$$\boxed{Signaling\ rate\ in\frac{PAM}{TDM}system : r \geq 2NW}$$

The RF transmission of TDM needs modulation. That is TDM signal should modulate some carrier. Before modulation, the pulsed signal in TDM is converted to baseband signal. That is pulsed TDM signal is converted to smooth modulating waveform $x_b(t)$; the baseband signal that modulates the carrier. The baseband signal$x_b(t)$ passes through all the individual sample values baseband signal is obtained bypassing pulsed TDM signal through lowpass filter. The bandwidth of this lowpass filter is given by half of the signalling rate. i.e.,

$$B_b = \frac{1}{2}r = \frac{1}{2}Nf_s$$

∴ Transmission bandwidth of TDM channel will be equal to bandwidth of the lowpass filter,

$$\therefore \quad B_T = \frac{1}{2}Nf_s$$

If sampling rate becomes equal to Nyquist rate i.e.,

$$f_s(min) = Nyquist\ rate = 2W, \qquad then$$

$$B_T = \frac{1}{2}N \times 2W$$

$$\boxed{Minimum\ transmission\ bandwidth\ of\ TDM\ channel : B_T = NW}$$

This equation shows that if there are total 'N' channels in TDM which are bandlimited to 'W' Hz, then minimum bandwidth of the transmission channel will be equal to 'NW'.

2 MARKS WITH ANSWERS

1. Define Nyquist rate

Let the signal be bandlimited to $'W'Hz$. Then Nyquist rate is given as,

$$Nyquist\ rate = 2\ W\ sample/sec$$

Aliasing will not take place if sampling rate is greater than Nyquist rate

2. What is meant by aliasing effects?

Aliasing effect takes place when sampling rate is greater than Nyquist rate. Under such condition, spectrum of the sampled signal overlaps with itself. Hence higher frequencies take form of lower frequencies. This interference of the frequency components is called aliasing effect.

3. State sampling theorem.

Sampling theorem states that, a bandlimited signal of finite energy, which has no frequency components higher than $W\ Hz$, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$ seconds and,

A bandlimited signal of finite energy, which has no frequency components higher than WHz, may be completely recovered from the knowledge of its samples taken at the rate of $2W$ samples per second.

The statement of above theorem can also be written as,

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency $f_s \geq 2W$. Here f_s is the sampling frequency and W is the maximum frequency present in the signal.

4. What is meant by quantization ?

While converting the signal value from analog to digital, quantization is performed. The analog value is assigned to the nearest digital level. This is called quantization. The quantized value is then converted to equivalent binary value. The quantization levels are fixed depending upon the number of bits. Quantization is performed in every analog to digital conversion.

5. State bandpass sampling theorem.

The bandpass signal *x(t)* whose maximum bandwidth is 2Wcan be completely represented into and recovered from its samples. If it is sampled at the minimum rate of twice the bandwidth.

6. What should be the passband for antialiasing and smoothing filters used with pulse modulation / demodulation systems.

(i) Antialiasing filter used before sampling. It should bandlimit the signal to maximum signal frequency of *'W'* Hz. Hence its passband should be 'W'Hz .

(ii) Smoothing filter is used after reconstruction or interpolation. It should successfully pass all the frequencies of 0 to *'W'* Hz and block frequencies greater than 'W'Hz.

7. What is the interpolatory property for sine function ?

The sine function is used for interpolation of signal from its samples.

The interpolated signal is given as,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\ \mathrm{sinc}(2Wt - n)$$

Here $F_s = 2W$ Hence $T_s = \frac{1}{2W}$

$$\therefore \quad x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \left(\frac{t}{T_s} - n \right)$$

Above equation shows that sine function will assume zero value when $t = \pm T_s, \pm 2T_s, \pm 3T_s, \dots \dots \dots$ and so on.

Thus the main lobe of sine function contributes for interpolation. It is weighted by $x(nT_s)$, i.e. sample value.

8. What do you understand by the term aliasing? [Q.no 2]

9. A bandpass signal has the spectral range that extends from 20 kHz to 82 kHz. Find the acceptable range of sampling frequency f_s

$$\begin{aligned} \text{BW} &= 82 \text{ kHz} - 20 \text{ kHz} = 62 \text{ kHz} \\ f_s &= 2 \times \text{BW} \\ &= 2 \times 62 \text{ kHz} = 124 \text{ kHz} \end{aligned}$$

10. What is the SNR of PCM system if number of quantization levels is 2^8?

$$\begin{aligned} \text{Quantization levels} &= 2^v = 2^8 \\ v &= 8 \\ \left(\frac{S}{N}\right) \text{dB} &= 4.8 + 6v \text{ dB} \\ &= 4.8 + 6 \times 8 = 52.8 \text{ dB} \end{aligned}$$

11. State sampling theorem. (Refer answer of Q.3.)

12. Define quantization error.

Quantization error : Because of quantization, inherent errors are introduced in the signal. This error is called quantization error. It is expressed mathematically as,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

Here $x_q(nT_s)$ is quantized value of the signal.
$x(nT_s)$ is the value of the sample, before quantization.

13. A message has zero mean value and a peak value of 10 V. It is to be quantized using a step size of 0.1 V with one level coinciding to 0 V. Find number of bits required for encoding the quantized signal.

$$\text{Step size } (\delta) = \frac{2x_{max}}{q}$$

Here x_{max} is peak amplitude of the signal.

$$\therefore \quad 0.1 = \frac{2 \times 10}{q} \Rightarrow q = 200 \text{ levels}$$

$$\begin{aligned} \text{Number of bits } (v) &= \log_2 q = \frac{\log_{10} q}{\log_{10} 2} = \frac{\log 200}{\log 2} \\ &= 7.643 \simeq 8 \end{aligned}$$

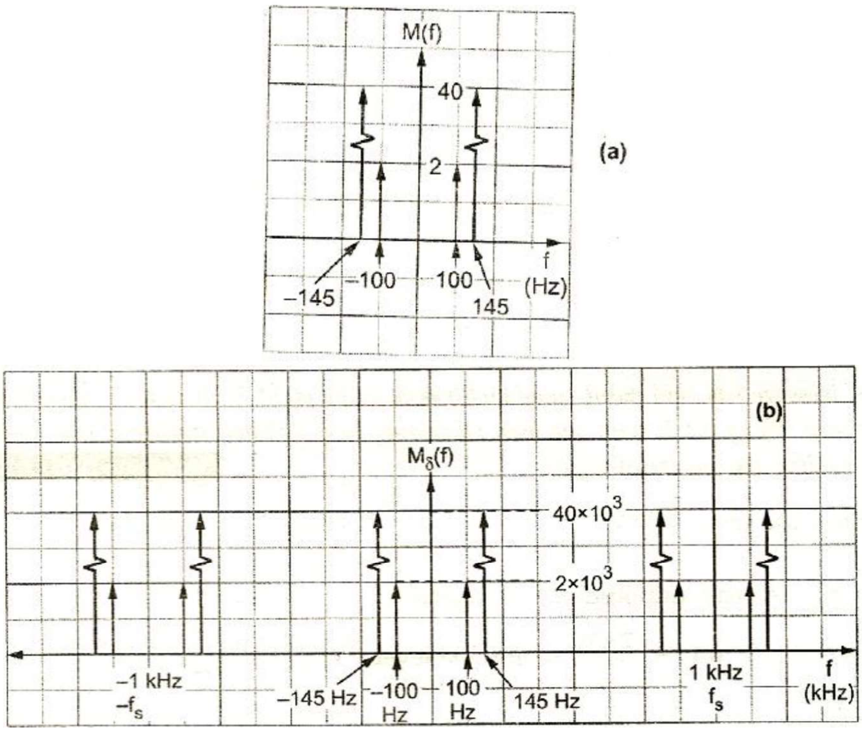
14. Plot the magnitude spectrum of the ideally sampled version of the signal m(t) = 2 cos (200 n t) + 40 sin (290 1t t). Assuming that the sampling rate is 1kHz.

Here *met)* = 2 cos (200 *n* t) + 40 sin (290 rt *t)*

Let us compare above equation with,

$$\begin{aligned} m(t) &= A_1 \cos(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) \\ A_1 &= 2, \quad f_1 = 100 \text{ Hz and} \\ A_2 &= 40, \quad f_2 = 145 \text{ Hz} \end{aligned}$$

Fig. (a) shows the spectrum of $m(t)$ and Fig. (b) shows the spectrum of sampled signal.



15. For a uniform quantizer, discuss the way in which the number of quantization levels (L) influence the bandwidth and the quantization noise.

$$\text{Number of levels } L = 2^v \quad \Rightarrow \quad v = \log_2 L$$

$$\text{Bandwidth} = v\,W = W \log_2 L$$

$$\text{Quantization noise} = \frac{\delta^2}{12}$$

$$= \frac{(2x_{max} / q)^2}{12} \quad \text{since} \quad \delta = \frac{2x_{max}}{q}$$

$$= \frac{x^2{}_{max}}{3q^2}$$

$$= \frac{x^2{}_{max}}{3L^2}, \quad \text{since levels } q = L.$$

16. Define quantization noise power.

Quantization noise power is the noise power due to quantization noise. Let the quantization noise has the pdf of $f_\epsilon(\varepsilon)$.Then quantization noise power is given as,

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

17. Why is prefiltering done before sampling?

• The signal must be limited to some highest frequency 'W' Hz before sampling. Then the signal is sampled at the frequency of $f_s = 2W$ or higher.

• Hence the signal should be prefiltered (low pass filtered) to eliminate any frequency components higher than 'W' Hz.

• If the signal is not prefiltered, then frequency components higher than 'W' Hz will generate aliasing in the sampled signal spectrum.

18. Define quantization noise.

When the signal is converted from analog to digital form, the analog sample amplitude is assigned the nearest available quantization amplitude level. The difference between quantized value and actual value of the sample introduces permanent distortion in the signal. It is called quantization error or quantization noise.

Thus,

$$Quantization\ error\ or\ noise, \varepsilon = x_q(nT_s) - x(nT_s)$$

Hence, $x_q(nT_s)$ is quantized value of sample and $x(nT_s)$ is actual value of the sample.

19. Write an expression for bandwidth of binary PCM with N messages each with a maximum frequency of f_m Hz.

If 'v' number of bits are used to code each input sample, then bandwidth of PCM is given as,

$$B_T \geq N \cdot v \cdot f_m$$

Here $v.f_m$ is the bandwidth required by one message.

20. The signal to quantization noise ratio in a PCM system depends on ...

The signal to quantization noise ratio in PCM is given as,

$$\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6\,v)\ \text{dB}$$

Here v is the number of bits used to represent samples in PCM. Hence signal to quantization noise ratio in PCM depends upon number of bits or quantization levels

21. For the transmission of normal speech signal in the PCM channel needs the B. W. of ...

Speech signals have the maximum frequency of 3.4 kHz. Normally 8 bits PCM is used for speed. The transmission bandwidth of PCM is given as,

$$\begin{aligned} B_T &\geq vW \\ &\geq 8 \times 3.4 \text{ kHz} \qquad \text{i.e. } 27.2 \text{ kHz} \end{aligned}$$

22. It is required to transmit speech over PCM channel with 8-bit accuracy. Assume the speech in baseband limited to 3.6 kHz. Determine the bit rate.

The signaling rate in PCM is given as,

$$r = vf_s$$

Here v number of bits i.e. 8

The maximum signal frequency is W = 3.6 kHz. Hence minimum sampling frequency will be,

$$\begin{aligned} f_s &= 2\text{ W} = 2 \times 3.6 \text{ kHz} = 7.2 \text{ kHz} \\ r &= 8 \times 7.2 \times 10^3 \\ &= 57.6 \text{ kbits/sec} \end{aligned}$$