude Modulation) waveform as the amplitude of the sampled pulses can vary continuously. The original analog waveform can be recovered from these PAM type samples simply by lowpass filtering them. From Sampling theorem, it is noted that if we undersample (i.e.$fs < f_{Nyquist}$), then aliasing (i.e. overlapping of adjacent spectrum replicates) occurs. The aliased spectral components represent ambiguous data that appear in the frequency band between $(f_s - f_m)$and f_m. Figure 1.5 shows the aliasing problem in the frequency domain. Due to practical difficulties in achieving Nyquist rate of sampling, sometimes undersampling is done purposely.

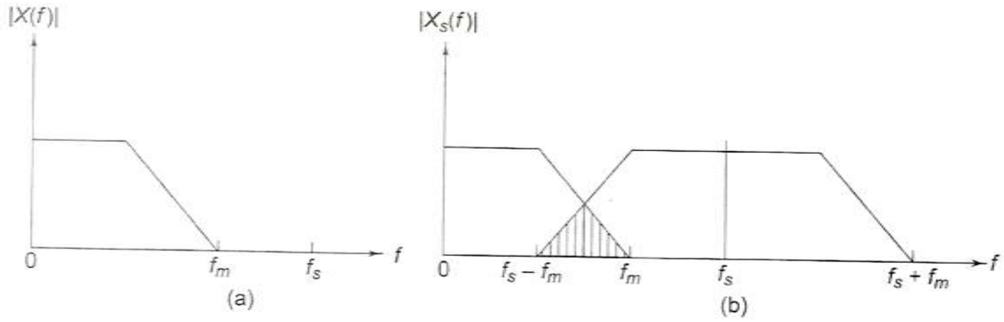

Fig. 1.5 Aliasing: The Spectral Representation, (a) Signal (b) Sampled Signal

There two ways to solve the problem of aliasing; both of these methods make use of **antialiasing filters.**

- In the first method, shown in fig. 1.5, called **prefiltering antialiasing filter**, the analog signal itself is prefiltered so that the new maximum frequency$f_s/2$or less. Therefore, the sampled spectrum does not show aliasing.
- The other method, shown in fig. 1.6, called **postfiltering anti-aliasing filter**. In this method, the aliased terms are eliminated after sampling with the help of a low pass filter operating on the sampled data. The cutoff frequency $f_m''$for this filter needs to be less than $(f_s - f_m)$.

The disadvantage of both these antialiasing filters is that some information is always lost due to filtering. However, if the spectrum of the signal is well known, and the lost signal components do not carry significant spectral information, the performance degradation due to antialiasing filter may be tolerable for most of the applications.

All realizable filters require a nonzero bandwidth for the transition between the passband and the stopband, commonly known as the *transition bandwidth.* Filter complexity and cost rise sharply with narrower transition bandwidth. However, if we want to keep the sample rate down, we should go for a very narrow transition bandwidth. Therefore, a trade-off is required between the cost of a small transition bandwidth and the cost of the higher sampling rate (i.e. cost of more storage and higher transmission rate). In many systems, the transition bandwidth is between 10% and 20% of the signal bandwidth.

The antialiasing filters described so far are analog filters. Instead of analog filters, **digital filters** are also used for filtering the large number of samples. It turns out that this third method is the most **economic solution** for sampling. This is so because signal processing performed with high performance analog equipment is typically more expensive than using digital signal processing equipment to perform the same task. We enumerate the steps required in A/Dconversion for boththe cases of undersampling and oversampling

Without Oversampling

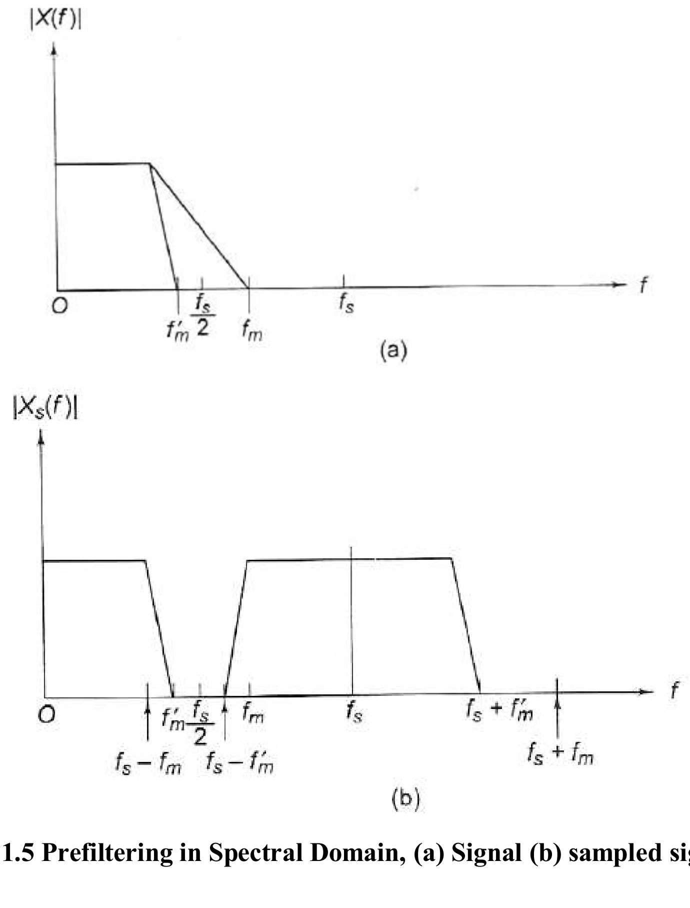

Fig. 1.5 Prefiltering in Spectral Domain, (a) Signal (b) sampled signal

(a) The signal passes through a high performance analog lowpass antialiasing filter to limit its bandwidth. However, the antialiasing filter has a passband equal to the signal bandwidth plus the transition bandwidth. For a transition bandwidth of f_t, the Nyquist sampling rate, $2f_m$, becomes $2fm + f_t$. This additional spectral interval does not represent any useful signal content, rather, it protects the signal by reserving free spectral interval between two spectral replicas. This overhead in spectral content increases the sampling rate by 10-20% from the Nyquist rate

For example, in Compact Disk (CD) digital audio system, the two-sided signal bandwidth is 40 kHz and the sampling rate is 44.1 ksamp1es/sec. In Digital Audio Tape (OAT) system, the corresponding figures are 40 kHz and 48 k samples/sec respectively.

Such narrow transition bandwidths in analog antialiasing filters (like 4.1 kHz in a 40 kHz signal bandwidth of CD system) introduce distortion (nonlinear phase versus frequency) and makes the filters fairly expensive because of the need to go for higher order filter with more number of L or C elements. As an example, in the Nyquist sampling case, the requirement may be for a sophisticated tenth order elliptic filter, whereas in the oversampled case, a fourth order elliptic filter suffices.

(b) The filtered signal is sampled at the Nyquist rate for the approximately bandlirnited signal obtained in previous step.

(c) The samples are processed by an analog-to-digital converter that maps the continuous-valued samples to a finite list of discrete output levels.

With Oversampling

When this same task is performed with the benefit of over-sampling, the process is best described in 5 steps,

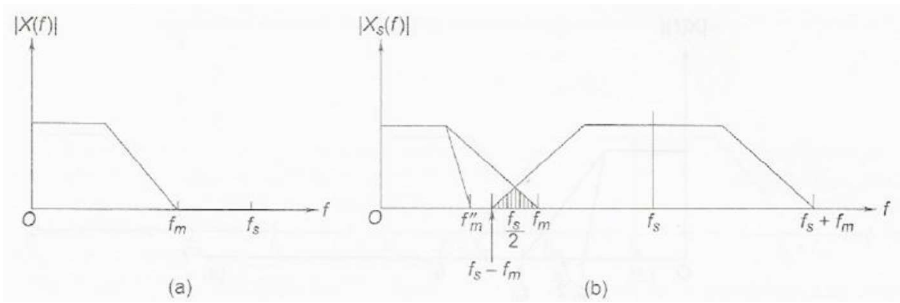

Fig. 1.6 Postfiltering, (a) Signal Spectrum, (b) Sampled Signal Spectrum

(a) The signal is passed through a low performance (less costly) analog low-pass filter (prefilter) to limit its bandwidth. Here a much wider bandwidth compared to the Nyquist sampling is used.

(b) The pre-filtered signal is oversampled at a much higher sampling rate than the Nyquist rate for the bandlimited signal. For example, with 4 times oversampling, the CD system can be sampled at 176.4 kHz with a transition bandwidth of 136.4 kHz.

(c) The samples are processed by an *A/D* converter that maps the continuous-valued samples to a finite list of discrete output levels.

(d) The digital samples are then processed by a high performance, low cost digital filter to reduce the bandwidth of the digital samples. This antialiasing filter can realise the fairly wide transition bandwidth without the distortion associated with analog filters.

(e) The sample rate at the output of the digital filter is reduced in proportion to the bandwidth reduction obtained by this digital filter. Good digital signal processing techniques combine the filtering and the resampling in a single structure. DSP techniques can also compensate for the distortion introduced by the analog prefilter employed in the first step, thereby improving the quality of the sampled signal to any desired level. DSP hardware, though quite complex, is becoming cheaper day by day, and provides a definitive cost-advantage in comparison with the analog signal processing blocks.

1.2.4 Effects of Undersampling (Aliasing)

While providing sampling theorem we considered that $f_s = 2W$. Consider the case of $f_s < 2W$. Then the spectrum of $X_\delta(f)$ shown in Fig. 1.7 will be modified as follows:

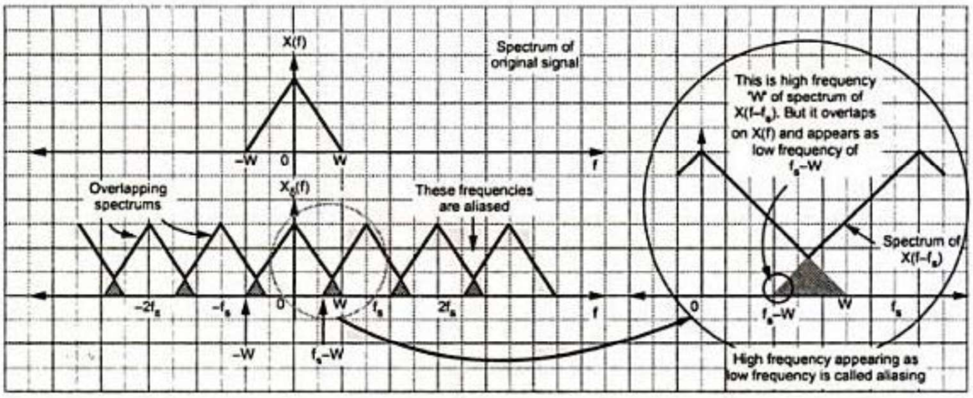

Fig. 1.7 Effects of Undersampling or aliasing

i) The spectrums located at $X(f), X(f - f_s), X(f - 2f_s)$overlap on each other.

ii) Consider the spectrums of $X(f)$ and $X(f - f_s)$shown as magnified in abovefigure. The frequencies from $(f_s - W)\ to\ W$ are overlapping in these spectrums.

iii) The high frequencies near *'w'* in $X(f - f_s)$overlap with low frequencies $(f_s - W)\ in\ X(f)$.

Definition of aliasing: When the high frequency interferes with low frequency and appears as low frequency, then the phenomenon is called aliasing.

Effects of aliasing:

i) Since high and low frequencies interfere with each other, distortion is generated.

ii) The data is lost and it cannot be recovered.

Different ways to avoid aliasing:

Aliasing can be avoided by two methods:

i) Sampling rate $f_s \geq 2W$.

ii) Strictly bandlimit the signal 'W',

i) Sampling rate $f_s \geq 2W$.

When the sampling rate is made higher than 2W, then the spectrums will not overlap and there will be sufficient gap between the individual spectrums. This is shown in Fig. 1.8.

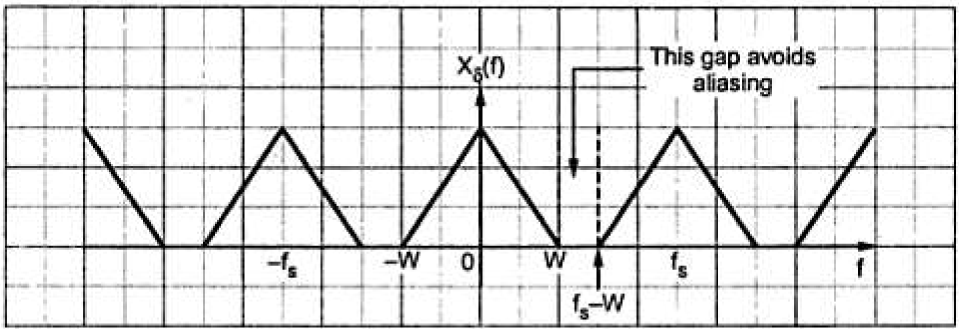

Fig.1.8 $f_s \geq 2W$ avoids aliasing by creating a bandgap

ii) Bandlimiting the signal

The sampling rate is, $f_s = 2W$. Ideally there should be no aliasing. But there can be few components higher than 2W. These components create aliasing. Hence a lowpass filter is used before sampling the signals as shown in Fig. 1.9.Thusthe output of lowpass filter is strictly bandlimited and there are no frequency components higher than W'. Then there will be no aliasing.

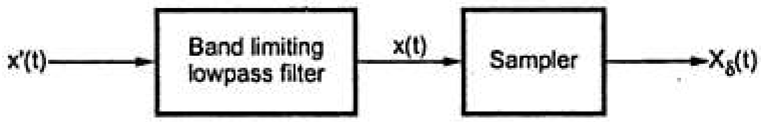

Fig.1.9 Bandlimiting the signal. The bandlimiting LPF is called prealins filter

1.2.5 Nyquist Rate and Nyquist Interval

Nyquist rate : When the sampling rate becomes exactly equal to '2W' samples/sec, for a given bandwidth of W Hertz, then it is called Nyquist rate.

Nyquist interval : It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

$$Nyquist\ rate = 2W\ Hz$$
$$Nyquist\ interval = \frac{1}{2W}\ seconds$$

1.2.6 Reconstruction Filter (Interpolation Filter)

Definition

The reconstructed signal is the succession of sine pulses weighted by $x(nT_s)$**.** These pulses are interpolated with the help of a lowpass filter. It is also called *reconstruction* filter or *Interpolation* filter.

Ideal filter

Fig. 1.9 shows the spectrum of sampled signal and frequency response of required filter. When the sampling frequency is exactly 2W, then the spectrums just touch each other as shown in Fig. 1.9. The spectrum of original signal, $X(f)$ can befiltered by an ideal filter having passband from $-W \leq f \leq W$

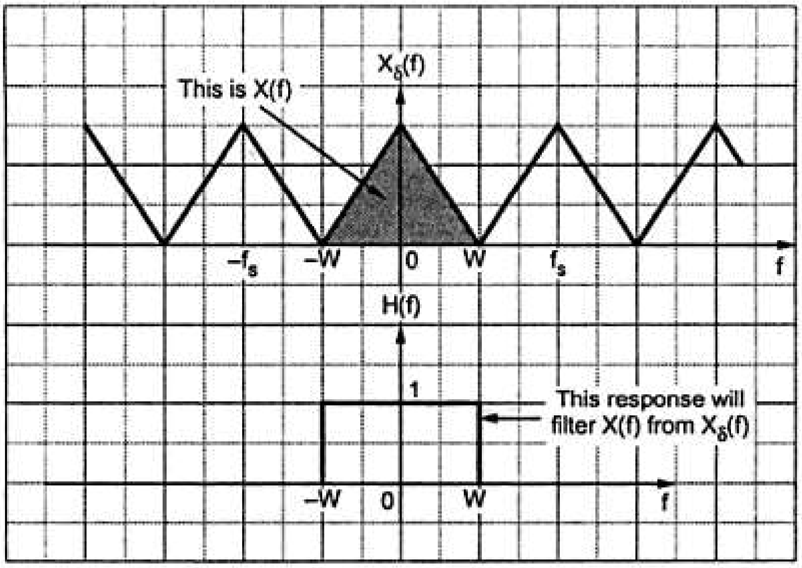

Fig.1.9 Ideal reconstruction filter

Non-Ideal filter

An ideal filter of bandwidth 'W' filters out an original signal. But practically ideal filter is not realizable. It requires some transition band. Hence $f_s$must be greater than 2W. It creates the gap between adjacent spectrums of $X_\delta(f)$. Thisgap can be used for the transition band of the reconstruction filter. The spectrum X(f)is then properly filtered out from $X_\delta(f)$. Hence the sampling frequency must begreater than '2W' to ensure sufficient gap for transition band.

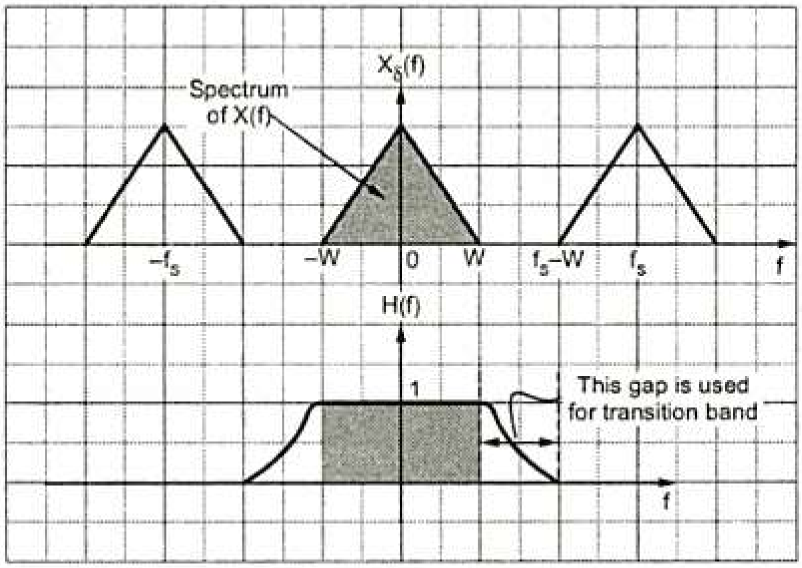

Fig.1.10 Practical reconstruction filter

Zero-Order Hold for Practical Reconstruction

• The zero-order hold circuit is used for practical reconstruction. It simply hold the value $x(n)$ for T seconds.Here T is the sampling period .

• The output of the zero-order hold is staircase signal that approximates $x(n)$.This is shown in Fig. 1.11.

• Let the impulse response of zero-order hold be represented as,

$$h(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

Then the output $y(t)$ of thezero-order hold will be convolution of$h(t)$and sampled input$x_\delta(t)$. i.e,

$$y(t) = h(t) * \sum_{h=-\infty}^{\infty} x(n)\delta(t - nT_s)$$

$$= h(t) * x_\delta(t)$$

$$\therefore \; Y(\omega) = H(\omega) \,.\, X(\omega)$$

$$Here, \quad H(\omega) = 2e^{-j\omega T/2}\ \frac{\sin\frac{\omega T}{2}}{\omega}$$

$$\therefore \quad Y(\omega) = 2e^{-j\omega T/2}\ \frac{\sin\frac{\omega T}{2}}{\omega}\ .X(\omega)$$

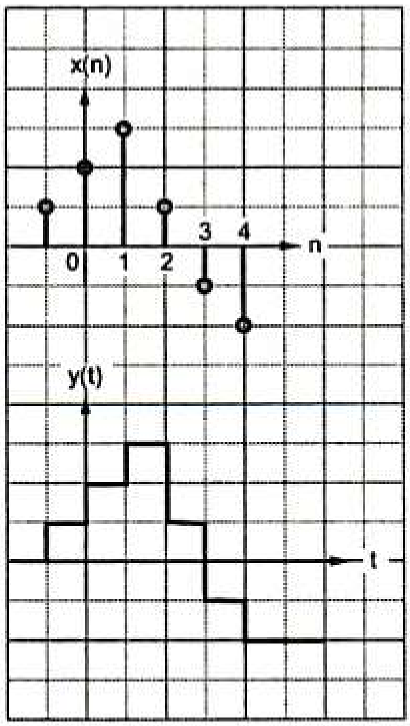

Fig.1.11 Input and output of zero order hold

The above equation shows that spectrum $X(\omega)$is changed due to convolution orpassing through zero order hold. These changes are,

(i) There is linear phase shift corresponding to time delay of $\frac{T}{2}$ sec

(ii) The main lobe of $\frac{\sin\frac{\omega T}{2}}{\omega}$modifies the shape of $X(\omega)$.

• The above modifications can be reduced by increasing the sampling frequency $\omega_s$or reducing the time T.

• Sometimes anti-imaging filter is used for compensating the modifications. It's spectrum is given as,

$$H_c(\omega) = \begin{cases} \frac{\omega T}{2\sin\frac{\omega T}{2}}, & |\omega| \le \omega_m \\ 0, & |\omega| > \omega_s - \omega_m \end{cases}$$

This filter provides reverse action to that of zero-order hold. Fig. 1.12 shows the block diagram with anit-imaging filter.

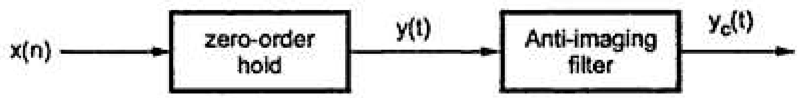

Fig.1.12 Block diagram of practical reconstruction

1.3 QUANTISATION

Quantiser converssts the analog form of the signal to discrete form. The sampled analog signal is still analog, because though discretised in time, the signal amplitude can take any value as it may wish. The quantiser forces the signal to take some discrete values from the continuum of amplitude values.

1.3.1 Uniform Quantisation (or Linear Quantization)

We know that input sample value is quantized to nearest digital level. This quantization can be uniform or nonuniform. In uniform quantization, the quantization step or difference between two quantization levels remains constant over the complete amplitude range.

1.3.1.1 MidtreadQuantizer

The transfer characteristic of the midtread quantizer is shown in Fig. 1.13.

As shown in this figure, when an input is between $-\delta/2\ and\ +\delta/2$ then thequantizer output is zero. i.e

$$For\quad \frac{\delta}{2} \leq x(nT_s) < \frac{\delta}{2};\quad x_q(nT_s) = 0$$

Here ' δ' is the step size of the quantizer.

$$For\quad \frac{\delta}{2} \leq x(nT_s) < 3\frac{\delta}{2};\quad x_q(nT_s) = \delta$$

Similarly other levels are assigned. It is called midtread because quantizer output is zero when $x(nT_s)$ is zero. Fig.1.13 (b) shows the quantization error of midtread quantizer. Quantization error is given as,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

In Fig. 1.13 (b) observe that when $x(nT_s) = 0, x_q(nT_s) = 0$.Hence quantization erroris zero at origin. When $x(nT_s) = \delta/2$, quantizer output is zero just before this level. Hence error is $\delta/$ 2near this level. From Fig. 1.13 (b) it is clear that,

$$-\frac{\delta}{2} \leq \varepsilon \leq \frac{\delta}{2}$$

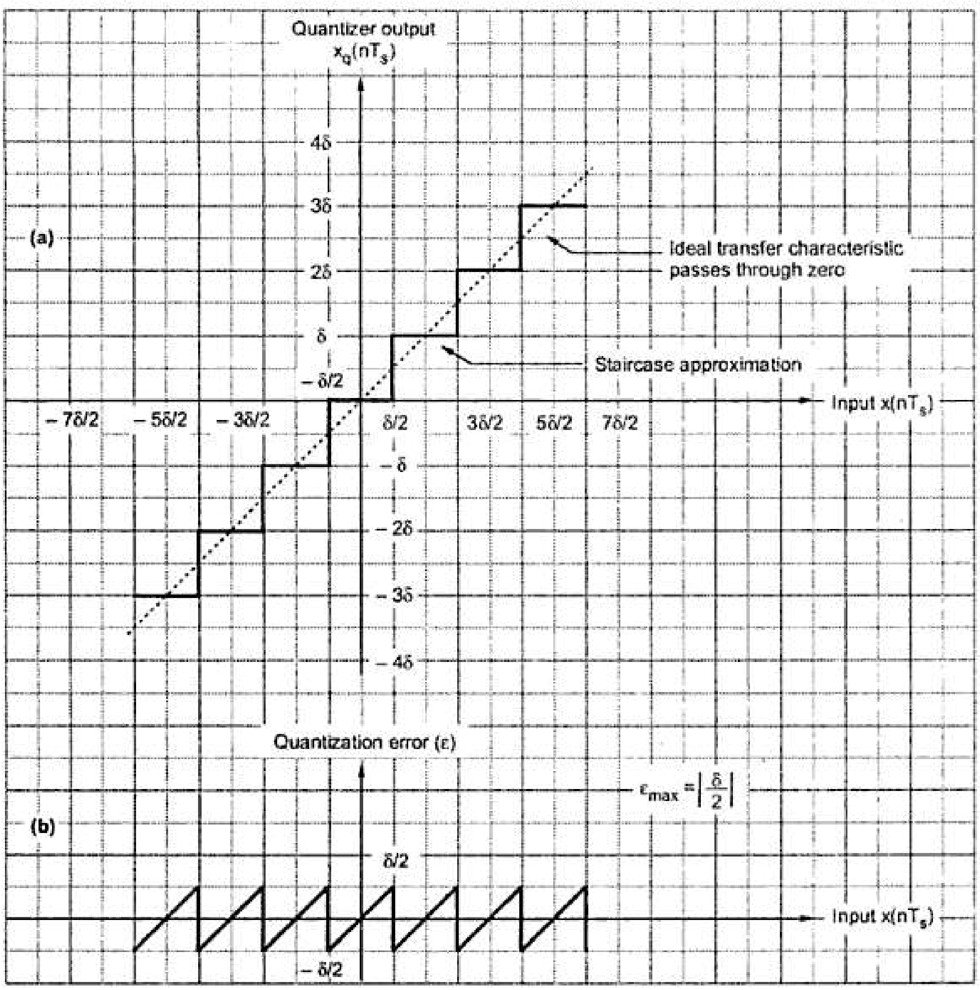

Fig. 1.13 (a) Quantization characteristic of midtread quantizer

(b) Quantization error

Thus quantization error lies between $-\delta/2$ *and* $+\delta/2$. And maximum quantization error is,

$$maximum\ quantization\ error, \varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

1.3.1.2 Midriser Quantizer

The transfer characteristic of the midriser quantizer is shown in Fig. 1.14.

In Fig. 1.14 observe that when an input is between 0 and δ, the output is $\delta/2$.Similarly when an input is between 0 and $-\delta$, the output is $-\delta/2$.. i.e.,

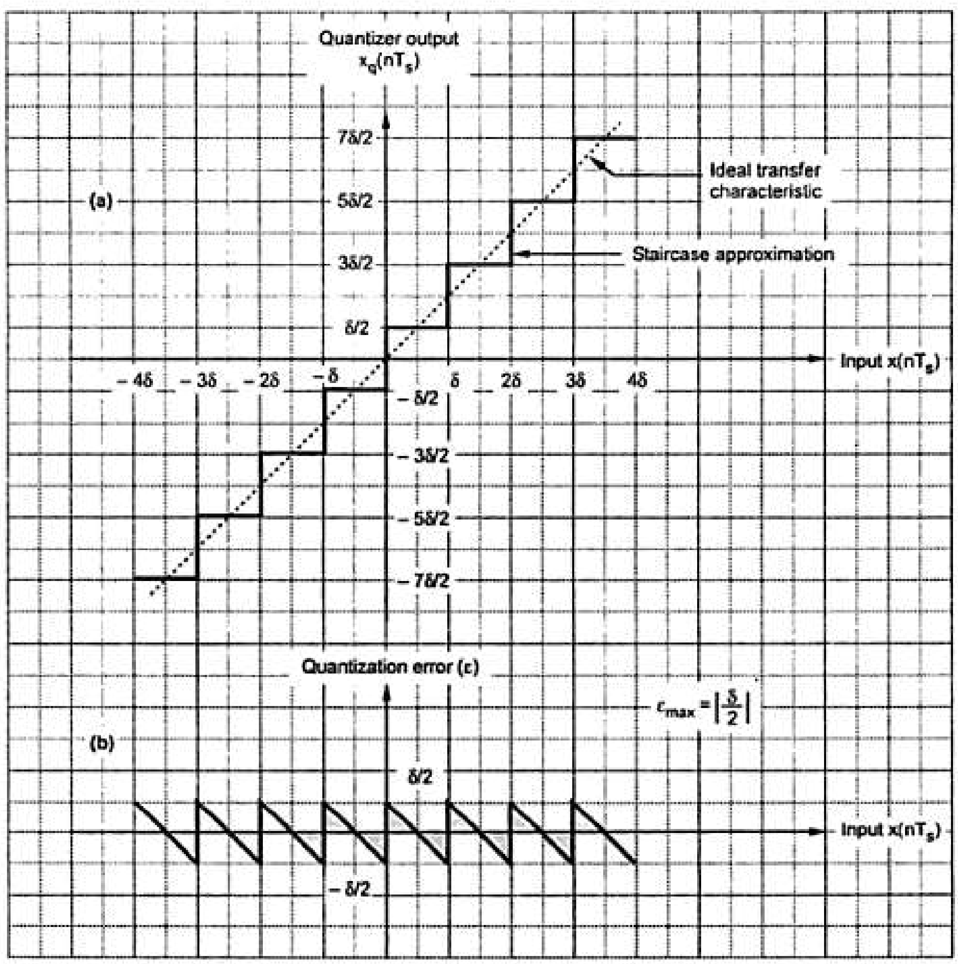

Fig.1.14 (a) Transfer characteristic of midriser quantizer (b) quantization error

$$For\ \ 0 \le x(nT_s) < \delta; \quad x_q(nT_s) = \delta/2$$

$$-\delta \le x(nT_s) < 0; \quad x_q(nT_s) = -\delta/2$$

Similarly when an input is between 3 δ and 4 δ, the output is 7 $\delta/2$. This is called midriser quantizer because its output is either + $\delta/2$ or - $\delta/2$ when input is zero

Fig. 1.14 (b) shows the quantization error in midriser quantization. When input$x(nT_s) = 0$, the quantizer will assign the level of $\delta/2$. Hence quantization error will be,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= \frac{\delta}{2} - 0 = \frac{\delta}{2}$$

Thus the quantization error lies between - $\delta/2$ and + $\delta/2$. i.e.,

$$-\frac{\delta}{2} \le \varepsilon \le \frac{\delta}{2}$$

And the maximum quantization error is,

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

In both the midriser and midtread quantizers, the dotted line of unity slope pass through origin. It represents ideal nonquantized input output characteristic. The staircase characteristic is an approximation of this line. The difference between the staircase and unity slope line represents the quantization error.

1.3.1.3 Biased Quantizer

Fig. 1.15shows the transfer characteristic of biased uniform quantizer

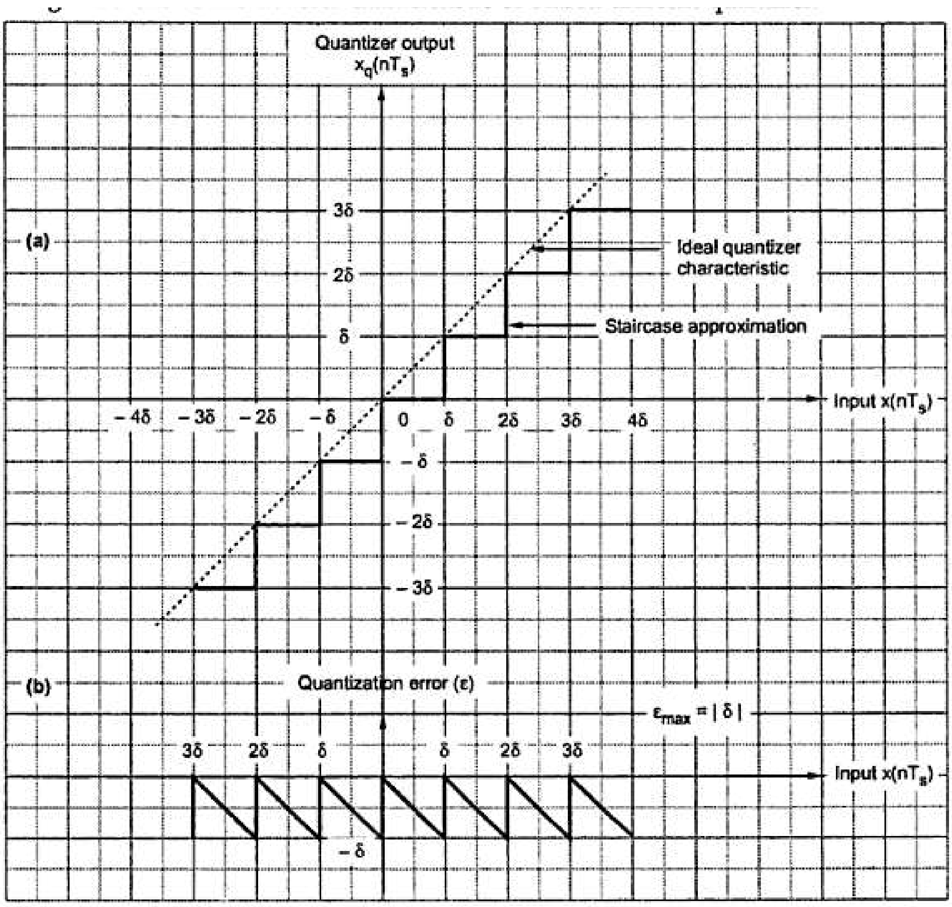

Fig.1.15 (a) Biased quantized transfer characteristic (b) Quantization error

The midriser and mid tread quantizers are rounding quantizers. But biased quantizer is truncation quantizer. This is clear from above diagram. When input is between 0 and δ, the output is zero. i.e.,

$$For\ \ 0 \leq x(nT_s) < \delta; \quad x_q(nT_s) = 0$$

$$-\delta \leq x(nT_s) < 0; \quad x_q(nT_s) = -\delta$$

Fig. 1.15 shows quantization error. When input is δ, output is zero. Hence quantization error is,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= 0 - \ \delta = -\delta$$

Thus the quantization error lies between 0 and $-\delta$i.e.,

$$-\delta \leq \varepsilon \leq 0$$

And the maximum quantization error is,

$$\varepsilon_{max} = |\delta|$$

Thus the quantization error is more in biased quantizer compared to midriser and mid tread quantizers, The unity slope dotted line passes through origin. It represents ideal nonquantized transfer characteristic. The difference between staircase and dotted line gives quantization error.

1.3.2 Non-Uniform Quantization

In nonuniform quantization, the step size is not fixed. It varies according to certain law or as per input signal amplitude. Fig. 1.16 shows the transfer characteristic and error in nonuniform quantization.

In this figure observe that step size is same at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noisepower ratio is improved at low signal levels. Step size is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the dynamic range of quantizer.

1.3.2.1 Necessity of Nonuniform Quantization

In uniform quantization, the quantizer has linear characteristics. The step size also remains same throughout the range of quantizer. Therefore over the complete range of inputs, the maximum quantization error also remains same. We know that, the quantization error is,

$$maximum\ quantization\ error = \varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

We also know that, step size $'\delta'$ is,

$$\delta = \frac{2x_{max}}{q}$$

If $x(t)$ is normalized, its maximum value i.e. $x_{max} = 1$.

$$\therefore \quad \delta = \frac{2}{q}$$

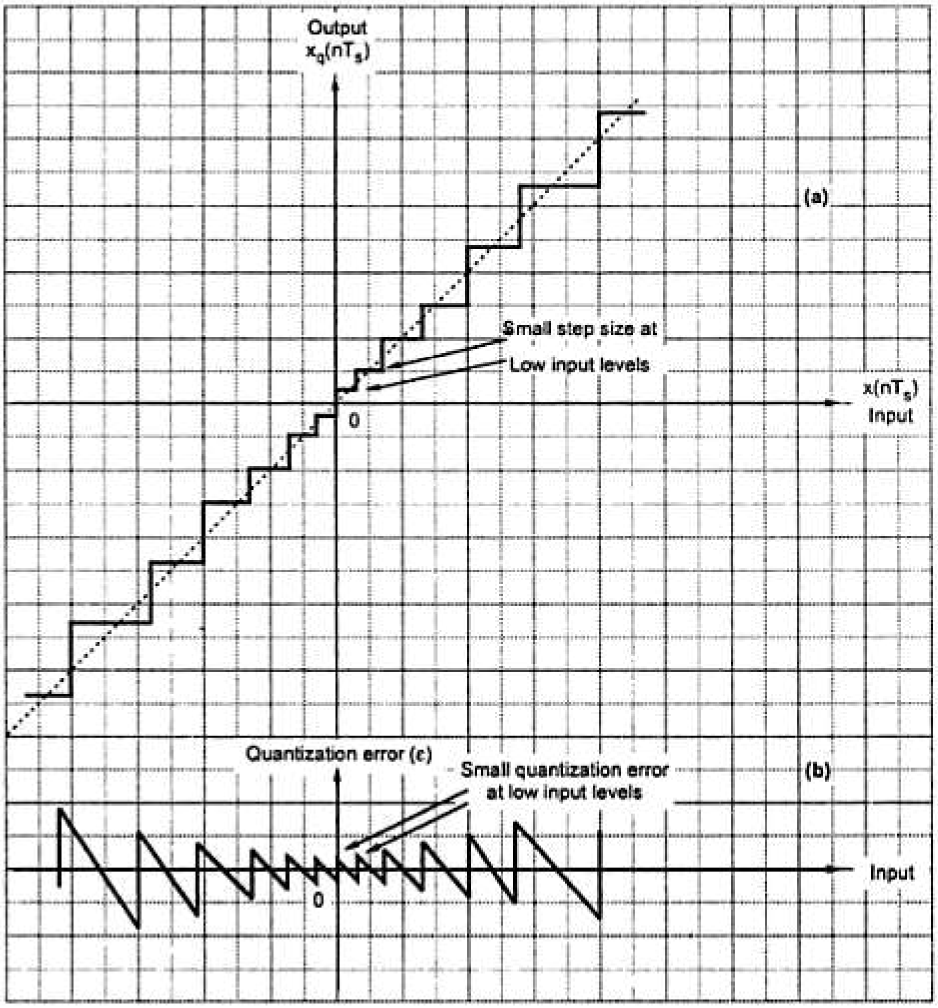

Fig.1.16 (a) Nonuniform quantization transfer characteristic (b) Quantization error

Let us consider an example of PCM system in which v =4 bits.

Then number of levels q will be,

$$q = 2^4 = 16 \; levels$$

∴ the step size δ will be,

$$\delta = \frac{2}{q} = \frac{2}{16} = \frac{1}{8}$$

∴Quantization error is,

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

$$= \left|\frac{1}{2 \times 8}\right| = \frac{1}{16}$$

Thus the quantization error $\frac{1}{16}$ volts of the full range voltage. For simplicity, assume that full range voltage is 16 volts. Then maximum quantization error will be1 volt. For the low signal amplitudes like 2 volts, 3 volts etc., the maximum quantization error of 1 volt is quite high i.e. about 30 to 50%. But for signal amplitudes near 15 volts, 16 volts etc., the maximum quantization error (which is same throughout the range) of 1 volt can be considered to be small. This problem arises because of uniform quantization. Therefore nonuniform quantization should be used in such cases.

1.3.2.2 Necessity of Nonuniform Quantization for Speech Signal

We know that speech and music signals are characterized by large crest factor. That is for such signals the ratio of peak to rms value is very high.

$$Crest\ factor = \frac{Peak\ value}{RMS\ value}$$

$$= very\ high\ for\ speech\ and\ music$$

We know that the signal to noise ratio is given as,

$$\frac{S}{N} = (3 \times 2^{2v} \times P)$$

Expressing in decibels,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}(3 \times 2^{2v} \times P)$$

If we normalize the signal power i.e. if $P = 1$, then above equation becomes

$$\left(\frac{S}{N}\right) dB \geq (4.8 + 6v) dB$$

Here power P is defined as, $P = \frac{V_{signal}^2}{R} = \frac{x^2(t)}{R}$

$$V_{signal}^2 = mean\ square\ of\ signal\ volatge$$

$$= x^2(t)$$

∴ Normalized power will be,$p = \frac{x^2(t)}{1}$ $[with\ R = 1]$

$$P = x^2(t)$$

Crest factor is given as,

$$Crest\ factor = \frac{Peak\ value}{RMS\ value} = \frac{x_{max}}{[x^2(t)]^{1/2}}$$

$$= \frac{x_{max}}{\sqrt{P}} \qquad since\ P = x^2(t)$$

When we normalize the signal $x(t)$,then

$$x_{max} = 1$$

Putting above value of x_{max} in equation $crest\ factor = \frac{x_{max}}{\sqrt{P}}$,

$$Crest\ factor = \frac{1}{\sqrt{P}}$$

For a large crest factor of voice (speech) and music signals P should be very veryless than one in above equation.

$$ie., \qquad P \ll 1 \qquad for\ large\ crest\ factor$$

Therefore actual signal to noise ratio will be significantly less than the value that isgiven by equation $\left(\frac{S}{N}\right) dB \geq (4.8 + 6v)dB$, since in this equation $P = 1$.

Consider,

$$\frac{S}{N} = (3 \times 2^{2v} \times P)$$

$$(3 \times 2^{2v} \times P)|_{p\ll 1} \ll (3 \times 2^{2v} \times P)|_{p=1}$$

This equation shows that the signal to noise ratio for large crest factor signal (P<< 1) will be very very less than that of the calculated theoretical value. The theoretical value is obtained for normalized power (P =1)by equation $\frac{S}{N} = (3 \times 2^{2v} \times P)$

Therefore such large crest factor signals (speech and music) should use nonuniform quantization to overcome the above problem. At low signal levels *(P<< 1)*noise can be kept low by keeping step size low. This means that at low signal levels signal to noise ratio can be increased by decreasing step size' δ'. This means step sizeδshould be varied according to the signal level to keep signal to noise ratio at therequired value. This is nothing but nonuniform quantization.

1.4 QUANTIZATION NOISE

1.4.1 Derivation of Quantization Error/Noise or Noise Power for Uniform (Linear) Quantization

Step 1 : Quantization Error

Because of quantization, inherent errors are introduced in the signal. This error is called **quantization error**. We have defined quantization error as,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

Step 2 : Step size

Let an input $x(nT_s)$be of continuous amplitude in the range $-x_{max}\ to\ +x_{max}$

From Fig. 1.14 (a) we know that the total excursion of input $x(nT_s.)$is mappedinto $'q'$ levels on vertical axis. That is when input is 4δ,output is $\frac{7}{2}\delta$ and when inputis -4δ, output is $-\frac{7}{2}\delta$. That is $+x_{max}$represents $\frac{7}{2}\delta$ and $-x_{max}$represents $-\frac{7}{2}\delta$. Therefore the total amplitude range becomes,

$$Total\ amplitude\ range = x_{max} - (-x_{max})$$

$$= 2x_{max}$$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ'is given as,

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$\frac{2x_{max}}{q}$$

If signal $x(t)$is normalized to minimum and maximum values equal to 1, then

$$x_{max} = 1$$

$$-x_{max} = -1$$

Therefore step size will be,

$$\delta = \frac{2}{q}\ \ (for\ normalized\ signal)$$

Step 3: Pdf of Quantization error

If step size 'δ' is sufficiently small, then it is reasonable to assume that the quantization error $'\varepsilon'$ will be uniformly distributed random variable. The maximumquantization error is given as,

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right|$$

$$i.e.,\quad -\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2}$$

Thus over the interval $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ quantization error is uniformly distributed random variable.

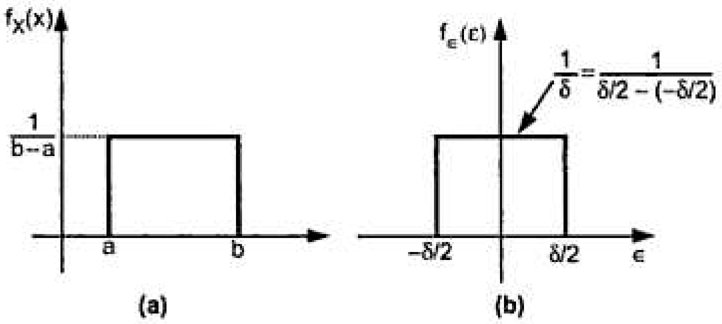

Fig.1.17 (a) Uniform distribution (b) Uniform distribution for quantization error

In above figure, a random variable is said to be uniformly distributed over an interval (a, b). Then PDF of 'X' is given by, (from equation of Uniform PDF).

$$f_X(x) = \begin{cases} 1, & for\ x \leq a \\ \frac{1}{b-a}, & for\ a < x \leq b \\ 0, & for\ x > b \end{cases}$$

Thus with the help of above equation we can define the probability density function for quantization error ' ε' as,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0, & for\ \varepsilon \leq \frac{\delta}{2} \\ \frac{1}{\delta}, & for -\frac{\delta}{2} < \varepsilon \leq \frac{\delta}{2} \\ 0, & for\ \varepsilon > \frac{\delta}{2} \end{cases}$$

Step 4: Noise Power

From Fig. 1.14 (b) we can see that quantization error ' ε' has zero average value.That is mean $'m_\varepsilon'$of the quantization error is zero.

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{Signal\ power\ (normalized)}{Noise\ power\ (normalized)}$$

If type of signal at input i.e., $x(t)$is known, then it is possible to calculate signal power.

The noise power is given as,

$$Noise\ Power = \frac{V_{noise}^2}{R}$$

Here V^2_{noise} is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$, its mean square value is given as,

$$mean\ square\ value = E\{\varepsilon^2\} = \varepsilon^{-2}$$

The mean square value of a random variable 'X' is given as,

$$\bar{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx \quad By\ definition$$

$$Here, \quad E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)$$

From equation $f_\varepsilon(\varepsilon) = \begin{cases} 0, & for\ \varepsilon \le \frac{\delta}{2} \\ \frac{1}{\delta}, & for\ -\frac{\delta}{2} < \varepsilon \le \frac{\delta}{2} \\ 0, & for\ \varepsilon > \frac{\delta}{2} \end{cases}$ we can write above equation as,

$$E\{\varepsilon^2\} = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 \times \frac{1}{\delta}\, d\varepsilon$$

$$= \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}} = \frac{1}{\delta}\left[\frac{\left(\frac{\delta}{2}\right)^3}{3} - \frac{\left(\frac{\delta}{2}\right)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12}$$

From equation $mean\ square\ value = E[\varepsilon^2] = \varepsilon^{-2}$,

the mean square value of noise voltage is,

$$V^2_{noise} = mean\ square\ value = \frac{\delta^2}{12}$$

When load resistance, R = 1 ohm, then the noise power is normalized i.e.,

$$Noise\ power\ (normalized) = \frac{V^2_{noise}}{1} \qquad [with\ R = 1]$$

$$= \frac{\delta^2/12}{1} = \frac{\delta^2}{12}$$

Thus we have,

For normalized power or Quantization noise power $= \frac{\delta^2}{12}$;

For linear quantization or Quantization error (in terms of power)

1.4.2 Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization

Signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{normalized\ Signal\ power}{normalized\ Noise\ power}$$

$$= \frac{normalized\ Signal\ power}{\delta^2/12} \qquad --(1)$$

The number of bits $'v'$ and quantization levels 'q' are related as,

$$q = 2^v$$

Putting this value in equation $\delta = \frac{2x_{max}}{q}$ we have,

$$\delta = \frac{2x_{max}}{2^v}$$

Putting this value in equation (1) get,

$$\frac{S}{N} = \frac{Normalizaed\ signal\ power}{\left(\frac{2x_{max}}{2^v}\right)^2 \times \frac{1}{12}}$$

Let normalized signal power be denoted as ‘P’.

$$\frac{S}{N} = \frac{P}{\frac{4x^2{}_{max}}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x^2{}_{max}} . 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus,

$$\boxed{Maximum\ signal\ to\ quantization\ noise\ ratio : \frac{S}{N} = \frac{3P}{x^2{}_{max}} . 2^{2v}}$$

This equation shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input *x(t)* is normalized, i.e.,

$$x_{max} = 1$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P$$

If the destination signal power 'P' is normalized, i.e.,

$$P \leq 1$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v}$$

Since$x_{max} = 1$ and $P \leq 1$, the Signal to noise ratio given by above equation is normalized.

Expressing the signal to noise ratio in decibels,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \qquad since\ power\ ratio$$

$$\leq 10 \log_{10}[3 \times 2^{2v}]$$

$$\leq (4.8 + 6v) dB$$

Thus, Signal to Quantization noise ratio for normalized values of power $=\left(\frac{S}{N}\right) dB \leq (4.8 + 6\,v) dB$ 'P' and amplitude of input $x(t)$

1.5 LOGARITHMIC COMPANDING OF SPEECH SIGNAL

Normally the variation of signal level will not be known in advance. Therefore the nonuniform quantization (variable step size$'\delta'$) becomes difficult to implement.Therefore the signal is amplified at low signal levels and attenuated at high signallevels. After this process, uniform quantization is used. This is equivalent to more step size at low signal levels and small step size at high signal levels. At the receiver a reverse process is done. That is signal is attenuated at low signal levels and amplified at high signal levels to get original signal. Thus the compression of signal at transmitter and expansion at receiver is called combinely as **companding.**

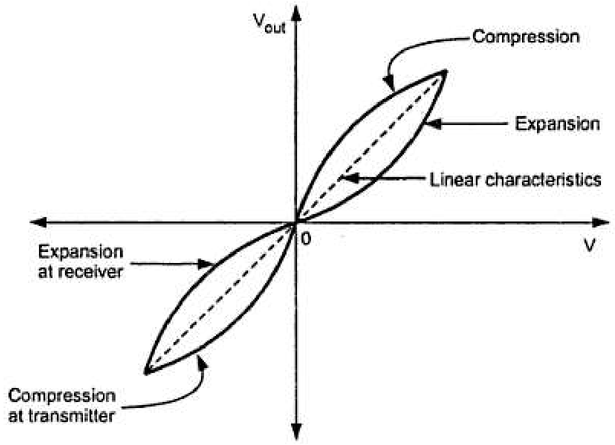

Fig. 1.18 Companding curves for PCM

Fig. 1.18 shows compression and expansion curves. From Fig. 1.18, at the receiver, the signal is expanded exactly opposite to compression curve at transmitter to get original signal. A dotted line in the Fig. 1.18 shows uniform quantization. The compression and expansion is obtained bypassing the signal through the amplifier having nonlinear transfer characteristic as shown in Fig. 1.18. That is nonlinear transfer characteristic means compression and expansion curves.

1.5.1 $\mu -$ LawCompandingfor Speech Signals

Normally for speech and music signals a μ - law compression is used. Thiscompression is defined by the following equation,

$$Z(x) = (sgn\ x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)}\ |x| \leq 1$$

Fig. 1.19 shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

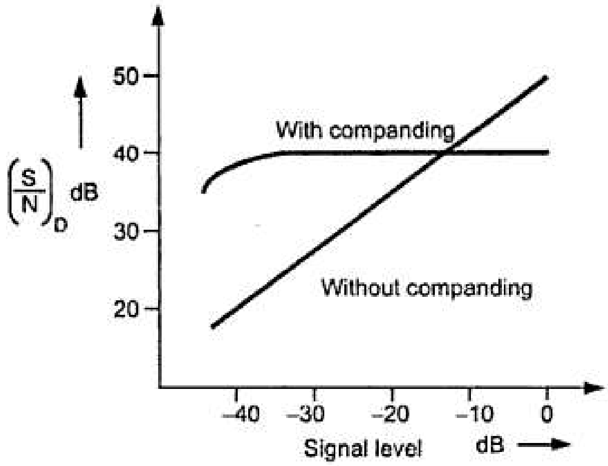

Fig. 1.19 PCM performance with μ – law companding

It can be observed from above figure that signal to noise ratio of PCM remains almost constant with companding.

1.5.2 A - Law for Companding

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

$$Z(x) = \begin{cases} \frac{A|x|}{1+\ln A}, & for\ 0 \leq |x| \leq \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln A}, & for \frac{1}{A} \leq |x| \leq 1 \end{cases}$$

When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ–law companding is used for PCM telephone systems.

1.5.3 Signal to Noise Ratio of Companded PCM

The signal to noise ratio of companded PCM is given as,

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2}$$

Here $q = 2^v$ is number of quantization levels.

1.6 PULSE CODE MODULATION

1.6.1 PCM Generator

The pulse code modulator technique samples the input signal $x(t)$at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog todigital converter. The parallel bits obtained are converted to a serial bit stream.Fig.1.20 shows the PCM generator

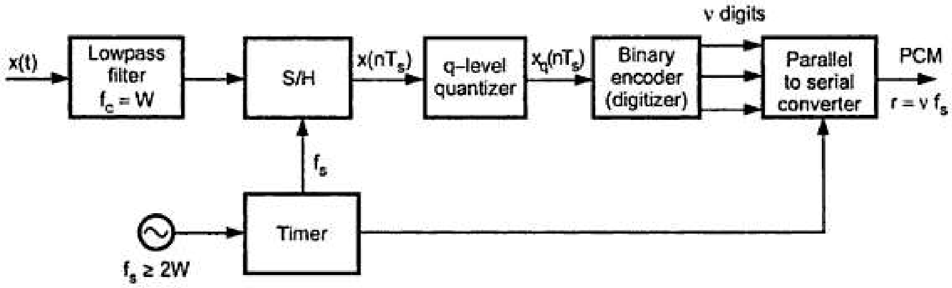

Fig.1.20 PCM generator

In the PCM generator of above figure, the signal $x(t)$is first passed through thelowpass filter of cutoff frequency 'W' Hz. This lowpass filter blocks all the frequencycomponents above 'W' Hz. Thus $x(t)$is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency $f_s$is selectedsufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 1.20 output of sample and hold is called $x(nT_s)$.This $x(nT_s)$is discrete intime and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$with itsfixed digital levels, It assigns anyone of the digital level to $x(nT_s)$with its fixeddigital levels. It then assigns anyone of the digital level to $x(nT_s)$which results inminimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

The quantized signal level$x_q(nT_s)$is given to binary encoder. This encoder converts input signal to 'v' digitsbinary word. Thus $x_q(nT_s)$is converted to 'V' binary bits. The encoder is also called digitizer.

It is not possible to transmit each bit of the binary word separately on transmission line. Therefore *'v'* binary digits are converted to serial bit stream to generate single baseband signal. The output of PCM generator is thus a single baseband signal of binary bits.

An oscillator generates the clocks for sample and hold an parallel to serial converter. In the pulse code modulation generator sample and hold quantizer and encoder combinely form an analog to digital converter.

1.6.2 Transmission Bandwidth in PCM

Let the quantizer use $'v'$ number of binary digits to represent each level Then the number of levels that can be represented by $'v'$ digits will be,

$$q = 2^v \qquad --(1)$$

Here $'q'$ represents total number of digital levels of q-level quantizer. For example if v =3 bits, then total number of levels will be,

$$q = 2^3 = 8\ levels$$

Each sample is converted to $'v'$ binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per .second = f_s

∴ Number of bits per second is given by,

(Number of bits per second) = (Number of bits per samples) x (Number of samples per second)

= v bits per sample x $f_s$samples per second --(2)

The number of bits per second is also called signaling rate of PCM and is denoted by 'r' i.e.,

$$\boxed{Signaling\ rate\ in\ PCM: r = vf_s} \qquad --(3)$$

Here, $f_s \geq 2W$

Bandwidth needed for PCM transmission will be given by half of the signaling **rate** i.e.,

$$Transmission\ bandwidth\ of\ PCM: \begin{cases} B_T \geq \frac{1}{2}r \\ B_T \geq \frac{1}{2}v\,f_s \\ B_T \geq vW \end{cases} \quad since\ f_s \geq 2W \qquad --(4,5,6)$$

1.6.3 PCM Receiver

Fig. 1.21 (a) shows the block diagram of PCM receiver and Fig. 1.21 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.

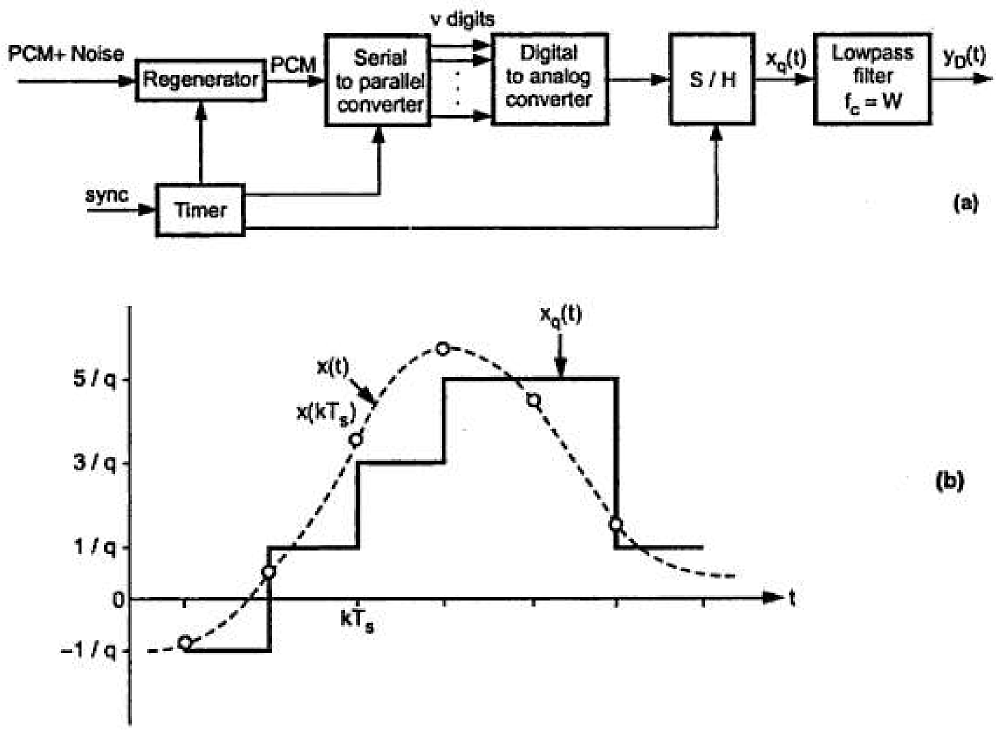

Fig.1.21 (a) PCM receiver (b) Reconstructed waveform

The digital word is converted to its analog value $x_q(t)$ along with sample andhold. This signal, at the output of S/H is passed through lowpass reconstruction filterto get $y_D\ (t)$.As shown in reconstructed signal of Fig. 1.21 (b), it is impossible to reconstruct exact original signal $x\ (t)$ because of permanent quantization error introduced during quantization at the transmitter. This quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits $'v'$ increases the signaling rate as well as transmission bandwidth as we have seen in equation (3) and equation (6).Therefore the choice of these parameters is made, such that noise due to quantization error (called as quantization noise) is in tolerable limits.

1.7 TIME DIVISION MULTIPLEXING (TDM SYSTEM)

In PAM, PPM and PDM the pulse is present for short duration and form most of the time between the two pulses, no signal is present. This free space between the pulses can be occupied by pulses from other channels. This is called Time Division Multiplexing (TDM). It makes maximum utilization of the transmission channel.

1.7.1Block Diagram of TDM

Fig.1.22 (a) shows the block diagram of a simple TDM system and Fig. 1.22 (b) shows the waveforms of the system. The system shows the time division multiplexing of 'N' PAM channels. Each channel to be transmitted is passed through the lowpass filter. The outputs of the lowpass filters are connected to the rotating sampling switch or commutator. It takes the sample from each channel per revolution and rotates at the rate of f_s.

Thus the sampling frequency becomes f_s.The single signal composed due tomultiplexing of input channels is given to the transmission channel. At the receiver the decommutator separates (decodes) the time multiplexed input channels. These channel signals are then passed through lowpass reconstruction filters

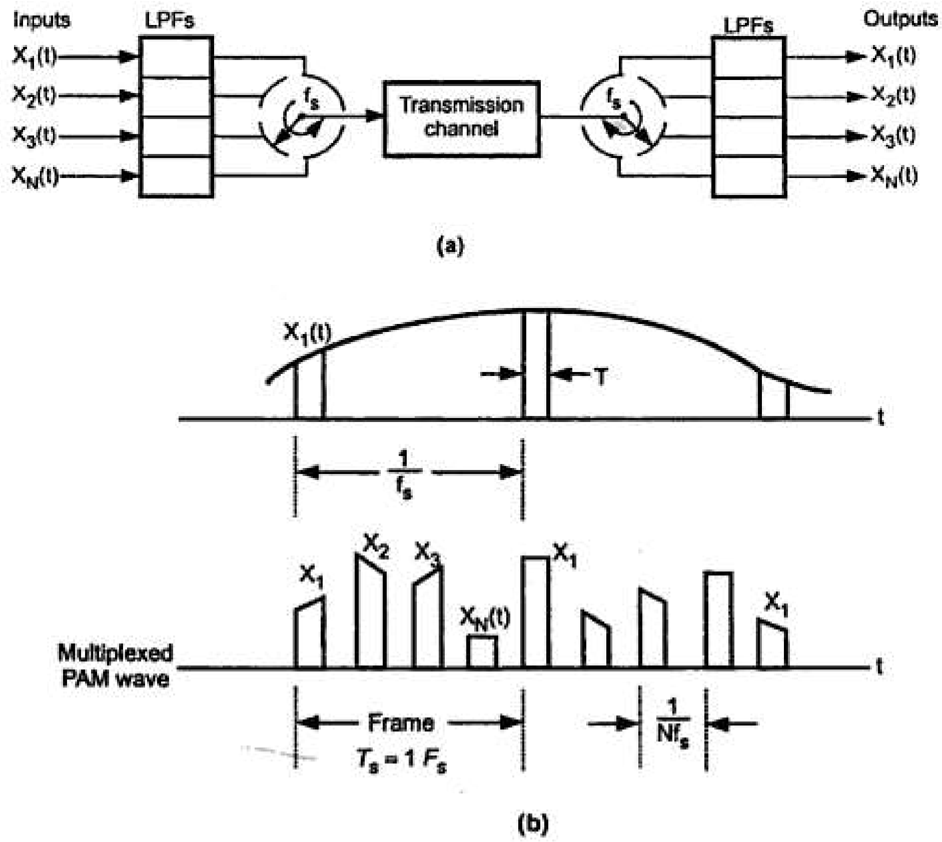

Fig.1.22 TDM system (PAM/TDM system) (a) Block diagram (b) Waveforms

If the highest signal frequency present in all the channels is W, then by sampling theorem the sampling frequency $f_s$should be,

$$f_s \geq 2W$$

Therefore the time space between successive samples from any one input will be

$$T_s = \frac{1}{f_s}$$

$$\therefore \quad T_s \leq \frac{1}{2W}$$

Thus the time interval T_s contains one sample from each input. This time interval is called frame. Let there be 'N' input channels. Then in each frame there will be one sample from each of the 'N' channels. That is one frame of Ts seconds contain total 'N' samples. Therefore pulse to pulse spacing between two samples in the frame will be equal to $\frac{T_s}{N}$

:. Spacing between two samples $\frac{T_s}{N}$.

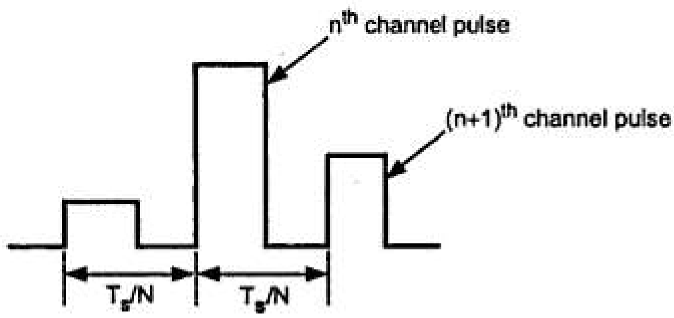

Fig.1.23 Calculation of number of pulses per second in TDM

From the above figure we can very easily calculate the number of pulses per second or pulse frequency as,

$$Number\ of\ pulses\ per\ second = \frac{1}{spacing\ between\ two\ pulses}$$

$$= \frac{1}{T_s/N}$$

$$= \frac{N}{T_s}$$

$$we\ know\ that, \qquad T_s = \frac{1}{f_s}$$

$$\therefore \quad Number\ of\ pulses\ per\ second = \frac{N}{1/f_s} = Nf_s$$

These number of pulses per second is also called signaling rate of TDM signal and is denoted by 'r' i.e.,

$$Signalling\ rate = r = Nf_s$$

since, $f_s \geq 2W$, then signalling rate becomes,

$$\boxed{Signaling\ rate\ in \frac{PAM}{TDM} system : r \geq 2NW}$$

The RF transmission of TDM needs modulation. That is TDM signal should modulate some carrier. Before modulation, the pulsed signal in TDM is converted to baseband signal. That is pulsed TDM signal is converted to smooth modulating waveform $x_b(t)$; the baseband signal that modulates the carrier. The baseband signal$x_b(t)$ passes through all the individual sample values baseband signal is obtained bypassing pulsed TDM signal through lowpass filter. The bandwidth of this lowpass filter is given by half of the signalling rate. i.e.,

$$B_b = \frac{1}{2} r = \frac{1}{2} N f_s$$

∴ Transmission bandwidth of TDM channel will be equal to bandwidth of the lowpass filter,

$$\therefore \quad B_T = \frac{1}{2} N f_s$$

If sampling rate becomes equal to Nyquist rate i.e.,

$$f_s(min) = Nyquist\ rate = 2W, \qquad then$$

$$B_T = \frac{1}{2} N \ \times 2W$$

$$\boxed{Minimum\ transmission\ bandwidth\ of\ TDM\ channel : B_T = NW}$$

This equation shows that if there are total 'N' channels in TDM which are bandlimited to 'W' Hz, then minimum bandwidth of the transmission channel will be equal to 'NW'.

2 MARKS WITH ANSWERS

1. Define Nyquist rate

Let the signal be bandlimited to $'W'Hz$. Then Nyquist rate is given as,

$$Nyquist\ rate = 2\ W\ sample/sec$$

Aliasing will not take place if sampling rate is greater than Nyquist rate

2. What is meant by aliasing effects?

Aliasing effect takes place when sampling rate is greater than Nyquist rate. Under such condition, spectrum of the sampled signal overlaps with itself. Hence higher frequencies take form of lower frequencies. This interference of the frequency components is called aliasing effect.

3. State sampling theorem.