UNIT 4

DIGITAL MODULATION SCHEME

Geometric Representation of signals - Generation, detection, PSD & BER of Coherent BPSK, BFSK & QPSK - QAM - Carrier Synchronization - structure of Non-coherent Receivers - Principle of DPSK.

4.1 GEOMETRICAL REPRESENTATION OF SIGNALS

The geometric representation of signals greatly simplifies the signal detection process. The geometric or vector view of signal waveforms is highly useful for detection of transmitted signals.

The complete set of all signals is called a **signal space**. Let an arbitrary signal 's', have any arbitrary number of components *M.* So, it can be represented by a linear combination of *M* number of functions $\hat{e}_i$, along the respective component directions:

$$s = a_1\hat{e}_1 + a_2\hat{e}_2 + \cdots + a_M\hat{e}_M$$

There are *M* no. of a_i, variables to completely describe the signal 's'. If there is another representation of the same signal s in terms of another set of *N* number of functions, say $\hat{f}_i$, where *N* is less than *M.* Then we can conclude that the second representation is better because lesser number of variables are required to represent the signal.

$$s = b_1\hat{f}_1 + b_2\hat{f}_2 + \cdots + b_N\hat{f}_N$$

Any processing done on the signal will require lesser number of steps if the second representation is selected. Moreover, if we need to store the signal, lesser storage space would be required. Further, if we want to transmit the signal, we need to transmit lesser number of unknowns thereby preserving the communication bandwidth. So, in communication signal theory, a fundamental question is: at the minimum what is the number of variables that is required to represent a signal? To answer this question, the concept of basis function is introduced.

Basis functions are the collection of the minimum number of functions necessary to represent a given signal. It is clear that the basis functions are independent i.e., no basis function is derivable by the linear combination of any other group of basis functions. An important corollary of this property is that the basis functions are always orthogonal to each other, i.e., the projection of one basis function on any other basis function is zero. Mathematically

$$\int_0^T \psi_j(t)\psi_k(t)\,dt = K_j\delta_{jk}, \qquad 0 \le t \le T, \qquad j,k = 1,\ldots\ldots,N \quad --(1)$$

where K_j is a non-zero constant and δ_{jk} is the well known Kronecker delta function given by

$$\delta_{jk} = \begin{cases} 1, & j = k \\ 0, & otherwise \end{cases}$$

For any two arbitrary time signal $s_1(t)$ and $s_2(t)$, the operation $\int_0^T s_1(t)s_2(t)\,dt$ is called the inner product of the two time signals over the interval $[0, T]$. Therefore, the orthogonality of the basis functions requires that the inner product of two different basis function should always be zero.

In signal-space theory, all signals belonging to a signal space requires the same minimum number of basis functions to represent them. This minimum number of basis functions is called the **dimension of the signal space**. We define an N-dimensional signal space as a space characterized by a set of N linearly independent functions $\psi_j(t)$, called **basis functions**. Any arbitrary signal in the space can be generated by a linear combination of these basis functions. The collection of the basis functions is called **basis set**. A signal in a signal space can be characterised by many basis sets. All of them are orthogonal among themselves and the number of their member functions are same and equal to the dimension of the signal space.

We note that in Eq. (1), when $k = j$, the LHS represents the 'energy' (i.e. projection of the basis function $\psi_j(t)$ upon itself). When the 'energy' for all the basis functions are normalised so that each $K_j = 1$, the basis set is called an **orthonormal basis set**. So, orthonormal basis set is a special case of basis sets.

The advantage of the orthogonal property of the basis set is that each basis function $\psi_j(t)$ is independent of the other members of the set. So, no basis function interferes with any other basis function in the detection process. From a geometric point of view, each basis function is mutually perpendicular to each of the other basis functions. Hence, Euclidean distance calculations get simplified the basis set is used.

In a digital communication system, only a few logical levels of the input signal are supported. A particular signal waveform is transmitted for each of these levels. The set of all these signal waveforms is called **signalling set**. If signalling set is identical to a basis set, then the signaling waveforms become orthogonal and the detector can easily detect them. However, even if the signaling waveforms do not form such an orthogonal set, they can be transformed into linear combination of orthogonal waveforms.

A**ny arbitrary** finite set of waveforms $s_k(t)$ $(k = 1, \ldots, M)$, where each member of the set is physically realizable and of duration T, can be expressed as a linear combination of N orthogonal waveforms $\psi_1(t), \psi_2(t), \ldots, \psi_j(t)$, where $N \leq M$, such that

$$s_1(t) = a_{11}\psi_1(t) + a_{12}\psi_2(t) + \cdots + a_{1N}\psi_N(t)$$
$$s_2(t) = a_{21}\psi_1(t) + a_{22}\psi_2(t) + \cdots + a_{2N}\psi_N(t)$$
$$\vdots \qquad\qquad\qquad \vdots$$
$$s_M(t) = a_{M1}\psi_1(t) + a_{M2}\psi_2(t) + \cdots + a_{MN}\psi_N(t)$$

These relationships are expressed in notation as

$$s_k(t) = \sum_{j=1}^{N} a_{kj}\psi_j(t), \quad k = 1, \ldots, M \quad j = 1, \ldots\ldots, N, \quad N \leq M \quad --(2)$$

where

$$a_{kj} = \frac{1}{K_j}\int_0^T s_k(t)\psi_j(t)dt, \quad k = 1, \ldots\ldots, M, \quad j = 1, \ldots\ldots, N, \quad 0 \leq t \leq T$$

The basis coefficient a_{kj} is the value of the $\psi_j(t)$ component of signal $s_k(t)$. The set of signal waveforms, $\{s_k(t)\}$, can be viewed as a set of vectors, $\{s_k\} = \{a_{k1}, a_{k2} \ldots\ldots a_{kN}\}$. If, for example, $N = 3$, we can plot the vector s_k corresponding to the waveform

$$s_k(t) = a_{k1}\psi_1(t) + a_{k2}\psi_2(t) + a_{k3}\psi_2(t)$$

as a point in a three-dimensional Euclidean space with coordinates (a_{k1}, a_{k2}, a_{k3}). In general, any signal waveform can be viewed as a point in the N-dimensional Euclidean space. The orientation among the signal vectors describes the relation of the signals to one another (with respect to phase or frequency) and the amplitude of each vector in the set $\{s_k\}$ is a measure of the signal energy transmitted during a symbol duration. In general, once a set of N basis functions has been adopted, each of the transmitted signal waveforms, $\{s_k\}$, is completely specified by the vector of its coefficients

$$s_k = (a_{k1}, a_{k2}, \ldots\ldots a\_kN), \quad k = 1, \ldots\ldots M$$

In the detection process for digitally modulated signals, this vector view of a signal is more convenient than the conventional waveform view. Using Eqs (1) and (2), the normalized energy E_k associated with the waveform $s_k(t)$ over a symbol interval T can be written as,

$$E_k = \int_0^T s_k^2(t)d\,t = \int_0^T \left[\sum_j a_{kj}\psi_j(t)\right]^2 dt$$

$$= \int_0^T \sum_j a_{kj}\psi_j(t) \sum_i a_{ki}\psi_i(t)\,dt$$

$$= \sum_j \sum_i a_{kj}a_{ki}\int_0^T \psi_j(t)\psi_i(t)dt$$

$$= \sum_j \sum_i a_{kj}a_{ki}\,K_j\delta_{jk} = \sum_{j=1}^{N} a_{kj}^2 K_j, \quad k = 1, \ldots\ldots M \quad --(3)$$

Equation (3) is a special case of Parseval's theorem relating the integral of the square of the waveform $s_k(t)$ to the sum of the square of the basis coefficients. If orthonormal functions are used (i.e. $K_j = 1$), the normalised energy over a symbol duration T is given by

$$E_k = \sum_{j=1}^{N} a_{kj}^2$$

Example 4.1 Figure 4.1 shows a set of three waveforms $s_1(t), s_2(t)$ *and* $s_3(t)$.

(a) Demonstrate that these waveforms do not form an orthogonal set.

(b) Show that $\psi_1(t)$ and $\psi_2(t)$ form a basis set

(c) Express the signal set $s_i(t)$ in terms of basis set $\psi_j(t)$

(d) Verify that $\psi_3(t)$ and $\psi_4(t)$ also form a basis set.

(e) Express the signal set $s_i(t)$ in terms of basis set $\{\psi_3(t), \psi_4(t)\}$

(a)

$$\int_0^T s_1(t)s_2(t)dt = 0$$

$$\int_0^T s_1(t)s_3(t)dt = 3T$$

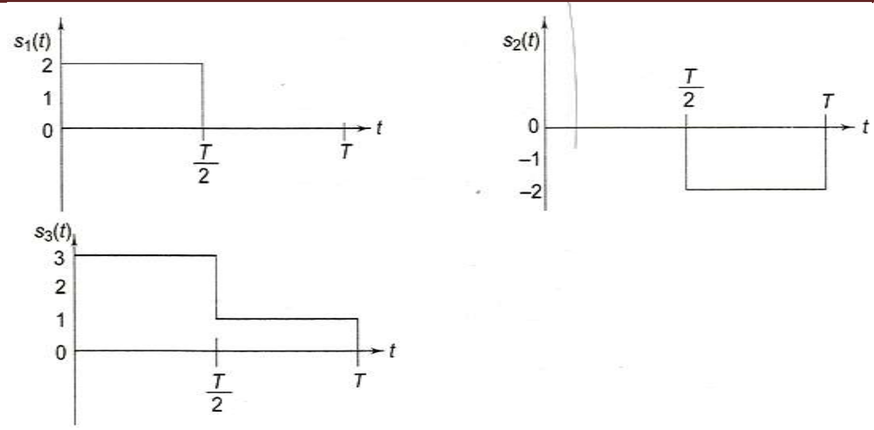
$$\int_0^T s_2(t)s_3(t)dt = -T$$

Since all the three inner products of the signal set are not identically zero, the signal set $s_1(t), s_2(t)$ *and* $s_3(t)$ are not orthogonal.

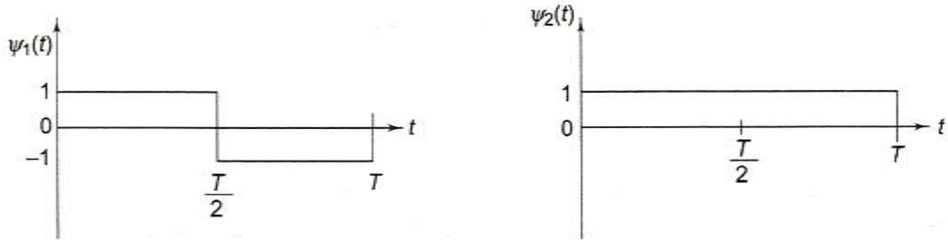
(b)

$$\int_0^T \psi_1(t)\psi_2(t)\,dt = 0$$

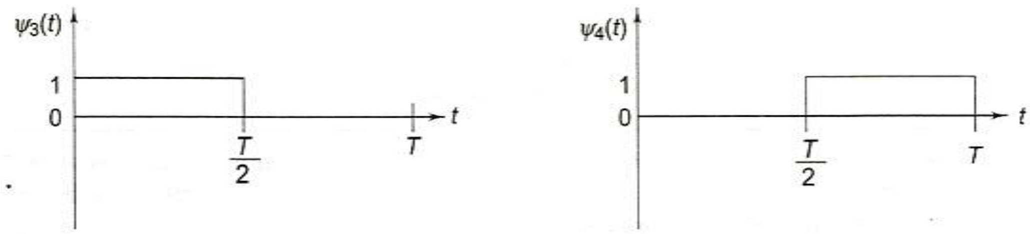
Therefore, the set of $\psi_1(t)$ and $\psi_2(t)$ form a basis set.



(a)



(b)



(c)

Fig.4.1 (a) Arbitrary Signal Set (b) An Orthogonal Basis Set, (c) Another Orthogonal Basis Set

(c)

$$K_1 = \int_0^T \psi_1(t)\psi_1(t) = T$$

$$K_2 = \int_0^T \psi_2(t)\psi_2(t) = T$$

$$a_{11} = \frac{1}{K_1}\int_0^T s_1(t)\psi_1(t)\,dt = 1$$

$$a_{12} = \frac{1}{K_1}\int_0^T s_2(t)\psi_1(t)\,dt = 1$$

$$a_{22} = \frac{1}{K_2}\int_0^T s_2(t)\psi_2(t)\,dt = -1$$

$$a_{31} = \frac{1}{K_1}\int_0^T s_3(t)\psi_1(t)\,dt = 1$$

$$a_{32} = \frac{1}{K_2}\int_0^T s_3(t)\psi_2(t)\,dt = 2$$

Thus, the signals s_i can be written as

$$s_1(t) = \psi_1(t) + \psi_2(t)$$

$$s_2(t) = \psi_1(t) - \psi_2(t)$$

$$s_3(t) = \psi_1(t) + 2\psi_2(t)$$

(d)

$$\int_0^T \psi_3(t)\psi_4(t)\,dt = 0$$

So, the waveform set $\psi_3(t)$ and $\psi_4(t)$ also form a basis set.

(e) Following the same procedure of part (c), we get $K_3 = K_4 = \frac{T}{2}$ and $a_{13}, = 2, a_{14} = 0$, $a_{23} = 0$, $a_{24} = -2$, $a_{33} = 3$ *and* $a_{34} = 1$.So, we may express the signal set in terms of this basis set as:

$$s_1(t) = 2\psi_3(t)$$

$$s_2(t) = -2\psi_4(t)$$

$$s_3(t) = 3\psi_3(t) + \psi_4(t)$$

The practical usefulness of the basis set is that if a communication system uses the three nonorthogonal signal waveforms $s_1(t), s_2(t)$ *and* $s_3(t)$ given above, the transmitter and the receiver need only be implemented using the two basis functions $\psi_1(t)$ and $\psi_2(t)$ instead of the three original waveforms. The procedure for obtaining the basis set from the original signal set is known as the **Gram-Schmidt orthogonalisation procedure**.

4.1.1 Gram-Schmidt Orthogonalisation Procedure (GSOP)

Suppose we have a set of finite energy signal waveforms $\{s_k(t), k = 1,2,\ldots, M\}$ and we need to construct a set of orthonormal basis functions. The Gram-Schmidt orthogonalisation procedure allows us to construct such a set. Consider the first waveform $s_1(t)$, which is having energy E_1 given by

$$E_1 = \int_0^T s_1^2(t)\, dt$$

The first basis function is constructed as

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

Thus, the first basis function $\psi_1(t)$ is the first signal waveform $s_1(t)$ normalized to unit energy.

To compute the second basis function, we first compute the projection of the first basis function $\psi_1(t)$ onto the second signal waveform $s_2(t)$. This projection is c_{12}, and its value is

$$c_{12} = \int_{-\infty}^{\infty} s_2(t)\psi_1(t)\, dt$$

Note that $c_{12}\psi_1(t)$ is the component of the second signal waveform $s_2(t)$ along the first basis function $\psi_1(t)$. So, to satisfy orthogonality of basis functions, we should subtract $c_{12}\psi_1(t)$ from $s_2(t)$ to yield

$$\psi_2'(t) = s_2(t) - c_{12}\psi_1(t)$$

This basis function $\psi_2'(t)$ is orthogonal to $\psi_1(t)$ but it does not have unit energy. If E_2 denotes its energy, we can easily evaluate the second normalized basis function $\psi_2(t)$ as

$$\psi_2(t) = \frac{\psi_2'(t)}{\sqrt{E_2}}$$

Now, generalize the steps of obtaining the orthonormal basis set. In general, the k^{th} basis function is obtained as

$$\psi_k(t) = \frac{\psi_k'(t)}{\sqrt{E_k}}$$

where

$$\psi_k'(t) = s_k(t) = \sum_{i=1}^{k-1} c_{ik}\psi_i(t)$$

and

$$c_{ik} = \int_{-\infty}^{\infty} s_k(t)\psi_i(t)\,dt, \qquad i = 1,2,\ldots..k-1$$

If the dimensionality of the signal space corresponding to the given *M* signal waveforms is also *M,* this procedure would lead to exactly *M* number of basis functions. However, if the dimensionality of the signal space is not *M,* but *N* where $N \leq M$ then automatically this GSOP generates only *N* basis functions. All the projections corresponding to the $(N+j)^{th}$ basis function turns out to be zero and so no basis functions are generated after obtaining the *N* basis functions. Hence, automatically GSOP detects the dimensionality of the signal set and generates same number of basis functions.

Example 4.2 Let us apply the (GSOP) to the set of three waveforms already illustrated in Fig. 4.1(a). And derive the basis functions instead of assuming them as known as was the case in Example 4.1. The energy of $s_1(t)$ is $E_1 = 2T$, so that $\psi_1(t) = \sqrt{1/2T}s_1(t)$. Next, we observe that $c_{12} = 0$; hence, $s_2(t)$ and $\psi_1(t)$ are orthogonal. So, $\psi_2'(t)= s_2(t)$ and the energy of $\psi_2'(t)$ is also *2T* i.e. $E_1 = 2T$. Therefore, $\psi_1(t) = \frac{s_2(t)}{\sqrt{E_2}} = \sqrt{\frac{1}{2T}}s_2(t)$. To obtain $\psi_3(t)$, we compute c_{13} and c_{23},

$$c_{13} = \int_{-\infty}^{\infty} \psi_1(t)\,s_3(t)dt = 3\sqrt{\frac{T}{2}}$$

$$c_{23} = \int_{-\infty}^{\infty} \psi_2(t)\,s_3(t)dt = -\sqrt{\frac{T}{2}}$$

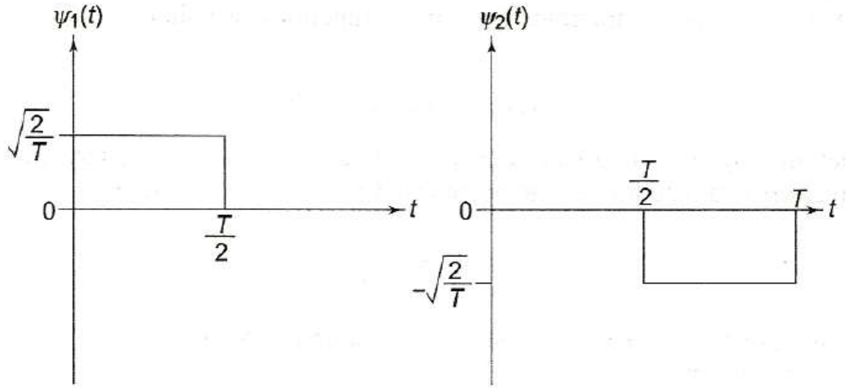


Fig. 4.2 Basis Set Determination by Gram-Schmidt Procedure

Thus,

$$\psi_3'(t) = s_3(t) - 3\sqrt{\frac{T}{2}}\psi_1(t) + \sqrt{\frac{T}{2}}\psi_2(t) = 0$$

Consequently, $s_3(t)$ is a linear combination of $\psi_1(t)$ and $\psi_2(t)$ and hence, $\psi_3(t) = 0$. The two orthonormal functions are shown in Fig. 4.2.

Once we have constructed the set of orthonormal waveforms $\{\psi_n(t)\}$ we can express the *M* signals $\{s_k(t)\}$ as linear combinations of the basis functions $\{\psi_n(t)\}$. Thus, we may write

$$s_k(t) = \sum_{n=1}^{N} s_{kn}\psi_n(t), \qquad k = 1,2,\ldots\ldots,M \qquad --(4)$$

and

$$E_k = \int_{-\infty}^{\infty} [s_k(t)]^2\,dt = \sum_{n=1}^{N} s_{kn}^2 = \|s_k\|^2$$

Based on Eq. (4) each signal may be represented by the vector

$$s_k = [s_{k1}, s_{k2}, \ldots . s_{kN}]$$

or, equivalently, as a point in the N-dimensional signal space with coordinates $s_{ki}, i = 1,2,\ldots,N\}$. The energy in the k^{th} signal is the square of the length of the vector, or the square of the Euclidean distance from the origin to the point in the N-dimensional space. Thus, we can conclude that any signal can be represented geometrically as a point in the signal space spanned by the orthonormal functions $\{\psi_n(t)\}$.

Example 4:3 Let us obtain the vector representation of the three signals shown in Fig. 4.1 (a) by using the orthonormal basis set shown in Fig. 4.2. Since the dimensionality of the signal space is $N = 2$, each signal is described by two components along the two basis functions $\psi_1(t)$ and $\psi_2(t)$. The first signal can be easily written in terms of the first basis function as

$$s_1(t) = \sqrt{E_1}\psi_1(t)$$

So, the signal $s_1(t)$ can be characterized by the vector $s_1 = (-\sqrt{2T}, 0)$. Exactly in the same way the signal *S2(t)* can be characterised by the vector S2 = (0, -/IT).For finding the vector corresponding to the third signal *S3,* we write,

$$s_3(t) = c_{13}\psi_1(t) + c_{23}\psi_2(t)$$

So, the vector corresponding to $s_3(t)$ is $(C_{13}, C_{23}) = \left(3\sqrt{\frac{T}{2}}, -\sqrt{\frac{T}{2}}\right)$

These vectors are shown in Fig.4.3. Their lengths are $|s_1| = -\sqrt{2T}$, $|s_2| = \sqrt{2T}$, $|s_3| = \sqrt{5T}$, and IS31 = ffi and the corresponding signal energies are $E_1 = |s_1|^2 = 2T = E_2, E_3 = 5T$.

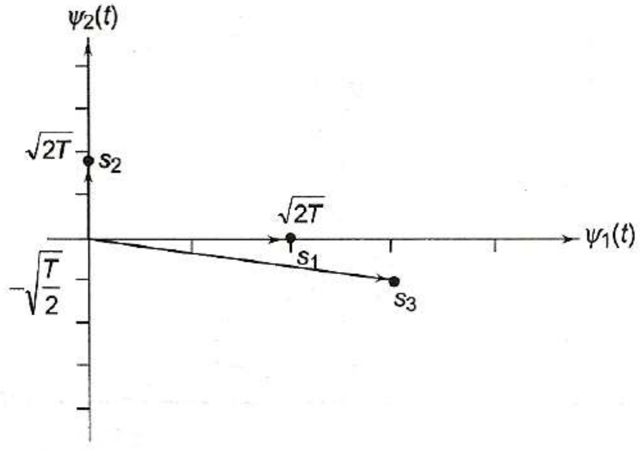
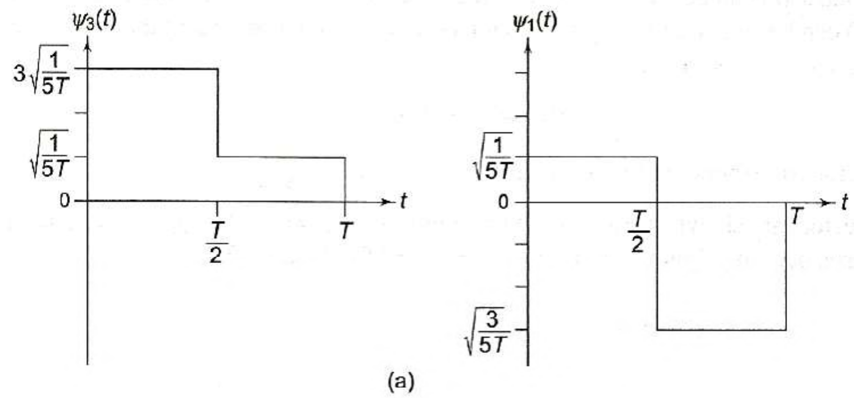


Fig. 4.3 Signal Vectors as Points in Signal Space

Therefore, a set of M finite energy waveforms $\{s_k(t)\}$ can be represented by a weighted linear combination of orthonormal functions $\{\psi_n(t)\}$ of dimensionality$N \leq M$. The functions $\{\psi_n(t)\}$ are obtained by applying the **Gram-Schmidt orthogonalisation procedure** on $\{s_k(t)\}$. However the functions $\{\psi_n(t)\}$obtained from **orthogonalisation procedure** are not unique. If we alter the order in which the orthogonalisation of the signals $\{s_k(t)\}$ is performed, the orthonormal waveforms would be different and the corresponding vector representation of the signals $\{s_k(t)\}$ would depend on the choice of the orthonormal functions $\{\psi_n(t)\}$. But, the vectors $\{s_k\}$ will retain their geometrical configuration and their lengths will be invariant to the choice of orthonormal functions $\{\psi_n(t)\}$. This is illustrated in the next example.

Example 4.4 Apply Gram-Schmidt orthogonalisation procedure to the signal set shown in Fig. 4.1(a) by starting from signal $s_3(t)$ instead of s_1 as was done in Example 4.2. Then obtain the vector representation of the signals in terms of this basis set.

The orthonormal basis set obtained by first considering the signal $s_3(t)$, then $s_1(t)$ and then $s_2(t)$ is shown in Fig. 4.4(a). By comparing this orthonormal set with the one shown in Fig. 4.2 it can be concluded that the orthonormal basis sets are not unique for a given signal set.



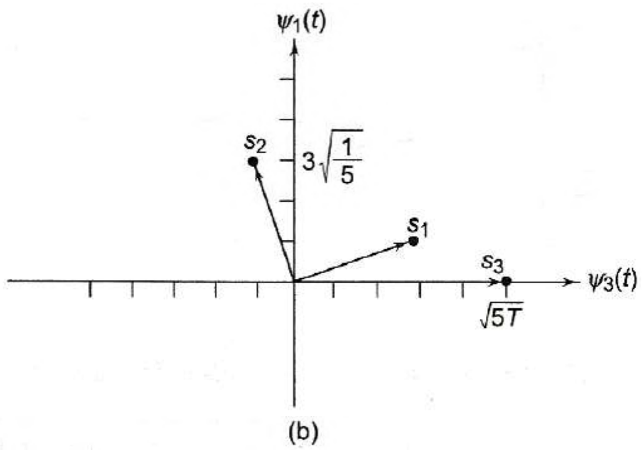


Fig.4.4 (a)Alternative Basis Set (b) Signal SpaceDiagram

Let us determine the components of the three signals along the basis functions $\psi_3(t)$ and $\psi_1(t)$ respectively. We start with signal $s_3(t)$. The energy of this signal is $E_3 = 5\,T$. It is clear that the signal vector for $s_3(t)$ is $(\sqrt{5T}, 0)$. Next, the signal $s_1(t)$ can be written as

$$s_1(t) = c_{31}\psi_3(t) + \sqrt{E_1}\psi_1$$

Noting that $EI = \frac{T}{2}$ and $c_{31} = \sqrt{\frac{9T}{5}}$, we can write the vector corresponding to signal $s_1(t)$ as $\left(3\sqrt{\frac{T}{5}}, \sqrt{\frac{T}{5}}\right)$. The last signal $s_2(t)$ can be expressed in terms of the above two basis functions as:

$$s_2(t) = c_{32}\psi_3(t) + c_{12}\psi_1(t)$$

Since, $c_{32} = -\sqrt{\frac{T}{5}}$ *and* $c_{12} = 3\sqrt{\frac{T}{5}}$, we may write the signal vector for *S2(t)* as $\left(-\sqrt{\frac{T}{5}}, 3\sqrt{\frac{T}{5}}\right)$. The graphical representation of these signals in terms of the alternative orthonormal basis set of Fig. 4.4(a) is shown in part (b) of the same figure. Their lengths are $|s_1| = \sqrt{2T}$, $|s_2| = \sqrt{2T}$, $|s_3| = \sqrt{5T}$ which are identical to the vector lengths of Fig. 4.3 verifying that signal vector lengths are invariant to the basis set.

4.2 DIGITAL MODULATION TECHNIQUES

4.2.1 Introduction

There are basically two types of transmission of digital signals:

1) Baseband data transmission: The digital data is transmitted over the channel directly. There is no carrier or any modulation. This is suitable for transmission over short distances.

2) Passband data transmission: The digital data modulates high frequency sinusoidal carrier. Hence it is also called digital CW modulation. It is suitable for transmission over long distances.

4.2.2 Types of Passband Modulation

The digital data can modulate phase, frequency or amplitude of carrier. This gives rise to three basic techniques :

1) Phase shift keying (PSK) : In this technique, the digital data modulates phase of the carrier.

2) Frequency shift keying (FSK) : In this technique, the digital data modulates frequency of the carrier.

3) Amplitude shift keying (ASK) : In this technique, the digital data modulates amplitude of the carrier.

4.2.3 Types of Reception for Passband Transmission

There are two types of methods for detection of passband signals.

1. Coherent (Synchronous) detection: In this method, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Hence it is also called synchronous detection.

2. Noncoherent (Envelope) detection: In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is also called envelope detection. Noncoherent detection is simple but it has higher probability of error.

4.2.4 Requirements of Passband Transmission Scheme

Any passband transmission scheme should satisfy following requirements -

1. Maximum data transmission rate.
2. Minimum probability of symbol error.
3. Minimum transmitted power
4. Minimum channel bandwidth
5. Maximum resistance to interfering signals.
6. Minimum circuit complexity.

4.2.5 Advantages of Passband Transmission over Baseband Transmission

1. Long distance transmission.
2. Analog channels can be used for transmission.
3. Multiplexing techniques can be used for bandwidth conservation.
4. Problems such as ISI and crosstalk are absent.
5. Passband transmission can take place over wireless channels also.
6. Large number of modulation techniques are available.

4.2.6 Drawback of Passband Modulation

1. Modulation and demodulation equipments, transmitting/receiving antennas, interference problems make the system complex.
2. It is not suitable for short distance communication.

4.2.7 Passband Transmission Model

Fig. 4.5 shows the model of passband data transmission system

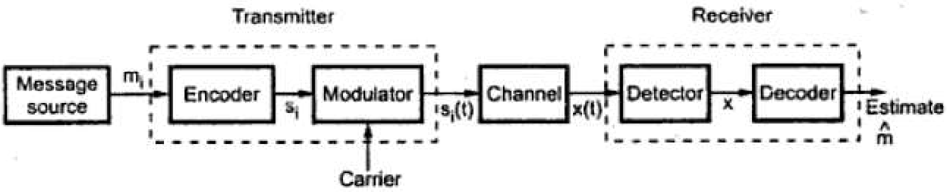


Fig. 4.5 Model of passband data transmission system

Message source: It emits the symbol at the rate of T seconds.

Encoder: It is signal transmission encoder. It produces the vector s_i made up of 'N' real elements. The vector s_i is unique for each set of 'M' symbols.

Modulator: It constructs the modulated carrier signal $s_i(t)$ of duration T seconds for every symbol m_i. The signal $s_i(t)$ is energy signal.

Channel: The modulated signal $s_i(t)$ is transmitted over the communication channel.

· The channel is assumed to be linear and of enough bandwidth to accommodate the signal silt).

· The channel noise is white Gaussian of zero mean and psd of $\frac{N_0}{2}$.

Detector: It demodulates the received signal and obtains an estimate of the signal vector.

Decoder: The decoder obtains the estimate of symbol back from the signal vector. Here note that the detector and decoder combinely perform the reception of the transmitted signal. The effect of channel noise is minimized and correct estimate of symbol $\hat{m}$ is obtained.

4.3 Binary Phase Shift Keying (BPSK)

4.3.1 Principle of BPSK

In binary phase shift keying (BPSK), binary symbol ‘1’ and '0' modulate the phase of the carrier. Let the carrier be,

$$s(t) = Acos\ (2\pi f_0 t)$$

'A'.represents peak value of sinusoidal carrier. In the standard 1Ω load register. The power dissipated will be,

$$P = \frac{1}{2}A^2$$

$$\therefore \quad A = \sqrt{2P}$$

When the symbol is changed, then the phase of the carrier is changed by 180 degrees (π radians).

Consider for example,

$$symbol\ 1 => s_1(t) = \sqrt{2P}\cos(2\pi f_0 t)$$

if next symbol is '0' then,

$$symbol\ 0 => s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$$

Since $cos\ (\theta + \pi) = -cos\ \theta$, we can write above equation as,

$$s_2(t) = -\sqrt{2P}\cos(2\pi f_0 t)$$

With the above equation we can define BPSK signal combinely as,

$$\boxed{s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)}$$

Here $b(t)$ =+1 when binary '1' is to be transmitted

= -1 when binary '0' is to be transmitted

4.3.2 Graphical Representation of BPSK Signal

Fig. 4.6 shows binary signal and its equivalent signal $b\ (t)$.

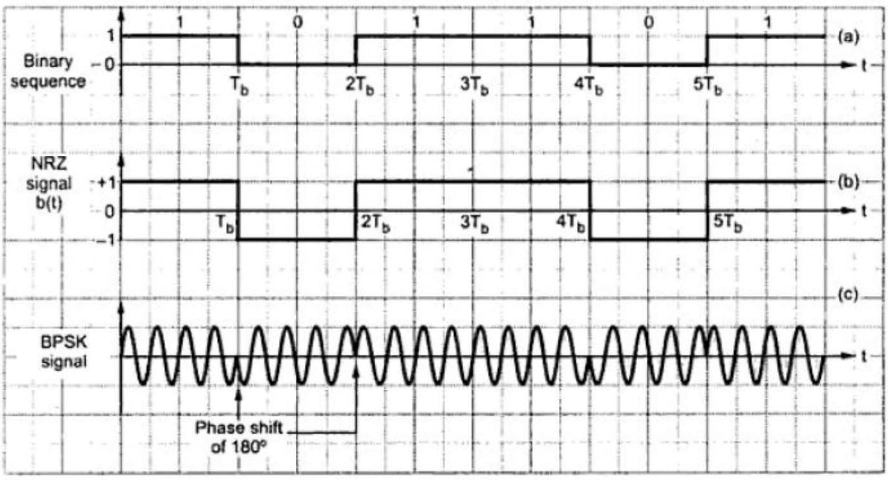


Fig. 4.6 (a) Binary sequence (b) Its equivalent bipolar signal *b(t)* (c) BPSK signal

As can be seen from Fig. 4.6 (b), the signal $b\ (t)$ is NRZ bipolar signal. This signal directly modulates carrier $cos\ (2\pi f_0 t)$.

4.3.3 Generation and Reception of BPSK Signal

Generator of BPSK Signal

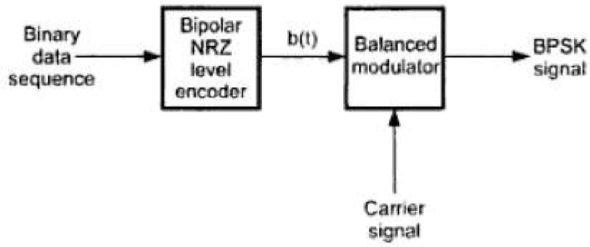


Fig. 4.7 BPSK generation scheme

- The BPSK signal can be generated by applying carrier signal to the balanced modulator.
- The baseband signal $b(t)$ is applied as a modulating signal to the balanced modulator. Fig. 4.7 shows the block diagram of BPSK signal generator.
- The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

Reception of BPSK Signal

Fig. 4.8 shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmitted BPSK signal is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

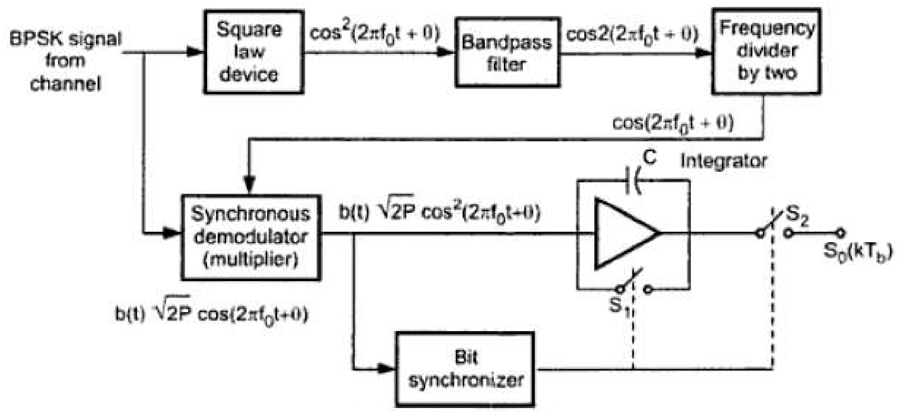


Fig. 4.8 Reception BPSK scheme

Phase shift in received signal : This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in the transmitted signal. Let the phase shift be θ. Therefore the signal at the input of the receiver is

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$$

Square law device: Now from this received signal, a carrier is separated since this is **coherent detection**. As shown in the figure, the received signal is passed through the square law device. At the output of the square law device the signal will be,

$$\cos^2(2\pi f_0 t + \theta)$$

Note here that we have neglected the amplitude, because we are only interested in the carrier of the signal.

We know that,

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\therefore\ \cos^2(2\pi f_0 t + \theta) = \frac{1 + \cos 2(2\pi f_0 t + \theta)}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2(2\pi f_0 t + \theta) \qquad here \frac{1}{2}\ represents\ a\ DC\ level$$

Bandpass filter: This signal is then passed through a bandpass filter whose passband is centered around $2f_0$. Bandpass filler removes the DC level of $\frac{1}{2}$ and at its output we get,

$$\cos 2(2\pi f_0 t + \theta) \qquad This\ signal\ has\ frequency\ of\ 2f_0$$

Frequency divider: The above signal is passed through a frequency divider by two. Therefore at the output of frequency divider we get a carrier signal whose frequency is f_0 i.e. $cos\ (2\pi f_0 t + \theta)$.

Synchronous demodulator : The synchronous (coherent) demodulator multiplies the input signal and the recovered carrier. Therefore at the output of multiplier we get,

$$b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta) \times \cos(2\pi f_0 t + \theta) = b(t)\sqrt{2P}\cos^2(2\pi f_0 t + \theta)$$

$$= b(t)\sqrt{2P} \times \frac{1}{2}[1 + \cos 2(2\pi f_0 t + \theta)]$$

Bit synchronizer and integrator: The above signal is then applied to the bit synchronizer and integrator. The integrator integrates the signal over one bit period. The bit synchronizer takes care of starting and ending times of a bit.

· At the end of bit duration T_b, the bit synchronizer closes switch S_2 ternperorily. This connects the output of an integrator to the decision device. It is equivalent to sampling the output of integrator.

· The synchronizer then opens switch S_2 and switch S, is closed temperorily. This resets the integrator voltage to zero. The integrator then integrates next bit.

· Let us assume that one bit period $'T_b'$ contains integral number of cycles of the carrier, That is the phase change occurs in the carrier only at zero crossing. This is shown in Fig. 4.6 (c). Thus BPSK waveform has full cycles of sinusoidal carrier.

To show that output of Integrator depends upon transmitted bit.

In the k^{th} bit interval we can write output signal as,

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b}[1+\cos 2(2\pi f_0 t+\theta)]\,dt$$

The above equation gives the output of an interval for k^{th} bit. Therefore integration is performed from $(k-1)T_b$ to kT_b. Here T_b is the one bit period.

We can write the above equation as,

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} 1\,dt + \int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_0 t+\theta)\,dt$$

Here $\int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_0 t+\theta)dt = 0$ because average value of sinusoidal waveform is zero if integration is performed over full cycles. Therefore we can write above equation as,

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} 1\,dt$$

$$= b(kT_b)\sqrt{\frac{P}{2}}\,[t]_{(k-1)T_b}^{kT_b}$$

$$= b(kT_b)\sqrt{\frac{P}{2}}\,\{kT_b-(k-1)T_b\}$$

$$\boxed{s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\,T_b}$$

This equation shows that the output of the receiver depends on input i.e.

$$s_0(kT_b) \propto b(kT_b)$$

Depending upon the value of $b(kT_b)$, the output $s_0(kT_b)$ is generated in the receiver.

This signal is then given to a decision device, which decides whether transmitted symbol was zero or one.

4.3.4 Spectrum of BPSK Signals

Step 1: Fourier transform of basic NRZ pulse.

We know that the waveform $b(t)$ is NRZ bipolar waveform. In this waveform there are rectangular pulses of amplitude $\pm V_b$. If we say that each pulse is $\pm\frac{T_b}{2}$ around its center as shown in Fig. 4.9, then it becomes easy to find fourier transform of such pulse.

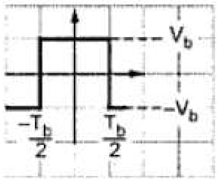


Fig: 4.9 NRZ pulse

The Fourier transform of this type of pulse is given as,

$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{\pi f T_b} \quad \textit{By standard relations} \qquad --(1)$$

Step 2: PSD of NRZ pulse.

For large number of such positive and negative pulses the power spectral density $S(f)$ is given as

$$S(f) = \frac{\overline{|X(f)|^2}}{T_s} \qquad --(2)$$

Here $\overline{X(f)}$ denotes average value of $X(f)$ due to all the pulses in $b(t)$. And T_s, is symbol duration. Putting value of $X(f)$ from equation (1) in equation (2) we get,

$$S(f) = \frac{{V_b}^2{T_b}^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

Step 3: PSD of baseband signal $b(t)$

For BPSK, since only one bit is transmitted at a time, symbol and bit durations are same i.e. $T_b = T_s$. Then above equation becomes,

$$S(f) = {V_b}^2{T_b}^2\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \qquad --(3)$$

The above equation gives the power spectral density of baseband signal $b(t)$.

Step 4: PSD of BPSK signal.

The BPSK signal is generated by modulating a carrier by the baseband signal $b(t)$. Because of modulation of the carrier of frequency f_0, the spectral components are translated from $f\ to\ f_0 + f\ and\ f_0 - f$. The magnitude of those components is divided by half.

Therefore from equation (3) we can write the power spectral density of BPSK signal as,

$$S_{BPSK}(f) = V_b{}^2 T_b \left\{ \frac{1}{2}\left[\frac{\sin(\pi(f_0 - f)T_b)}{\pi(f_0 - f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin(\pi(f_0 + f)T_b)}{\pi(f_0 + f)T_b}\right]^2 \right\}$$

The above equation is composed of two half magnitude spectral components of same frequency $'f'$ above and below f_0. Let us say that the value of $\pm V_b = \pm\sqrt{P}$. That is the NRZ signal is having amplitudes of $+\sqrt{P}$ and $-\sqrt{P}$. Then above equation becomes,

$$S_{BPSK}(f) = \frac{PT_b}{2}\left\{ \left[\frac{\sin(\pi(f_0 - f)T_b)}{\pi(f_0 - f)T_b}\right]^2 + \left[\frac{\sin(\pi(f_0 + f)T_b)}{\pi(f_0 + f)T_b}\right]^2 \right\} \quad --(4)$$

The above equation gives power spectral density of BPSK signal for modulating signal $b(t)$ having amplitudes of $\pm\sqrt{P}$. We know that modulated signal is given as,

$$s(t) = \pm\sqrt{2P}\cos(2\pi f_0 t) \qquad since\ A = \sqrt{2P}$$

If $b(t) = \pm\sqrt{P}$, then the carrier becomes,

$$\phi(t) = \sqrt{2}\cos(2\pi f_0 t) \qquad --(5)$$

Plot of PSD

Equation (3) gives power spectral density of the NRZ waveform. For one rectangular pulse, the shape of $S(f)$ will be a sinc pulse as given by equation (3). Fig.4.10 shows the plot of magnitude of $S(f)$

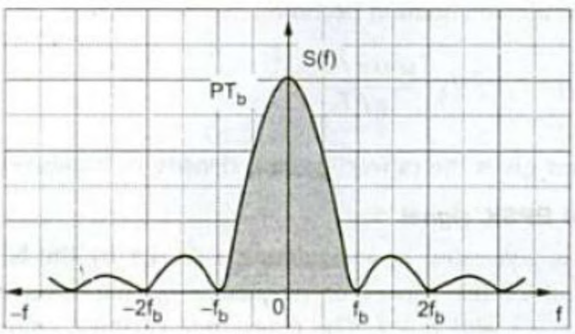


Fig. 4.10 Plot of power spectral density of NRZ baseband signal

Above figure shows that the main lobe ranges from $-f_b\ to + f_b$. Here $f_b = \frac{1}{T_b}$. Since we have taken $\pm V_b = \pm\sqrt{P}$ in equation (3), the peak value of the main lobe is PT_b

Now let us consider the power spectral density of BPSK signal given by equation (4).Fig. 4.11 shows the plot of this equation. The figure thus clearly shows that there are two lobes; one at f_0 and other at $-f_0$. The same spectrum of Fig. 4.10 is placed at $+f_0$ $and - f_0$. But the amplitudes of main lobes are $\frac{PT_b}{2}$. In Fig. 4.11

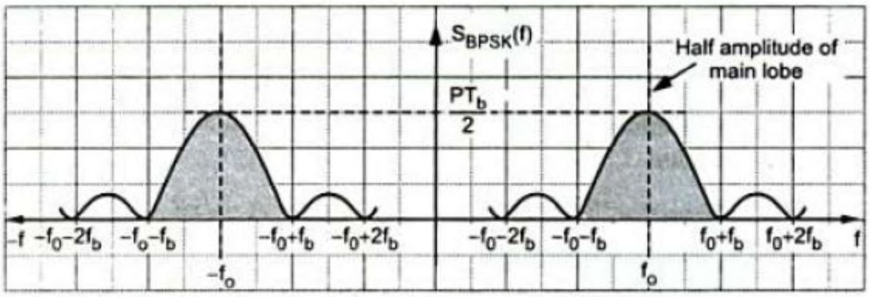


Fig. 4.11 Plot of power spectral density of BPSK signal

Thus they are reduced to half. The spectrums of $S(f)$ as well as $S_{BPSK}\ (f)$ extends over all the frequencies.

Interchannel Interference and ISI:

· Let's assume that BPSK signals are multiplexed with the help of different carrier frequencies for different baseband signals. Then at any frequency, the spectral components due to all the multiplexed channels will be present. This is because $S(f)$ as well as $S_{BPSK}(f)$ of every channel extends over all the frequency range.

· Therefore a BPSK receiver tuned to a particular carrier frequency will also receive frequency components due to other channels. This will make interference with the required channel signals and error probability will increase. This result is called **Interchannel Interference**

· To avoid interchannel interference, the BPSK signal is passed through a filter. This filter attenuates the side lobes and passes only main lobe. Since side lobes are attenuated to high level, the interference is reduced. Because of this filtering the phase distortion takes place in the bipolar NRZ signal, i.e. $b(t)$. Therefore the individual bits (symbols) mix with adjacent bits (symbols) in the same channel. This effect is called **intersymbol interference or ISI**.

· The effect of ISI can be reduced to some extent by using equalizers at the receiver. Those equalizers have the reverse effect to that filter's adverse effects. Normally equalizers are also filter structures.

4.3.5 Geometrical Representation of BPSK Signals

We know that BPSK signal carries the information about two symbols. Those are symbol '1' and symbol '0'. We can represent BPSK signal geometrically to show those two symbols.

We know that BPSK signal is given as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

Rearranging the above equation as,

$$s(t) = b(t)\sqrt{PT_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t) \qquad --(6)$$

Let $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)$ represents an orthonormal carrier signal. Equation (5) also gives equation for carrier. It is slightly different than $\phi_1(t)$ defined here. Then we can write equation (6) as,

$$s(t) = b(t)\sqrt{PT_b}\,\phi_1(t)$$

The bit energy E_b is defined in terms of power 'P' and bit duration T_b as,

$$E_b = PT_b$$

∴ Equation (6) becomes,

$$s(t) = \pm\sqrt{E_b}\,\phi_1(t)$$

Thus on the single axis of $\phi_1(t)$ there will be two points. One point will be located at $+\sqrt{E_b}$ and other point will be located at $-\sqrt{E_b}$. This is shown in Fig. 4.12,

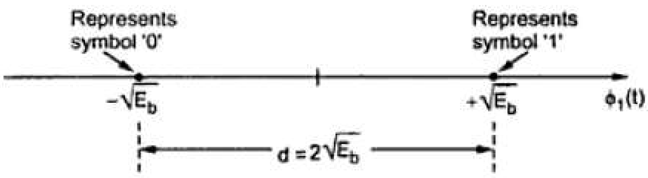


Fig. 4.12 Geometrical representation of BPSK signal

At the receiver the point at $+\sqrt{E_b}$ on $\phi_1(t)$ represents symbol '1' and point at $-\sqrt{E_b}$ represents symbol '0'. The separation between these two points represent the isolation in symbols '1' and '0' in BPSK signal. This separation is normally called distance 'd'. From Fig,4.12 it is dear that the distance between the two points is,

$$d = +\sqrt{E_b} - \left(-\sqrt{E_b}\right)$$

$$\therefore \quad d = 2\sqrt{E_b}$$

As this distance 'd' increases, the isolation between the symbols in BPSK signal is more. Therefore probability of error reduces.

4.3.6 Bandwidth of BPSK Signal

The spectrum of the BPSK signal is centered around the carrier frequency f_0. If $f_b = \frac{1}{T_b}$, then for BPSK the maximum frequency in the baseband signal will be f_b see Fig. 4.11. In this figure the main lobe is centered around carrier frequency f_0 and extends from $f_0 - f_b$ to $f_0 + f_b$. Therefore Bandwidth of BPSK signal is,

$$BW = Highest\ frequency - Lowest\ frequency\ in\ the\ main\ lobe$$
$$= f_0 + f_b - (f_0 - f_b)$$
$$\therefore\ BW = 2f_b$$

Thus the minimum bandwidth of BPSK signal is equal to twice of the highest frequency contained in baseband signal.

4.3.7 Drawbacks of BPSK

Fig. 4.8 shows the block diagram of BPSK receiver. To regenerate the carrier in the receiver, we start by squaring $b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$. If the received signal is $-b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$ then the squared signal remains same as before. Therefore the recovered carrier is unchanged even if the input signal has changed its sign. Therefore it is not possible to determine whether the received signal is equal to $b(t)\ or\ -b(t)$. This result in **ambiguity** in the output signal.

This problem can be removed if we use differential phase shift keying.

Other problems of BPSK are ISI and Interchannel interference. These problems are reduced to some extent by use of filters.

Example: Determine the minimum bandwidth for a BPSK modulator with a carrier frequency of 40MHz and all input bit rate of 500 kbps.

Solution: The input bit rate indicates highest frequency of the baseband signal.

$$f_b = 500\ kbps$$
$$= 500\ kHz$$

The bandwidth of the BPSK system is given as,

$$BW = 2f_b$$
$$= 2 \text{ x } 500\ kHz$$
$$= 1\ MHz$$

4.4 BINARY FREQUENCY SHIFT KEYING (BFSK)

In binary frequency shift keying, the frequency of the carrier is shifted according to the binary symbol. The phase of the carrier is unaffected. That is we have two different frequency signals according to binary symbols. Let there be a frequency shift by Ω. Then we can write following equations.

$$if\ \ b(t) = 1, \qquad s_H(t) = \sqrt{2P_s}\cos(2\pi f_0 + \Omega)t \qquad --(1)$$
$$if\ \ b(t) = 0, \qquad s_L(t) = \sqrt{2P_s}\cos(2\pi f_0 - \Omega)t \qquad --(2)$$

Thus there is increase or decrease in frequency by Ω. The following conversion table is used to combine above two FSK equations

$b(t)$ Input	$d(t)$	$P_H(t)$	$P_L(t)$
1	+ 1V	+1V	0V
0	− 1V	0V	+1V

Table 4.1 Conversion table for BPSK representation

We can write equation (1) and equation (2) combinely as,

$$s(t) = \sqrt{2P_s}\cos[(2\pi f_0 + d(t)\Omega)t]$$

Thus when symbol '1' is to be transmitted, the carrier frequency will be $f_0 + \left(\frac{\Omega}{2\pi}\right)$. If symbol '0' is to be transmitted, the carrier frequency will be $f_0 - \left(\frac{\Omega}{2\pi}\right)$ i.e.,

$$f_0 + \left(\frac{\Omega}{2\pi}\right) \quad for\ symbol\ '1'$$

$$f_0 - \left(\frac{\Omega}{2\pi}\right) \quad for\ symbol\ '0'$$

4.4.1 BFSK Transmitter

From the table 4.1, we know that $P_H(t)$ is same as $b\ (t)$. And $P_L(t)$ is inverted version of $b\ (t)$. The block diagram of BFSK transmitter is shown in Fig. 4.13.

We know that input sequence $b\ (t)$ is same as $P_H(t)$. An inverter is added after $b\ (t)$ to get $P_L(t)$. $P_H(t)$ and $P_L(t)$ are unipolar signals. The level shifter converts the *'+1'* level to $\sqrt{P_s T_b}$. Zero level is unaffected. Thus the output of the level shifters will be either $\sqrt{P_s T_b}$ (if '+1') or zero (if input is zero). Further there are product modulators after level shifter. The two carrier signals $\phi_1(t)\ and\ \phi_2(t)$ are used. $\phi_1(t)\ and\ \phi_2(t)$ are orthogonal to each other. In one bit period of input signal (i.e. T_b), $\phi_1(t)\ and\ \phi_2(t)$ have integral number of cycles.

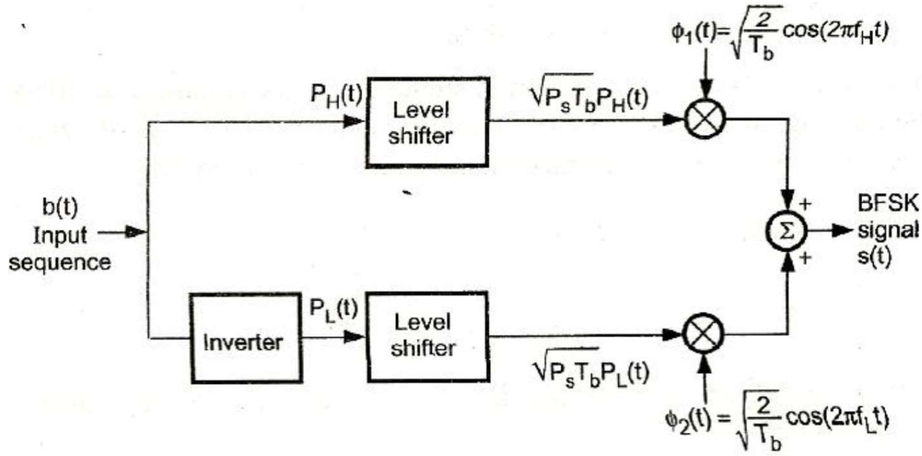


Fig. 4.13 Block diagram of BFSK transmitter

Therefore the modulated signal has continuous phase. Such BFSK signal is shown in Fig.4.14. The adder then adds the two signals.

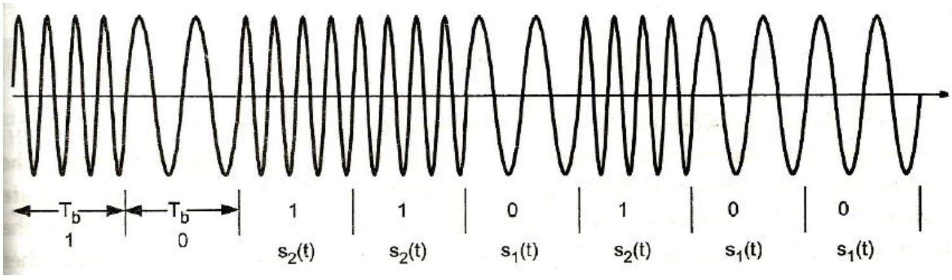


Fig. 4.14 BFSK signal

Here note that outputs from both the multipliers are not possible at a time. This is because $P_H(t)$ and $P_L(t)$ are complementary to each other. Therefore if $P_H(t) = 1$, then output will be only due to upper modulator and lower modulator output will be zero (since $P_L(t) = 0$).

4.4.2 Spectral Characteristics and Bandwidth of BFSK

In Fig. 4.13 we can write BFSK signal $s\,(t)$ as,

$$s(t) = \sqrt{2P_s}P_H \cos(2\pi f_H t) + \sqrt{2P_s}P_L\, cos(2\pi f_L t) \qquad --(3)$$

This is the BFSK signal equation. Let's compare this equation with BPSK equation of eqn (3) i.e.,

$$s_{BPSK}(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

We observe that the above equation is similar to BFSK equation. In BPSK equation $b\,(t)$ is a bipolar signal but in BFSK the similar coefficients $P_H(t)$ and $P_L(t)$ are unipolar. Therefore let us convert those coefficientsin bipolar form as follows

$$P_H(t) = \frac{1}{2} + \frac{1}{2}P'_H(t)$$

$$and \quad P_L(t) = \frac{1}{2} + \frac{1}{2}P'_L(t)$$

Here $P'_H(t)$ and $P'_L(t)$ will be bipolar (i.e. +1 or -1). Putting those equation (1) we get,

$$s(t) = \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P'_H(t)\right]\cos(2\pi f_H t) + \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P'_L(t)\right]cos(2\pi f_L t)$$

$$s(t) = \sqrt{\frac{P_s}{2}}cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}}cos(2\pi f_L t) + \sqrt{\frac{P_s}{2}}P'_H(t)\,cos(2\pi f_H t)$$

$$+ \sqrt{\frac{P_s}{2}}P'_L(t)\,cos(2\pi f_L t) \quad --(4)$$

In the above equation, the first term represent the single frequency impulse at f_H. The second term represent the pulse at f_L. Those are constant amplitude pulses. The last two terms are similar to BPSK equation of equation (4). Here $P_H(t)$ and $P_L(t)$ equivalent to $b\ (t)$. Therefore those last two terms in equation (4) produce spectrum which are similar to that of BPSK. One spectrum is located at f_H and at f_L. Thereforewe can write the power spectral density of BFSK as,

$$S(f) = \sqrt{\frac{P_s}{2}}\left\{\delta(f - f_H) + \delta(f - f_L) + \frac{P_s T_b}{2}\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2 + \frac{P_s T_b}{2}\left\{\frac{\sin(\pi f_L T_b)}{\pi f_L T_b}\right\}^2\right\}$$

Fig. 4.15 shows the plot of power spectral density of BFSK signal given by above equation.

f_H and f_L are slected. Such that,

$$f_H - f_L = 2f_b$$

With such selection, it is clear from the spectrums in the above figure that, the two frequencies f_H and f_L can be identified properly.

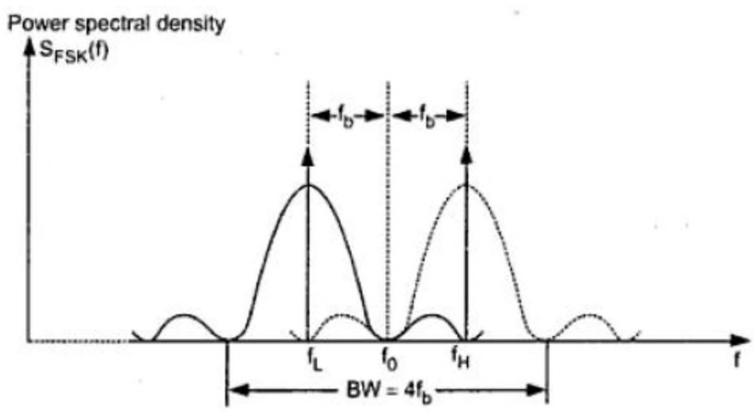


Fig. 4.15 Power spectral density of BFSK signal

The interference between the spectrums is not much with the above assumption.

Bandwidth of BFSK Signal:

From Fig. 4.15 it is clear that the width of one lobe is $2f_b$. The two main lobes due to f_H and f_L are place such that the total width due to both main lobes is $4f_b$. i.e.,

$$Bandwidth\ of\ BFSK = 2f_b + 2f_b$$

$$BW = 4f_b$$

If we compare this bandwidth with that of BPSK, we observe that,

$$BW(BFSK) = 2 \text{ x } BW(BPSK)$$

4.4.3 Coherent BFSK Receiver

Fig. 4.16 shows the block diagram of coherent BFSK receiver. There are two correlators for two frequencies of FSK signal. These correlators are supplied with locally generated carriers $\phi_1(t)$ and $\phi_2(t)$. If the transmitted frequency is f_H then output $s_1(t)$ will be higher than $s_2(t)$. Hence $y(t)$ will be greater than zero.

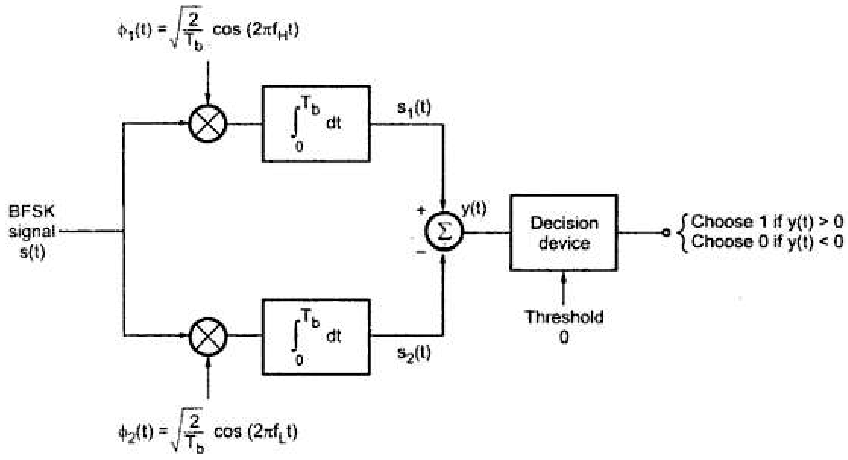


Fig. 4.16 Coherent BFSK reciever

The decision device then decides in favour of binary '1'. If $s_2(t) > s_1(t)$, then $y(t) < 0$ lind decision device decides in favour of 0. The coherent carriers are generated as same as previous methods.

4.4.4 Noncoherent BFSK Receiver

Fig. 4.17 shows the block diagram of BFSK receiver. The receiver consists of two bandpass filters; one with centre frequency / H and other with centre frequency h· Since I'' -h; 2/", the outputs of filters do not overlap. The bandpass filters pass their corresponding main lobes without much distortion.

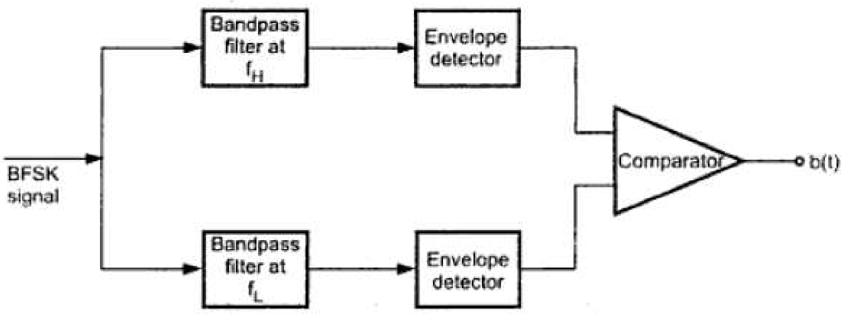


Fig. 4.17 Block diagram of BFSK receiver

The outputs of filters are applied to envelop detectors. The outputs of detectors are compared by the comparator. If unipolar comparator is used, then the output of comparator is the bit sequence $b\ (t)$.

4.4.5 Geometrical Representation of Orthogonal BFSK or Signal Space Representation of Orthogonal BFSK

Orthogonal carriers are used for M-ary PSK and QASK. The different signal points are represented geometrically in $\phi_1\phi_2$ plane. For geometrical representation of BFSK signals such orthogonal carriers are required. We know that, two carriers $\phi_1\ and\ \phi_2$ of two different frequencies $f_H\ and\ f_L$ are used for modulation. To make $\phi_1\ and\ \phi_2$ orthogonal, the frequencies $f_H\ and\ f_L$ should be some integer multiple of base band frequency f_b

$$f_H = mf_b \qquad --(5)$$

$$f_L = nf_b \qquad --(6)$$

Here, $f_b = \frac{1}{T_b}$, then the carriers will be,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi m f_b t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi m f_b t)$$

The carriers $\phi_1\ and\ \phi_2$ are orthogonal over the period T_b. We can write eqn (1) and (2) as,

$$s_H(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_H t)$$

$$s_L(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_L t)$$

$$f_H = f_0 + \frac{\Omega}{2\pi} \quad and \;\; f_L = f_0 - \frac{\Omega}{2\pi}$$

Using the relations of equation (5) and (6) we can write above equations as,

$$s_H(t) = \sqrt{P_s T_b}\,\phi_1(t)$$

$$s_L(t) = \sqrt{P_s T_b}\,\phi_2(t)$$

Based on the above two equations we can draw the signal space diagram as shown in Fig. 4.18.

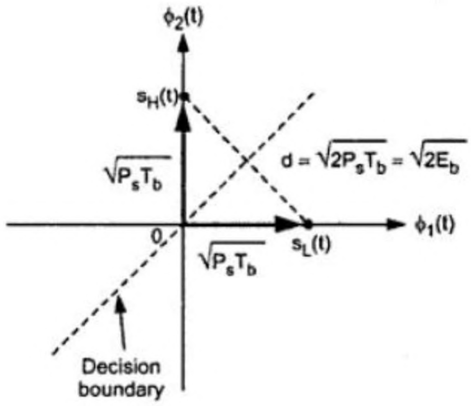


Fig. 4.18 Signal space representation of orthogonal BFSK

Distance between signal points ;

There are two signal points in the signal space. The distance between these two points can be obtained as,

$$d^2 = \left(\sqrt{P_sT_b}\right)^2 + \left(\sqrt{P_sT_b}\right)^2 = 2P_sT_b$$
$$d = \sqrt{P_sT_b}$$

Since $P_sT_b = E_b$, we can write above relation as,

$$d = \sqrt{2E_b}$$

As compared to the distance of BPSK, we observe that this distance is **smaller.**

4.4.6 Geometrical Representation of Non Orthogonal BFSK Signals

Whenever the carriers ϕ_1 *and* ϕ_2 are non orthogonal, then the signal point $S_H(t)$ *or* $S_L(t)$ will not lie exactly on the axes ϕ_1 *and* ϕ_2. Such a representation is shown in Fig. 4.19.

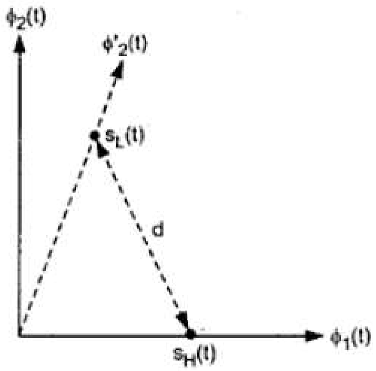


Fig. 4.19 Geometrical representation of non-orthogonal BFSK signals

The distance 'd' for non orthogonal signal is given approximately as,

$$d^2 = 2E_b\left[1 - \frac{\sin 2\pi(f_H - f_L)T_b}{2\pi(f_H - f_L)T_b}\right]$$

4.4.7 Advantages and Disadvantages of BFSK

Even though the generation of BFSK is easier it has many disadvantages compared to BPSK.

- The bandwidth of BFSK is greater than $4f_b$, which is almost double the bandwidth of BPSK.
- The distance between the signal points is less in BFSK. Hence the error rate of BFSK is more compared to BPSK.

We know that,

$$s(t) = \sqrt{2P_s}\cos\{d(t)\Omega t\}\cos(2\pi f_0 t) - \sqrt{2P_s}\sin\{d(t)\Omega t\}\sin(2\pi f_0 t)$$

$$Since, \quad d(t) = \pm 1$$

$$\therefore \cos\{\pm\Omega t\} = \cos(\Omega t)$$

$$and \ \ \sin\{\pm\Omega t\} = \pm\sin(\Omega t) = d(t)\sin(\Omega t)$$

By standard trigonometric relations

$$s(t) = \sqrt{2P_s}\cos(\Omega t)\cos(2\pi f_0 t) - \sqrt{2P_s}\,d(t)\sin(\Omega t)\sin(2\pi f_0 t)$$

In the above relation the first term, $\sqrt{2P_s}\cos(\Omega t)\cos(2\pi f_0 t)$ carries no information. The second term, $\sqrt{2P_s}\,d(t)\sin(\Omega t)\sin(2\pi f_0 t)$ carries the information signal $d(t)$. Thus only half of the transmitted energy carries the information signal.

4.5 QUADRATURE PHASE SHIFT KEYING (QPSK)

Principle

- In communication systems we know that there are two main resources, i.e. transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or Signalling rate f_b. In digital bandpass transmission, a carrier is used for transmission. This carrier is transmitted over a channel.
- If two or more bits are combined in some symbols, then the signalling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the transmission channel bandwidth. Thus because of grouping of bits in symbols, the transmission channel bandwidth is reduced.

- In quadrature phase shift keying, two successive bits in the data sequence are grouped together. This reduces the bits rate of signalling rate (i.e. f_b) and hence reduces the bandwidth of the channel.
- In BPSK we know that when symbol changes the level, the phase of the carrier is changed by 180^o. Since there were only two symbols in BPSK, the phase shift occurs in two levels only.
- In QPSK two successive bits are combined. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the phase of the carrier is changed by 45^o ($\pi/4$ $radians$). Table 4.2 shows these symbols and their phase shifts.

Sr.No.	Input successive bits		Symbol	Phase shift in carrier
$i = 1$	1(1V)	0(−1V)	S_1	$\pi/4$
$i = 2$	0(−1V)	0(−1V)	S_2	$3\pi/4$
$i = 3$	0(−1V)	1(1V)	S_3	$5\pi/4$
$i = 4$	1(1V)	1(1V)	S_4	$7\pi/4$

Table 4.2 Symbol and corresponding phase shifts in QPSK

Thus as shown in above table, there are 4 symbols and the phase is shifted by $\pi/4$ for each symbol.

4.5.1 QPSK Transmitter and Receiver

4.5.1.1 Offset QPSK (OQPSK) or Staggered QPSK Transmitter

Step 1 : Input Sequence Converted to NRZ type :

Fig. 4.20 shows the block diagram of OQPSK transmitter. The input binary sequence is first converted to a bipolar NRZ type of signal. This signal is called $b(t)$. It represents binary '1' by $+1\ V$ and binary '0' by $-1\ V$. This signal is shown in Fig.4.21(a).

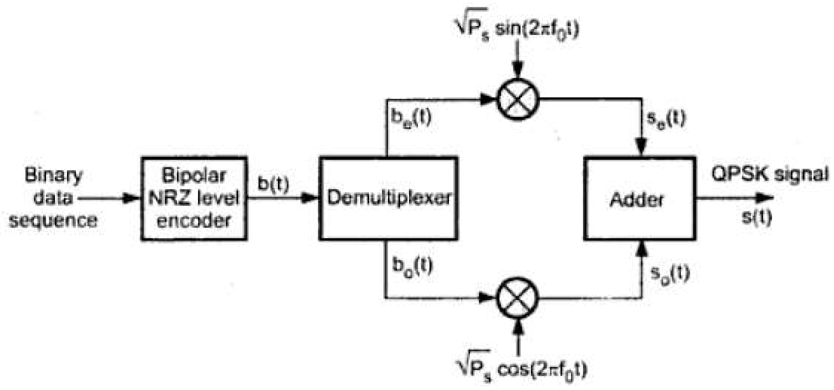


Fig. 4.20 An offset QPSK transmitter

Step 2 : Demultiplexing into odd and even numbered sequences

The demultiplexer divides $b(t)$ into two separate bit streams of the odd numbered and even numbered bits. $b_e(t)$ represents even numbered sequence and $b_o\ (t)$ represents odd numbered sequence. The symbol duration of both of these odd and even numbered sequences is $2T_b$. Thus every symbol contains two bits. Fig. 4.21(b) and (c) shows the waveforms of $b_e(t)\ and\ \ b_o(t)$.

Observe that the first even bit occurs after the first odd bit. Therefore even numbered bit sequence $b_e(t)$ starts with the delay of one bit period due to first odd bit. Thus first symbol of $b_e(t)$ is delayed by one bit period t_b with respect to first symbol of $b_o(t)$. This delay of T_b is called offset. Hence the name offset QPSK is given. This shows that the change in levels of $b_e(t)\ and\ b_o(t)$ cannot occur at the same time because of offset or staggering.

Step 3 : Modulation of quadrature carriers

The bit stream $b_e(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_0 t)$ and $b_0(t)$ modulates $\sqrt{P_s}\sin(2\pi f_0 t)$. These modulators are balanced modulator. The two carriers $\sqrt{P_s}\cos(2\pi f_0 t)$ and $\sqrt{P_s}\sin(2\pi f_0 t)$ are shown in Fig. 4.21 (d) and (e). These carriers arc also called quadrature carriers. The two modulated signals are,

$$s_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t) \qquad --(1)$$

$$and\ \ s_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) \qquad --(2)$$

Thus $s_e(t)$ and $s_o(t)$ are basically BFSK. The only difference is that $T\ =\ 2T_b$ here. The value of $b_e(t) and\ b_0(t)$ will be $+1V$ or $-1V$. Fig. 4.21 (f) and (g) shows the waveforms of $s_e(t)$ and $s_o(t)$.

Step 4 : Addition of modulated carriers

The adder of Fig. 4.20 adds these two signals $b_e(t)$ and $b_o(t)$. The output of the adder is OQPSK signal and it is given as,

$$s(t) = s_e(t) + s_o(t)$$

$$= b_e(t)\sqrt{P_s}\sin(2\pi f_0 t) + b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) \qquad --(3)$$

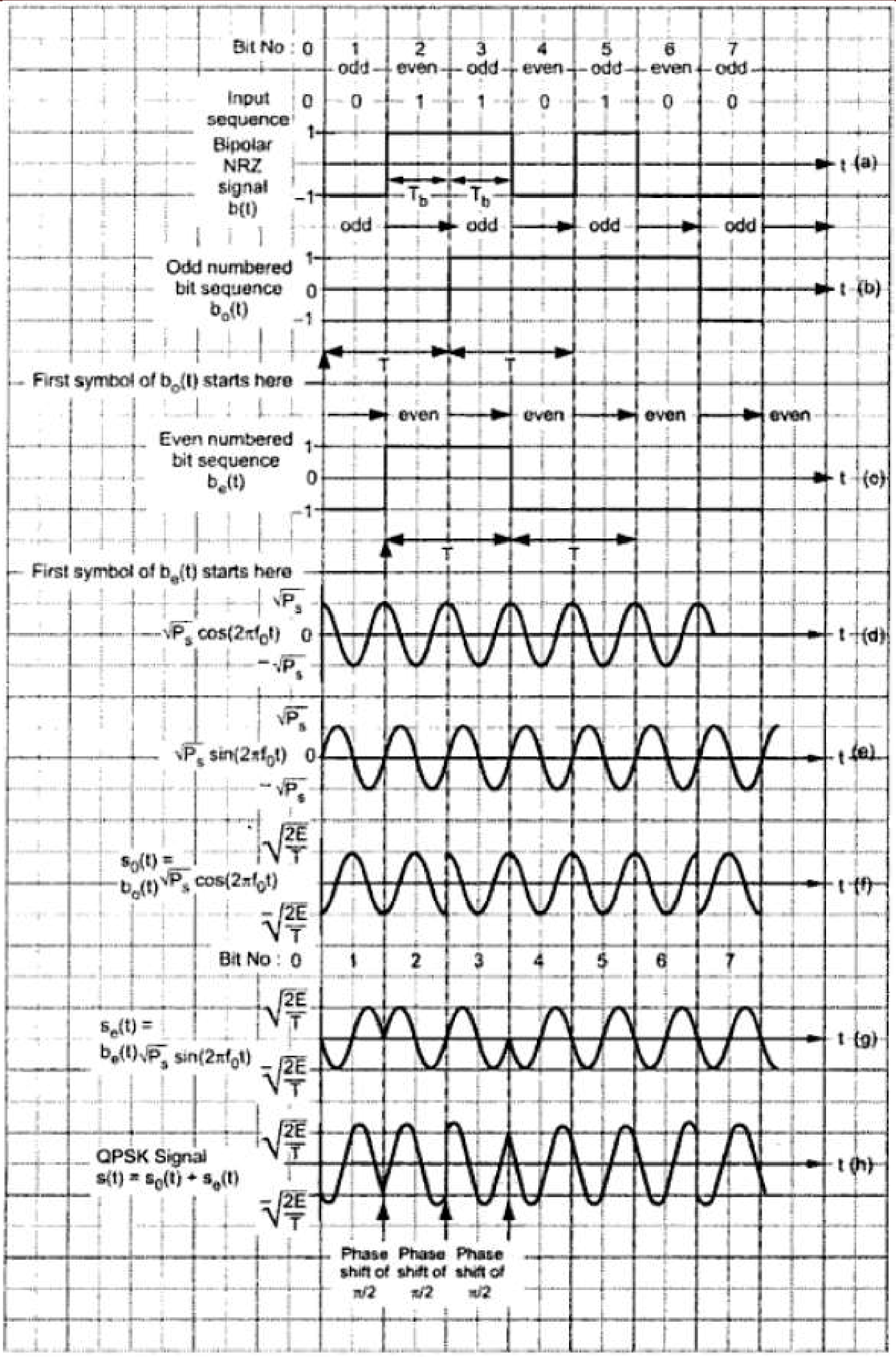


Fig. 4.21 QPSK waveforms (a) Input sequence and its NRZ waveform (b) Odd numbered bit sequence and its NRZ waveform (c) Even numbered bit sequence and its NRZ waveform (d) Basis function $\phi_1(t)$ (e) Basis function $\phi_2(t)$ (f) Binary PSK waveform for odd numbered channel (g) Binary PSK waveform for even numbered channel (h) final QPSK waveform representing equation

Step 5: QPSK signal and phase shift

Fig. 4.21(h) shows the QPSK signal represented by above equation. In QPSK signal of Fig. 4.21 (h). if there is any phase change, it occurs at minimum duration of T_b**.** This is because the two signals $s_e(t)$ *and* $s_o(t)$ have an offset of T_b. Because of this offset, the phase shift in QPSK signal is $\frac{\pi}{2}$. It is clear from the waveforms of Fig. 4.21 that $b_e(t)$ *and* $b_0(t)$ cannot change at the same time because of offset between them. Fig. 4.22 shows the phasor diagram of QPSK signal of equation (2). Since $b_e(t)$*and* $b_0(t)$ cannot change at the same time, the phase change in QPSK signal will be maximum $\pi/2$.

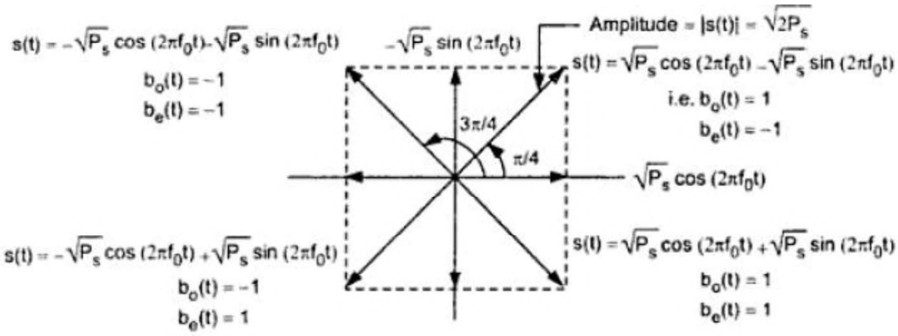


Fig. 4.22 Phasor diagram of QPSK signal

4.5.1.2 Non-Offset QPSK

- We known that there is an offset of T_b between $b_e(t)$*and* $b_0(t)$. If we delay $b_e(t)$ by T_b then there will be no offset. Then the sequences $b_e(t)$*and* $b_0(t)$ will change at the same time. This change will occur after minimum of $2T_b$
- As a result, the signals $s_e(t)$*and* $s_0(t)$ will have phase shifts at the same time. The individual phase shifts of $s_e(t)$*and* $s_0(t)$ are 180^o . Because of this the amplitude variations in the waveform will occur at the same time in $s_e(t)$*and* $s_0(t)$. Therefore these variations will be more pronounced in non offset QPSK than OQPSK.
- Filters are used to suppress side bands in QPSK. Since phase changes by 180^o in non offset QPSK, amplitude changes are more. Hence filtering affects the amplitude of non-offset QPSK. In OQPSK, the phase changes by 90^o, hence amplitude changes during filtering are less.
- Since amplitude variations an more in non-offset QPSK, the signal is affected if communication takes place through repeaters. These repeaters highly affect the amplitude and phase of the QPSK signal.

4.5.1.3 The QPSK Receiver

Fig. 4.23 shows the QPSK receiver. This is synchronous reception. Therefore coherent carrier is to be recovered from the received signal $s(f)$.

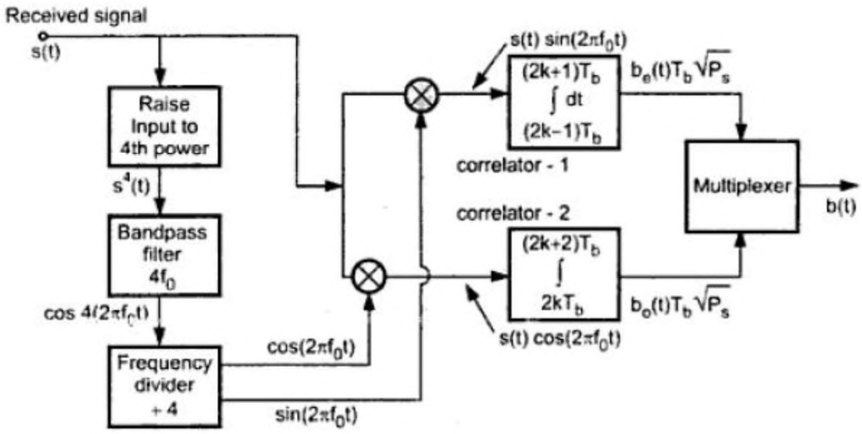


Fig. 4.23 QPSK receiver

Operation

Step 1: Isolation of carrier

The received signal $s\ (t)$ is first raised to its 4^{th} power, i.e. $s^4(t)$.Then it is passed through a bandpass filter centered around $4f_0$. The output of the bandpass filter is a coherent carrier of frequency $4f_0$. This is divided by 4 and it gives two coherent quadrature carriers $\cos(2\pi f_0 t)$ *and* $\sin(2\pi f_0 t)$.

Step 2: Synchronous detection

These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.

Step 3: Integration over two bits interval

The incoming signal is applied to both the multipliers. The integrator integrates the product signal over two bit interval (i.e, $T_s\ =\ 2T_b$).

Step 4: Sampling and multiplexing odd and even bit sequences

At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b. Hence the output of multiplexer is the signal $b\ (t)$. That is, the odd and even sequences are combined by multiplexer.

To show that output of Integrator depends upon respective bit sequence.

Let's consider the product signal at the output of upper multiplier

$$s(t)\sin(2\pi f_0 t) = b_0(t)\sqrt{P_s}\cos(2\pi f_0 t)\sin(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin^2(2\pi f_0 t)$$

This signal is integrated by the upper integrator in Fig. 4.23.

$$\therefore \int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = b_0(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt$$

$$+b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin^2(2\pi f_0 t)\;dt$$

$$since\ \frac{1}{2}\sin(2x) = \sin x\,.\cos x$$

$$and\ \sin^2 x = \frac{1}{2}[1-\cos 2x]$$

Using the above two trigonometric identities in the above equation,

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = \frac{b_0(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin(4\pi f_0 t)\,dt + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} 1.dt$$

$$-\frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} \cos(4\pi f_0 t).dt$$

In the above equation, the first and third integration terms involves integration of sinusoidal carriers over two bit period. They have full (integral number of) cycles over two bit period and hence integration will be zero.

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t).dt = \frac{b_e(t)\sqrt{P_s}}{2}[t]_{(2k-1)T_b}^{(2k+1)T_b}$$

$$= \frac{b_e(t)\sqrt{P_s}}{2}\times 2T_b$$

$$= b_e(t)\sqrt{P_s}T_b$$

Thus the upper integrator responds to even sequence only. Similarly we can obtain the output of lower integrator as $b_0(t)\sqrt{P_s}Tb$.

Eventhough bit synchronizer is not shown in Fig. 4.23, it is assumed to be present with the integrator to locate starting and ending times of integration. The multiplexer is also operated by bit synchronizer. The amplitudes of signals marked in Fig. 4.23 are arbitrary. They can change depending upon the gains of integrator.

Ambiguity in the output:

In Fig. 4.23 observe that even if the received signal is negative, the recovered carrier remains unaffected because of the 4th power conversion of the signal. Therefore it will not be possible to determine whether the transmitted signals were positive or negative [i.e. $+b_e(t)\ or - b_e(t)\ and\ + b_o(t)\ or - b_o(t)$]. This is phase ambiguity in output similar to BPSK. This problem can be recovered by using differential encoding and decoding of $b(t)$.

4.5.1.4 Carrier Synchronization in QPSK

Both the carriers are to be synchronized properly in coherent detection in QPSK. Fig. 4.24 shows the PLL system for carrier synchronization in QPSK.

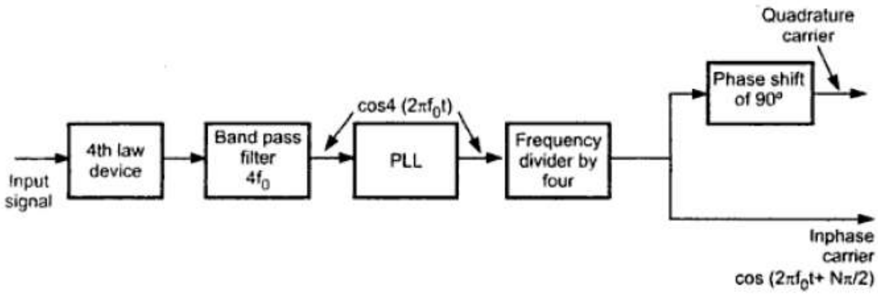


Fig. 4.24 PLL system for carrier synchronization

The fourth power of the input signal contains discrete frequency component at $4f_o$. We know that,

$$\cos 4(2\pi f_0 t) = \cos(8\pi f_0 t) + 2\pi N)$$

Here 'N' is the number of cycles over the bit period. It is always integer value. When the frequency division by four takes place, the RHS of above equation becomes $\cos\Big(2\pi f_0 t + N\pi 2$.This shows that the output has a fixed phase error of $N\pi 2$. Differential encoding can be used to nullify the phase error events. The PLL remains locked with the phase of $'4f_o'$ and then output of PLL is divided by 4. This gives a coherent carrier. A 90^o phase shift is added to this carrier to generate a quadrature carrier.

4.5.2 Signal Space Representation of QPSK Signals

Fig. 4.22 shows the phasor diagram of QPSK signal. Depending upon the combination of two successive bits, the phase shift occurs in carrier (see table 4.2).That is the QPSK signal of equation (3) can be written as,

$$s(t) = \sqrt{2P_s}\cos\left[2\pi f_0 t + (2m+1)\frac{\pi}{4}\right] \qquad m = 0,1,2,3$$

Here, the above equation takes four values arid they represent the phasors of Fig.4.22.

The above equation can be expanded as,

$$s(t) = \sqrt{2P_s}\cos(2\pi f_0 t)\cos(2m+1)\frac{\pi}{4} - \sqrt{2P_s}\sin(2\pi f_0 t)\sin(2m+1)\frac{\pi}{4}$$

Let's rearrange the above equation as,

$$s(t) = \left\{\sqrt{P_s T_s}\cos\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$$

$$-\left\{\sqrt{P_s T_s}\sin\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t) \qquad --(4)$$

$$let\ \ \phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t) \qquad --(5)$$

$$and\ \ \phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t) \qquad --(6)$$

The above two signals are called orthogonal signals and they are used as carriers in QPSK modulator.

$$Let\ \ b_0(t) = \sqrt{2}\cos\left[(2m+1)\frac{\pi}{4}\right]$$

$$Let\ \ b_e(t) = -\sqrt{2}\sin\left[(2m+1)\frac{\pi}{4}\right]$$

With the use of equation (5) and (6) we can write equation (4) as,

$$s(t) = \sqrt{P_s T_s}.\frac{1}{\sqrt{2}}.b_0(t)\phi_1(t) + \sqrt{P_s T_s}.\frac{1}{\sqrt{2}}.b_e(t)\phi_2(t)$$

$$= \sqrt{P_s\frac{T_s}{2}}.b_0(t)\phi_1(t) + \sqrt{P_s\frac{T_s}{2}}.b_e(t)\phi_2(t)$$

T_s= symbol duration and $T_s\ =\ 2T_b$

$$T_b = \frac{T_s}{2}$$

Then the above equation becomes,

$$s(t) = \sqrt{P_s T_b}.b_0(t)\phi_1(t) + \sqrt{P_s T_b}.b_e(t)\phi_2(t)$$

Since bit energy $E_b = P_s T_b$

$$s(t) = \sqrt{E_b}.\, b_0(t)\phi_1(t) + \sqrt{E_b}.\, b_e(t)\phi_2(t)$$

Comments

- The above equation gives signal space representation of QPSK signal. The two orthogonal signals, $\phi_1(t)$ and $\phi_2(t)$ form the two axes of the signal space. Fig. 4.25 shows the signal space representation.

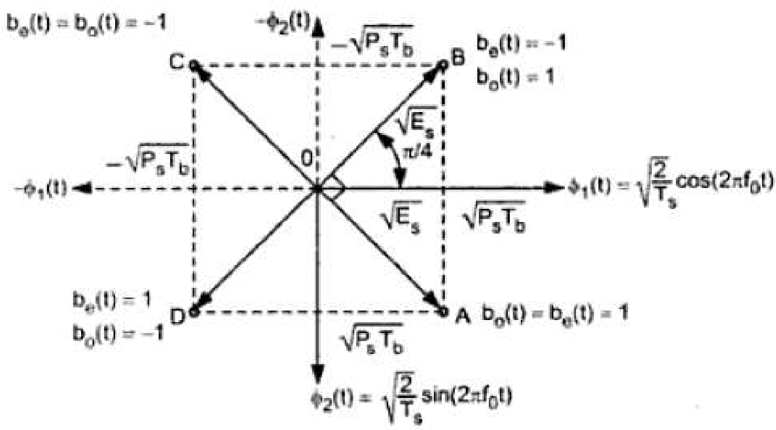


Fig. 4.25 Signal space representation of QPSK signals

- The possible 4 signal points are shown by small circles on $\phi_1(t)\phi_2(t)$ plane. From each signal point, we obtain two bits. For example from point 'A', we obtain two bits as (1, 1) and from 'B' we obtain bits, as (-1, 1).
- The distance of any signal point from origin '0' given as,

$$Distance, \quad OB = \sqrt{P_s T_b + P_s T_b}$$

$$= \sqrt{2 P_s T_b}$$

$$= \sqrt{P_s T_s} \qquad (\because 2T_b = T_s)$$

$$= \sqrt{E_s} \qquad (\because P_s T_s = E_s)$$

Thus the length of each signal point from origin is $\sqrt{E_s}$

- We know that $b_e(t)$ and $b_o(t)$ represent two successive bits. There is an offset of $'T_b'$ between $b_e(t)$ and $b_o(t)$. Therefore $b_e(t)$ and $b_o(t)$ both cannot change their levels simultaneously. Therefore either $b_e(t)$ or $b_o(t)$ can change at a time.
- Let's say that $b_e(t) = b_o(t) = 1$ representing signal point 'A' in Fig. 4.25. In the next bit interval if $b_o(t) = -1$, then signal point will be 'D'. Otherwise if $b_e(t)$ changes its level

(i.e. $b_e(t) = -1$), then next signal point will be 'B'.Thus from signal point 'A', then next signal points will be either 'D' or ·B'.

Distance between signal points:

Normally the ability to determine a bit without error is measured by the distance between two nearest possible signal points in the signal space. Such points differed in a single bit. For example signal points 'A' and 'B' are two nearest points since they differ by n single bit $b_e(t)$. As 'A' and 'B' becomes closer to each other, the possibility of error increases. Hence this distance should be as large as possible. This distance is denoted by 'd', In Fig. 4.25, the distance between signal points 'A' and 'B' is given as,

$$d^2 = \left(\sqrt{E_s}\right)^2 + \left(\sqrt{E_s}\right)^2$$

$$d = \sqrt{2E_s}$$

$$d = 2\sqrt{P_s T_b} = 2\sqrt{E_b}$$

Compare this distance with the distance of BPSK signals. This shows that the distance for QPSK is the same as that for BPSK. Since this distance represents noise immunity of the system, it shows that noise immunities of BPSK and QPSK are same.

4.5.3 Spectrum of QPSK Signal

Step 1: PSD of NRZ waveform

The input sequence $b(t)$ is of bit duration T_b. It is NRZ bipolar waveform. The power spectral density of such waveform as,

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

And $V_b = \sqrt{P_s}$, then above equation becomes,

$$S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \qquad --(7)$$

The above equation gives power spectral density of signal $b(t)$

Step 2: PSDs of even and odd numbered sequence

This signal is divided into $b_e(t)$ *and* $b_o(t)$ each of bit period $2T_b$. if we consider that symbols 1 and 0 are equally likely, then we can write power spectral densities of $b_e(t)$ *and* $b_o(t)$ as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$$S_0(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

In the above two equations we have just replaced T_b by T_s and T_s is the period of bit in $b_e(t)$ and $b_o(t)$.

Step 3: PSD of QPSK signal

Since inphase and quadrature components $[b_e(t)$ and $b_o(t)]$ are statistically independent, the baseband power spectral density of QPSK signal equals the sum of the individual power spectral densities of $b_e(t)$ and $b_o(t)$ i.e.,

$$S_B(f) = S_e(f) + S_0(f)$$
$$= 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

This equation gives baseband power spectral density of QPSK signal. Upon modulation of carrier of frequency f_0, the spectral density given by above equation is shifted at $\pm f_o$. Thus plots of power spectral density of QPSK will be similar to that BPSK.

4.5.4 Bandwidth of QPSK Signal

We have seen that the bandwidth of BPSK signal is equal of $2f_b$. Here $T_b = \frac{1}{f_b}$ is the one bit period. In QPSK the two waveforms $b_e(t)$ and $b_o(t)$ from the baseband signals. One bit period for both of these signals is equal to $2T_b$. Therefore bandwidth of QPSK signal is,

$$\boxed{BW = 2 \times \frac{1}{2T_b} \qquad or \;\; BW = f_b} \qquad --(8)$$

Thus the bandwidth of QPSK signal is half of the bandwidth of BPSK signal. We know that noise immunity of QPSK and BPSK is same. This shows that in spite of the reduction in bandwidth in QPSK, the noise immunity remains same as compared to BPSK. BW of QPSK can also be obtained by plotting equation (7) as shown in Fig. 4.26 below.

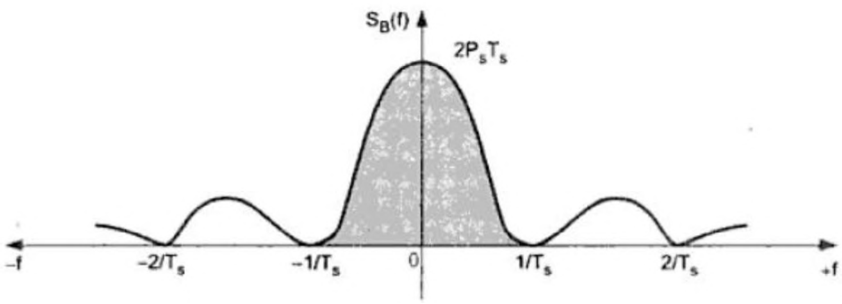


Fig. 4.26 Plot of power spectral density of QPSK signal

$$BW = Highest\ frequency - Lowest\ frequency\ in\ main\ lobe$$

$$= \frac{1}{T_s} - \frac{1}{T_s}\ since\ carrier\ frequency\ f_0\ cancels\ out$$

$$= \frac{2}{T_s}$$

We know that $T_s = 2T_b$

$$\therefore\ BW = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$$

which is same as equation (8).

4.5.5 Advantages of QPSK

QPSK has some definite advantages and disadvantages as compared to BPSK and DPSK.

Advantages:

1) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.

2) Because of reduced bandwidth, the information transmission rate of QPSK is higher.

3) Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

4.6 QUADRATURE AMPLITUDE MODULATION (QAM)

[OR] QUADRATURE AMPLITUDE SHIFT KEYING (QASK)

The correct detection of the signal depends upon the separation between the signal points in the signal space. In case of PSK systems all points lie on the circumference of the circle. This is because PSK signal has constant amplitude throughout. If amplitude of the signal is also varied, then the points will lie inside the circle also on the signal space diagram. This further increases the noise immunity of the system. Such system involves phase as well as amplitude shift keying. It is called quadrature amplitude phase shift keying or simply QASK. It is also called quadrature amplitude modulation or QAM.

4.6.1 Geometrical Representation and Euclidean Distance of QASK Signals (or Signal Space Representation or Signal Space Constellation)

Let us consider the case of 4 bit symbol. Then there will be $2^4 = 16$ possible symbols. In the QASK system, such 16 symbols are represented geometrically as shown in Fig. 4.27

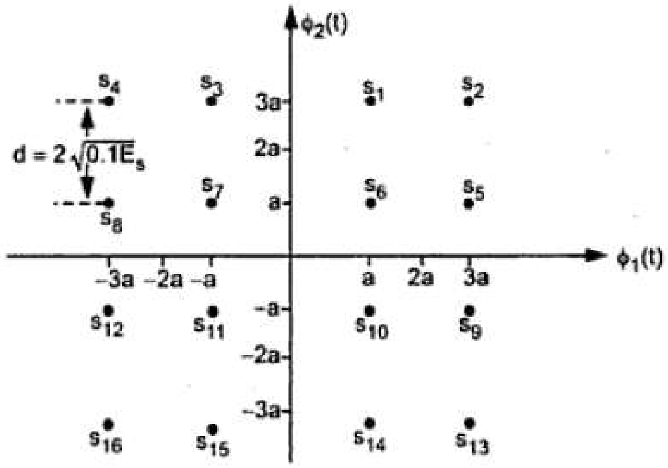


Fig. 4.27 Geometrical representation of 16 signals In QASK system

It shows Geometrical representation of 16 QASK signals. The distance from the neighboring points is $d = 2a$. Let the signals be equally likely. Then the average energy associated with the signal can be obtained as (Considering first quadrant),

$$E_s = \frac{1}{4}[(a^2 + a^2) + (9a^2 + a^2) + (a^2 + 9a^2) + (9a^2 + 9a^2)]$$

$$= 10a^2$$

$$a = \sqrt{0.1E_s}$$

Since $d = 2a$ we have,

$$d = 2\sqrt{0.1E_s}$$

$$= \sqrt{0.4E_s}$$

This gives the distance between two signal points in 16 QASK. In each symbol there are 4 bits. Hence bit energy and symbol energy are related as,

$$E_s = 4E_b$$

$$\therefore \; d = \sqrt{0.4 \text{ x } 4E_b}$$

$$= \sqrt{16E_b}$$

The distance for QPSK is given as,

$$d_{QPSK} = 2\sqrt{E_b}$$

$$= \sqrt{4E_b}$$

and the distance for 16-ary PSK is given from equation

$$d_{16PSK} = 2\sqrt{E}\sin\frac{\pi}{16}$$

$$= 2\sqrt{4E_b}\sin\frac{\pi}{16} \quad \because E_s = 4E_b$$

$$= 2\sqrt{0.15E_b}$$

$$= \sqrt{0.6E_b}$$

Thus the distance of 16-QASK is greater than 16-ary PSK where as it is less than QPSK.

4.6.2 Transmitter and Receiver of QAM

Transmitter of QAM

The signal in Fig. 4.27 is represented as,

$$s(t) = k_1 a\,\phi_1(t) + k_2 a\,\phi_2(t) \qquad --(1)$$

Here k_1 and k_2 will take values of ± 1 or ± 3. $\phi_1(t)$ *and* $\phi_2(t)$ are orthogonal carriers having the values as follows

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$$

We know that,

$$a = \sqrt{0.1E_s}$$

We can write equation (1) as,

$$s(t) = k_1\sqrt{0.2\,\frac{E_s}{T_s}}\cos(2\pi f_0 t) + k_2\sqrt{0.2\,\frac{E_s}{T_s}}\sin(2\pi f_0 t)$$

We know that $E - s = P_s T_s$

$$\therefore \frac{E_s}{T_s} = P_s$$

then the above equation becomes,

$$s(t) = k_1\sqrt{0.2\,P_s}\cos(2\pi f_0 t) + k_2\sqrt{0.2\,P_s}\sin(2\pi f_0 t) \qquad --(2)$$

This equation gives the QASK signal. Here k_1 and k_2 defined the amplitude of the modulated signal. Fig 4.28 shows the transmitter for 4 bit QASK (or 16-QASK) system. The input bit steam is applied to a serial to parallel converter. Four successive bits are applied to the digital to analog converters. These bits are applied after every T_s second. T_s is the symbol period and $T_s = 4T_b$. Bits b_k and b_{k+1} are applied to upper digital to analog converter and $b_{k+2}\ and b_{k+3}$ are applied to lower digital to analog converter. Depending upon two input bits, the output of digital to analog converter takes four output levels. Thus $A_e(t) and\ A_o(t)$ takes 4 levels depending upon combination of two inputs bits. $A_e(t)$ modulates the carrier $\sqrt{P_s}\cos(4\pi f_0 t)$ and $A_o(t)$ modulates $\sqrt{P_s}\sin(4\pi f_0 t)$. The adder combines two signals to give QASK signal

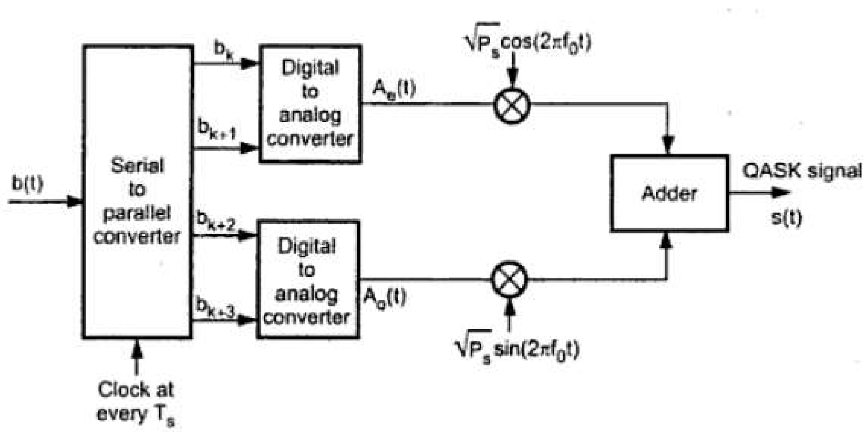


Fig. 4.28 Generation of QASK signal

It is given as,

$$s(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_0 t) + A_0(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

If we compare the above equation with equation (2), we can write

$$A_e(t)\ and\ \ A_0(t) = \pm\sqrt{0.2}\ or\ \pm 3\sqrt{0.2}$$

(depending upon input to *D*/A converter)

Receiver of QASK Signal

Fig. 4.29 shows the receiver of 16-QASK (4 bits QASK) system. The mput signal $s(t)$ is raised to 4th power. It then passed through a bandpass filter centered around the frequency $4f_0$ the signal is then divided in frequency by four. It gives a coherent carrier $cos\ (2\pi f_0 t)$. Quadrature carrier $\sin(2\pi f_0 t)$ is produced by phase shifting of 90^o. The inphase and quadrature coherent carriers are multiplied with QASK signal $s\ (t)$.

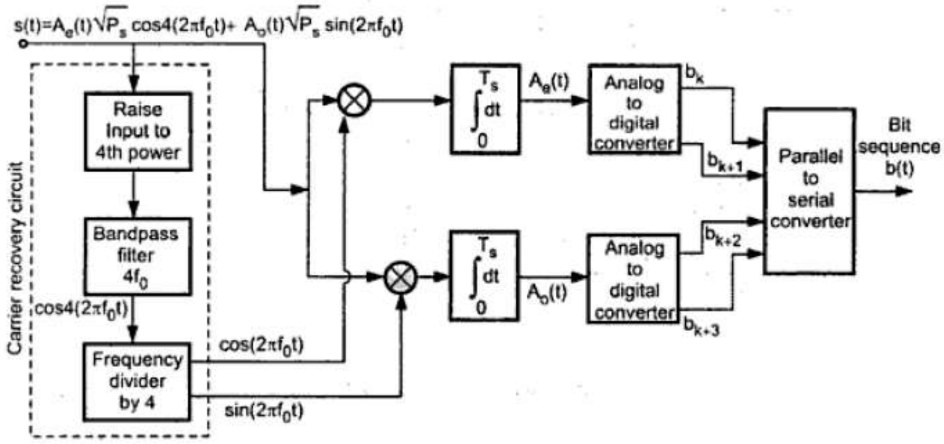


Fig. 4.29 4-bit QASK receiver block diagram

Since the amplitudes of $A_e(t)\ and\ A_o(t)$ are bit constant and equal, let us check whether we can really recover the carrier correctly. The 4^{th} power QASK signal is,

$$s^4(t) = P_s^2[A_e(t)\cos(2\pi f_0 t) + A_0(t)\sin(2\pi f_0 t)]^4$$

This signal is passed through a bandpass filter of $4f_0$. Therefore we will consider only the frequencies of $4f_0$ i.e.,

$$s^4(t) = \frac{P_s^2}{8}[A^4{}_e(t) + A^4{}_0(t) - 6A_e^2(t)A_0^2(t)]\cos 4(2\pi f_0 t)$$
$$+\frac{P_s^2}{2}[A_e(t)A_0(t)\{A_e^2(t) - A_0^2(t)\}]\sin 4(2\pi f_0 t)$$

The average value of second term will be zero, hence only first term is passed through a bandpass filter centered at $4f_o$. This happens because all power of $A_e(t)\ and\ A_o(t)$ in the first term are even. The integrators integrate the multiplied signals over one symbol period. The output of integrators at sampling period give $A_e(t)\ and\ A_o(t)$. The analog to digital converters gives the four bits $b_k, b_{k+1}, b_{k+2}\ and\ b_{k+3}$. The parallel to serial converter then generates the bit sequence $b\ (t)$.

4.6.3 Power Spectral Density and Bandwidth of QASK Signal

The QASK equation given by equation (2) is similar to that of M-ary PSK. Therefore power spectral density of baseband QASK signal will be,

$$S(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \qquad --(3)$$

The above equation gives power spectral density of $A_e(t)$ and $A_o(t)$. When tley modulate the carrier, the main lobe given by above equation is shifted at carrier frequency f_o.

$$S(f) = \frac{P_sT_s}{2}\left[\frac{\sin(\pi(f-f_0)T_s)}{\pi(f-f_0)T_s}\right]^2 + \frac{P_sT_s}{2}\left[\frac{\sin(\pi(f+f_0)T_s)}{\pi(f+f_0)T_s}\right]^2$$

This equation gives power spectral density of QASK signal.

Bandwidth of QASK Signal :

The main lobe of QASK signal given by equation (3) can be ploted.

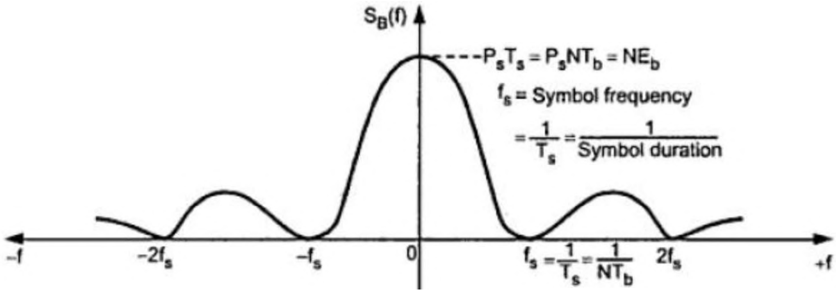


Fig 4.30 Plot of PSD of QASK signal

Therefore bandwidth will be,

$$BW = f_s - (-f_s) = 2f_s$$

$$= \frac{2}{T_s} \qquad since\ f_s = \frac{1}{T_s}$$

$$= \frac{2}{NT_b} \qquad since\ T_s = NT_b$$

$$= \frac{2f_b}{N} \qquad since\ f_b = \frac{1}{T_b}$$

4.6.4 Comparison between QASK and QPSK

QASK and QPSK are both quadrature modulation techniques. They have certain advantages and disadvantages over each other. Table 4.3 shows the comparison between QASKand QPSK.

Sr. No.	Parameter	QPSK	QASK
1	Modulation	Quadrature phase	Quadrature amplitude and phase
2	Location of signal points	All signal points placed on circumference of circle	Signal points are replaced symmetrically about origin
3	Distance between signal points	$2\sqrt{0.15 E_b}$ for 16 symbols and $2\sqrt{E_b}$ for 4 symbols	$2\sqrt{0.4 E_b}$ for 16 symbols
4	Complexity	Relatively simpler	Relatively complex
5	Noise immunity	Better than QASK	Poor than QPSK. But better than M-ary PSK
6	Error probability	Less than QASK.	Higher than QPSK. Lower than M-ary PSK.
7	Type of demodulation	Coherent	Coherent

Table 4.3 Comparison of QPSK and QASK

4.7 SYNCHRONIZATION

The signals from various sources are transmitted on the single channel by multiplexing. This requires synchronization between transmitter and receiver. Special synchronization bits are added in the transmitted signal for this purpose. Synchronization is also required for detectors to recover the digital data properly from the modulated signal. In this section various synchronization techniques are discussed.

There are three broad types of synchronization. They are,

1. Carrier Synchronization.
2. Symbol and Bit Synchronization.
3. Frame Synchronization.

4.7.1 Carrier Synchronization (Carrier Recovery)

The carrier synchronization is required in coherent detection methods to generate a coherent reference at the receiver. In this method the data bearing signal is modulated on the carrier in such a way that the power spectrum of the modulated carrier signal contains a discrete component at the carrier frequency. That is the Fourier transform of the modulated signal contains one component at f_c also. Then the phase lock loop can be used to track this component f_c. The output frequency of phase locked loop is thus locked to the carrier frequency f_c in the transmitted signal. This output frequency of phase locked loop is used as a coherent reference signal for detection in the receiver.

Carrier Synchronization using M^{th} Power Loop

Fig. 4.31 shows the block diagram of carrier recovery or carrier synchronization circuit.

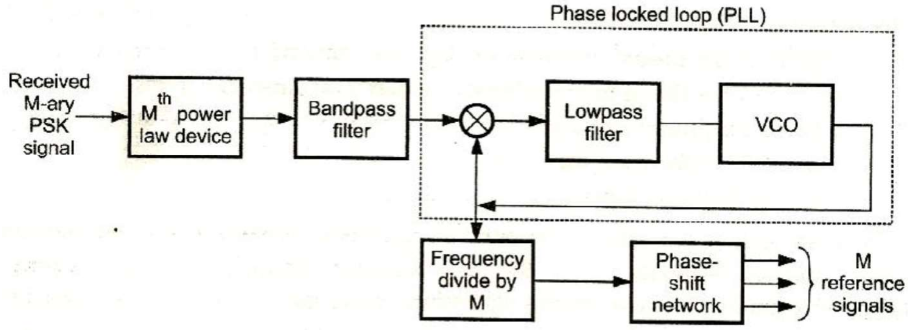


Fig. 4.31 Block diagram of M^{th} power loop

Fig. 4.31 shows the block diagram of carrier recovery circuit for M-ary PSK.This circuit is called the M^{th} power loop. When M = 2, then it is called squaring loop. When M= 2, the M-ary PSK is then called as binary PSK. As shown in diagram, the input signal is first raised to the M^{th} power by the M^{th} power law device. Then the signal is passed through a bandpass filter. The bandpass filter is tuned to the carrier frequency f_c. The phase locked loop consists of a phase detector, low-pass filter and VCO. The phase locked loop tracks the carrier frequency.Then the output of a voltage controlled oscillator (VCO) is the carrier frequency. The output frequency of VCO is divided by M. This is done because the M^{th} power of the input signal multiplies carrier frequency by M. The phase shift network then separates 'M' reference signals for the 'M' correlation receivers. In this technique the power of the input signal is raised to some power say 'M'. Let us say M = 2, then the input signal is squared. Because of this, the sign of the recovered carrier is always independent of sign of the input signal carrier since it is squared. Therefore there can be 180^0 error in the output.

Costas Loop for Carrier Synchroization

This is the alternative method for carrier synchronization.This is used for binary phase shift keying. The block diagram is shown in Fig. 4.32.

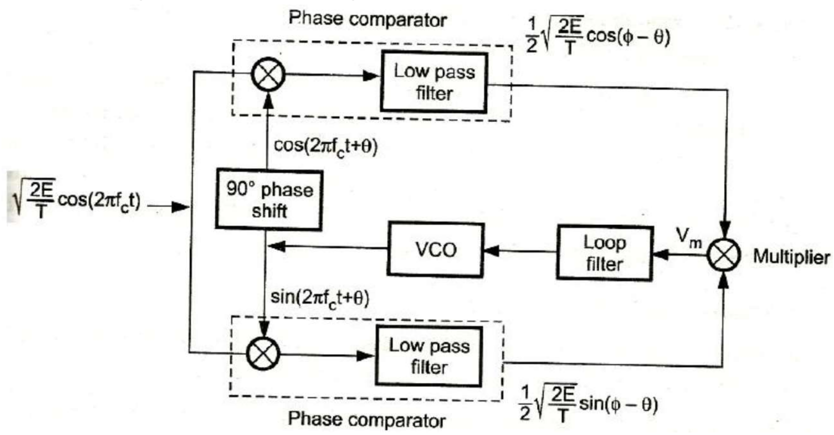


Fig. 4.32 The costas loop

As shown in Fig. 4.32 there are two phase locked loops. They have a common VCO and separate phase comparators. Let us assume that the VCO operates at the carrier frequency f_c with arbitrary phase angle θ. The BPSK signal is supplied to both the phase comparators. The low-pass filters remove the double frequency terms generated in the phase comparators and generate,

$$\frac{1}{2}\sqrt{\frac{2E}{T}}\cos(\phi-\theta) \quad and \quad \frac{1}{2}\sqrt{\frac{2E}{T}}\sin(\phi-\theta)$$

The multiplier output is given as,

$$V_m = \frac{1}{4}\times\frac{2E}{T}\sin(\phi-\theta)\cos(\phi-\theta)$$

$$= \frac{E}{2T}\cdot\frac{1}{2}\sin 2(\phi-\theta)$$

$$= \frac{E}{4T}\sin 2(\phi-\theta) \qquad --(1)$$

The power 'P' of the signal over the period T is given by

$$P = \frac{E}{T}$$

Therefore equation (1) can be written as,

$$V_m = \frac{P}{4}\sin 2(\phi-\theta)$$

If there is some difference between the VCO frequency and the input carrier frequency then the phase difference $(\phi-\theta)$ is changed proportionally. The change $\phi-\theta)$ causes V_m to increase or decrease VCO frequency such that synchronization maintained.

4.8 DIFFERENTIAL PHASE SHIFT KEYING (DPSK)

Principle:

Differential phase shift keying (DPSK) is differentially coherent modulation method. DPSK does not need a synchronous (coherent) carrier at the demodulator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

4.8.1 DPSK Transmitter and Receiver

4.8.1.1 Transmitter / Generator of DPSK Signal

Fig. 4.33 shows the scheme to generate DPSK signal.

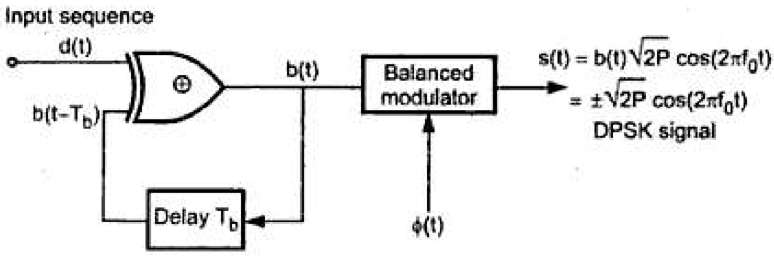


Fig. 4.33 Block diagram of DPSK generate or transmitter

Operation and waveform of transmitter

The input sequence is $d(t)$. Output sequence is $b(t)$ and $b(t - T_b)$ is the previous output delayed by one bit period. Depending upon values of $b(t)$ and $b(t - T_b)$, exclusive OR gate generates the output sequence $b(t)$. Table 4.4 shows the truth table of this operation.

$d(t)$	$b(t - T_b)$	$b(t)$
0 (−1V)	0 (−1V)	0 (−1V)
0 (−1V)	1(1V)	1(1V)
1(1V)	0 (−1V)	1(1V)
1(1V)	1(1V)	0 (−1V)

Table 4.4 Truth table of exclusive OR gate

An arbitrary sequence $d(t)$ is taken. Depending on this sequence, $b\ (t)$ and $b(t - T_b)$ are found. These waveforms are shown in Fig. 4.34. The above table 4.4 is used to derive the levels of these waveforms.

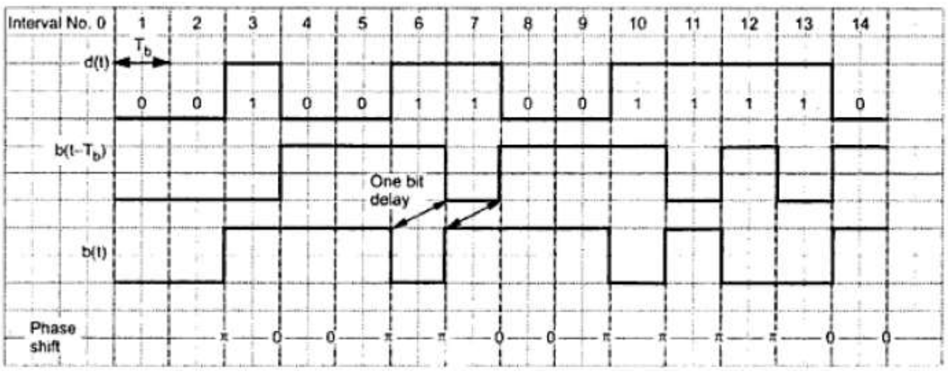


Fig. 4.34 DPSK waveforms

From the waveforms of Fig. 4.34 it is clear that $b(t-T_b)$ is the delayed version of $b\ (t)$ by one bit period T_b. The exclusive OR operation is satisfied in any interval i.e. in any interval $b\ (t)$ is given as,

$$b(t) = d(t) \oplus b(t-T_b)$$

While drawing the waveforms the value of $b(t-T_b)$ is not known initially in interval no.l. Therefore it is assumed to be zero and then waveforms are drawn.

Conclusions from the waveforms

1. Output sequence $b(t)$ changes level at the beginning of each interval in which $d\ (t) = 1$ and it does not changes level when $d\ (t) = 0$. Observe that $d\ (3) = 1$, hence level of $b\ (3)$ is changed at the beginning of interval 3. Similarly in intervals 10, 11, 12 and 13 $d\ (t) = 1$. Hence $b\ (t)$ is changed at the starting of these intervals. In interval 8 and 9 $d\ (t) = 0$. Hence $b\ (t)$ is not changed in these intervals

2. When $d(t) = 0,\ b(t) = b(t-T_b)$ and

When $d(t) = 1,\ b(t) = \overline{b(t-T_b)}$

3. In Interval no.1. we has assumed $b(t-T_b) = 0$ and we obtained the waveform as shown in fig. 4.34. If we assume $b(t-T_b) = 1$ in interval no. 1, then the waveform of $b\ (t)$ will be inverted. But still $b(t)$ changes the level at the beginning each interval in which $d(t) = 1$.

4. The sequence $b(t)$ modulates sinusoidal carrier.

5. When $b(t)$ changes the level, phase of the carrier is changed. Since $b(t)$ changes its level only if $d(t) = 1$; It shows that phase of the carrier is changed only if $d\ (t) = 1$.

In BPSK phase of the carrier on both the symbol '1' and '0'. Whereas in DPSK phase of the carrier changes only on symbol '1'. This is the main difference between BPSK and DPSK

6. Always two successive bits of $d(t)$ are checked for any change of level. Hence one symbol has two bits.

$$Symbol\ duration\ (T) = Duration\ of\ two\ bits\ (2T_b)$$

$$\therefore\ T = 2T_b$$

As shown in Fig. 4.33, the sequence $b\ (t)$ is applied to a balanced modulator. The balanced modulator is also supplied with a carrier $\sqrt{2P}\cos(2\pi f_0 t)$.

The modulator output is,

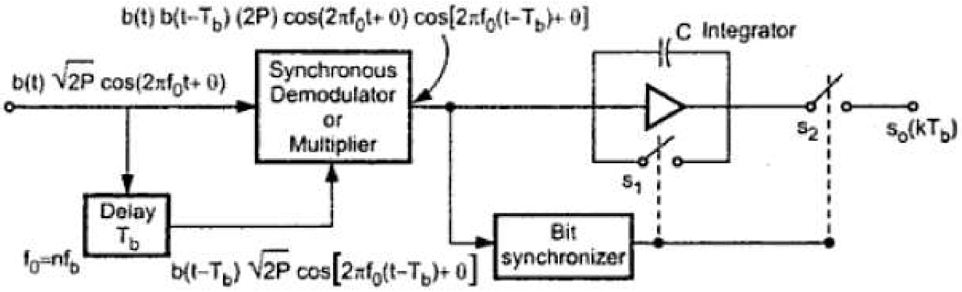
$$s(t) = b(t)\sqrt{2P}\,cos(2\pi f_0 t)$$

$$= \pm\sqrt{2P}\, cos(2\pi f_0 t)$$

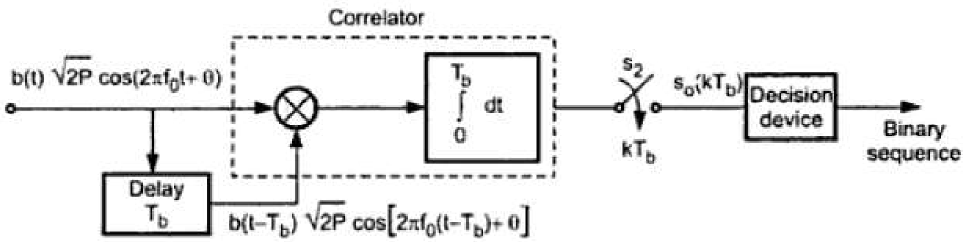
The above equation gives DPSK signal. Fig. 4.34 shows this DPSK waveforms. As shown in the waveforms the phase changes only when $d(t) = 1$.

4.8.1.2 DPSK Receiver

Fig. 4.35 shows the method to recover the binary sequence from DPSK signal. Fig. 4.35 (a) and (b) are equivalent to each other. Fig. 4.35(b) represents DPSK receiver using correlator. Fig. 4.35(3) shows multiplier and integrators separately.



(a)



(b)

Fig. 4.35 (a) DPSK receiver (b) Equivalent diagram of DPSK receiver using correlator

Operation of Receiver

1. Phase shift in received signal: During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the input of the receiver is,

$$Received\ signal = b(t)\sqrt{2P}\, cos(2\pi f_0 t + \theta)$$

2. Multiplier output: This signal is multiplied with its delayed version by one bit. Therefore the output of the multiplier is,

$$Multiplier\ output = b(t)b(t - T_b)(2P)\cos(2\pi f_0 t + \theta)\cos[2\pi f_0 (t - T_b) + \theta]$$

$$we\ know\ that, \quad cosA \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

Here,

$$A = 2\pi f_0 t + \theta \quad and \quad B = 2\pi f_0 (t - T_b) + \theta$$

$$\therefore Multiplier\ output = b(t)b(t-T_b)P\left\{\cos 2\pi f_0 T_b + \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right]\right\}$$

f_0 is the carrier frequency and T_b is one bit period. T_b contains integral number of cycles of f_o. We know that,

$$f_b = \frac{1}{T_b}$$

If T_b contains 'n' cycles of f_o then we can write,

$$f_0 = nf_b \quad => f_0 = \frac{n}{T_b}$$

$$\therefore \quad f_0 T_b = n$$

Putting $f_0 T_b = n$ in first cosine term in equation (1) we get

$$Multiplier\ output = b(t)b(t-T_b)P\left\{\cos 2\pi n + \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right]\right\}$$

Since $\cos 2\pi n = 1$, the above equation wil be,

$$Multiplier\ output = b(t)b(t-T_b)P + b(t)b(t-T_b)P\cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right]$$

3. Integrator: The above signal is given to the integrator. In the k^{th} bit interval, the integrator output can be written as,

$$s_0(kT_b) = b(kT_b)b[(k-1)T_b]P\int_{(k-1)T_b}^{kT_b} dt$$
$$+ b(kT_b)b[(k-1)T_b]P\int_{(k-1)T_b}^{kT_b} \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right]dt$$

The integration of the second term will be zero since it is integration of carrier over one bit duration. The carrier has integral number of cycles over one bit period hence integration is zero. Therefore we can write,

$$s_0(kT_b) = b(kT_b)b[(k-1)T_b]\,P[kT_b - (k-1)T_b]$$

$$= b(kT_b)b[(k-1)T_b]\ PT_b$$

Here know that $PT_b = E_b$; i.e. energy of one bit. The product $b(kT_b)b[(k-1)T_b]$ decides the sign of PT_b.

The transmitted data bit $d\ (t)$ can be verified easily from product $b(kT_b)b[(k-l)T_b)$. We know from Fig. 4.34 when $b(t) = b(t-T_b),\ d(t) = 0$. That is if both are $+1V\ or\ -1V$ then $b(t)b(t-T_b) = 1$. Alternately we can write,

$$\boxed{if\ \ b(t)b(t-T_b) = 1V \quad then\ \ d(t) = 0}$$

We know that $b(t) = \overline{b(t-T_b)}$ then $d\ (t) = 1$. That is $b(t) = -1V,\ b(t-T_b) = +1V$ and vice versa. Therefore $b(t)b(t-T_b) = -1$. Alternately we can write,

$$\boxed{if\ \ b(t)b(t-T_b) = -1V \quad then\ \ d(t) = 1}$$

4. Decision device: The decision device is shown in Fig. 4.35 (b). We know that,

$$s_0(kT_b) = b(kT_b)b[(k-1)T_b]\ PT_b$$

$$if\ \ s_0(kT_b) = \begin{cases} -PT_b, & then\ \ d(t) = 1\ \ and \\ +PT_b, & then\ \ d(t) = 0 \end{cases}$$

4.8.2 Bandwidth of DPSK Signal

We know that one previous bit is used to decide the phase shift of next bit. Change in $b\ (t)$ occurs only if input bit is at level '1'. No change occurs if input bit is at level '0'.

Since thee previous bit is always used to define the phase shift in next bit, the symbol call be said to have two bits. Therefore one symbol duration (T) is equivalent to two bits duration $(2T_b)$.

$$i.e. Symbol\ duration\ T = 2T_b$$

Bandwidth is given as,

$$BW = \frac{2}{T}$$

$$= \frac{1}{T_b}$$

$$or\ \ BW = f_b$$

Thus the minimum bandwidth in DPSK is equal to f_b; i.e, maximum baseband signal frequency.

4.8.3 Advantages and Disadvantages of DPSK

DPSK has some advantages over BPSK, but at the same time it has some drawbacks.

Advantages:

1) DPSK does not need carrier at its receiver, Hence the complicated circuitry for generation of local carrier is avoided.

2) The bandwidth requirement of DPSK is reduced compared to that of BPSK.

Disadvantages:

1) The probability of error or bit error rate of DPSK is higher than that of BPSK.

2) Since DPSK uses two successive bits for its reception, error in the first bit creates error in the second bit. Hence error propagation in DPSK is more. Whereas in PSK single bit can go in error since detection of each bit is independent.

3) Noise interference in DPSK is more. In DPSK previous bit is used to detect next bit. Therefore if error is present in previous bit, detection of next can also go wrong. Thus error is created in next bit also. Thus there is tendency of appearing errors in pairs in DPSK.

2 MARKS

1. **Mention the need of optimum transmitting and receiving filter in baseband data transmission.**

When binary data is transmitted over the baseband channel, noise interferes with it. Because of this noise interference, errors are introduced in signal detection. Optimum filter performs two functions while receiving the noisy signal:

i) Optimum filter integrates the signal during the bit interval and checks the output at the time instant where signal to noise ratio is maximum.

ii) Transfer function of the optimum filter is selected so as to maximize signal to noise ratio.

iii) Optimum filter minimizes the probability of error.

2. **Define ASK.**

In ASK, carrier is switched on when binary '1' is to be transmitted and it is switched off when binary 'D' is to be transmitted ASK is also called on-off keying.

3. **What is meant by DPSK ?**