UNIT – 4

BASICS OF MEASUREMENT AND INSTRUMENTATION

4.1 INTRODUCTION

The measurement of any quantity plays very important role not only in science but in all branches of engineering, medicine and in almost all human day to day activities.

4.2 MEASUREMENT

The measurement of a given parameter or quantity is the act or result of a quantitative comparison between a predefined standard and an unknown quantity to be measured.

4.3 METHODS OF MEASUREMENT

The methods of measurement are classified as,

1. Direct method

2. Indirect method

1. Direct method

In direct method, the quantity to be measured is used to produce certain effects which directly give the indication on the meter.

Examples: ammeters, voltmeter, wattmeter, ohmmeter etc.

2. Indirect method

In the indirect method of measurement, the quantity to be measured is not directly measured but other parameters related to the quantity are measured. Example is the measurement of resistance. Instead of using ohmmeter for direct measurement, the voltage across the resistance and the current through the resistance are measured. Then resistance can be calculated using Ohm's law as R = V/I.

4.4 APPLICATIONS OF MEASURING SYSTEMS

- Monitoring of process and operation.
- Control of process and operation.
- Experimental engineering analysis.

4.5 INSTRUMENT

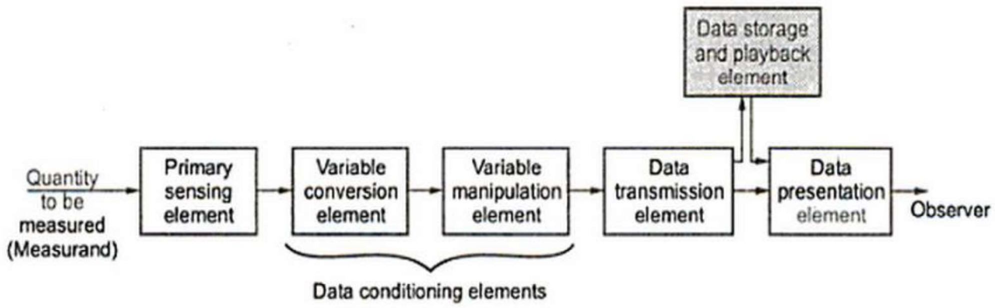
The measuring instrument may be defined as a device for determining the value or magnitude of a quantity or variable.

4.6 INSTRUMENT CLASSIFICATION

The instruments are classified as,

1. Active/Passive instruments
2. Null/Deflection type instruments
3. Monitoring/Control instruments
4. Analog/Digital instruments
5. Absolute/Secondary instruments

4.7 FUNCTIONAL ELEMENTS OF AN INSTRUMENT



Functional elements of an instrument

The functional elements of an instrument are,

1. Primary Sensing Element
2. Variable Conversion Element
3. Variable Manipulation Element
4. Data Transmission Element
5. Data Presentation Element

1. Primary Sensing Element

The quantity to be measured or measurand is first detected by the primary sensing element. A transducer follows primary sensing element which converts the measurand into a corresponding electrical signal. A transducer is a device which converts a physical quantity into an electrical quantity. The quantity to be measured is sensed and detected by an element which gives the output in different analogous form. The output is then converted into an electrical quantity by a transducer. The first stage of measurement system is known **as detector transducer stage**.

2. Variable Conversion Element

The output of the primary sensing element is in electrical form such as voltage, frequency or any other electrical parameters. Sometimes this output may not be suitable for actual measurement system. For example the measurement of a system is digital then the analog signal obtained from the primary sensing element is not suitable for the digital systems. In such case analog to digital converter is required.

Many instruments do not need any variable conversion element, while others need more than one variable conversion element.

3. Variable Manipulation Element

The function of this element is to manipulate the signal, preserving the original nature of the signal. The manipulation means a change in numerical value of the signal.

For example an amplifier which just amplifies the magnitude of the input, at its output retaining the original nature of the signal. In some cases the levels of outputs of the previous stage are high and required to be lowered. In such case attenuators are used as variable manipulation element.

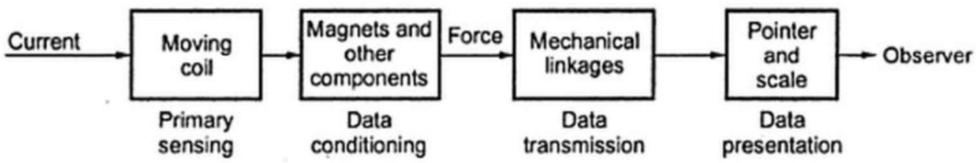
Sometimes the output of the transducer may be affected due to unwanted signals like noise. Some process like modulation, clipping, clamping etc. is done to obtain the signal in pure or acceptable form. Such a process is called signal conditioning. The term signal conditioning includes many other functions in addition to variable conversion and variable manipulation. The second stage is called **data conditioning or signal conditioning.**

4. Data Transmission Element

When the elements of an instrument are physically separated, it is necessary to transmit the data from one stage to the other. This is done by data transmission element. The signal conditioning and data transmission together is called **intermediate stage** of an instrument.

5. Data Presentation Element

The transmitted data may be used by the system, finally for monitoring, controlling and analyzing purpose. The person handling the instrument must get the information in proper form. If the data is to be monitored then visual display devices are used. If the data is to be recorded for analysis purpose then magnetic tape recorders, high speed cameras are used. For control and analysis purpose, microprocessors, computers and microcontrollers are used as data presentation element. This stage may be called **terminating stage** of an instrument.



Block schematic of an ammeter

For example consider a simple analog meter used to measure current or voltage. The moving coil is primary sensing element. The magnets and coil together acts as data conditioning stage to convert current in a coil to a force. This force is transmitted to the pointer through mechanical linkages which acts as data transmission element. The pointer and the scale act as data presentation element.

4.8 PERFORMANCE CHARACTERISTICS

The performance characteristics of an instrument are mainly divided into two categories.

1. Static Characteristics
2. Dynamic Characteristics

4.8.1 Static Characteristics

Static characteristics are defined for the instrument which measure the quantities which do not vary with time.

The various Static characteristics are,

1. Accuracy
2. Precision

3. Resolution
4. Error
5. Sensitivity
6. Threshold
7. Reproducibility
8. Zero drift
9. Stability
10. Linearity

1. Accuracy

It is the degree of closeness with which the instrument reading approaches the true value of the quantity to be measured. It indicates the true value of the quantity.

(a) Accuracy as a percentage of full scale reading

In case of instruments having uniform scale , the accuracy can be expressed as percentage of full scale reading. For example the accuracy of an instrument having full scale reading of 50 units may be expressed as ±0.01% of full scale reading.

(b) Accuracy as a percentage of true value

This the best method of specifying the accuracy in terms of the true value of quantity being measured. It can be specified as ±0.1% of true value. This indicates, as readings get smaller error gets reduced.

(c) Accuracy as a percentage of scale span

For an instrument if a_{max} is the maximum point for which the scale is calibrated and a_{min} is the lowest reading on scale. Then (a_{max} - a_{min}) is called scale span or span of the instrument.

(d) Point Accuracy

Point accuracy is specified at only one particular point of scale. It does not give any information about the accuracy at any other point on the scale.

2. Precision

It is the measure of consistency or repeatability of measurements.

The precision may be composed of,

a. Conformity

b. Number of significant figures

a. Conformity

A resistor whose true resistance is 1,385,692 Ω is measured by an ohmmeter. The ohmmeter repeatedly indicates the true value. But observer estimates the scale reading to 1.4 M Ω. The error created due to limitation of scale reading is a precision error. Conformity is necessary but not sufficient condition for accuracy.

b. Number of significant figures

The precision of measurement is obtained from the number of significant figures in which the reading is expressed. The Number of significant figures convey actual information about the magnitude and the measurement precision of the quantity.

For e.g a resistance of 110Ω, specified by an instrument may be closer to 109 or 111Ω. There are three significant figures.

The precision can be expressed as,

$$P = 1 - \left| \frac{Xn - \overline{Xn}}{Xn} \right|$$

P = Precision

X_n = Value of n^{th} measurements

X_n = Average of the set of measured values.

3. Resolution

It is the smallest increment of quantity being measured which can be detected with certainty by an instrument. The resolution means the smallest measurable input change. The resolution can affect the accuracy of measurement.

4. Error

The algebraic difference between the indicated value and the true value of the quantity to be measured is called an error.

$e = A_t - A_m$

e = error (absolute error)

A_m = measured value of the quantity

A_t = true value of the quantity

Relative error $e_r = \frac{absoluteerror}{truevalue} = \frac{At - Am}{At}$

$\%e_r = \frac{At - Am}{At} \times 100$

Accuracy $A = 1 - e_r = 1 - e_r = 1 - \left| \frac{At - Am}{At} \right|$

A=relative accuracy

a = A x 100

a = percentage accuracy

error as a percentage of full scale reading.

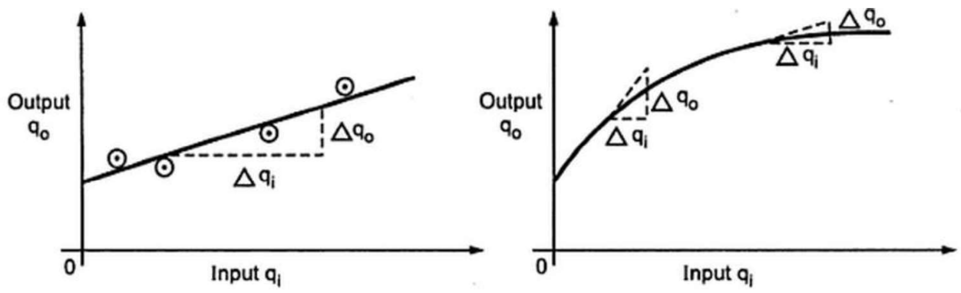
$$= \frac{At - Am}{fsd} \times 100$$

5. Sensitivity

The sensitivity denotes the smallest change in the measured variable to which the instrument responds.

Sensitivity $= \frac{changeinoutput}{changeininput}$

$$= \frac{\Delta qo}{\Delta qi}$$



Sensitivity

If the calibration curve is linear, the sensitivity of the instrument is the slope of the calibration curve.

If the calibration curve is non - linear, the sensitivity of the instrument varies with the input.

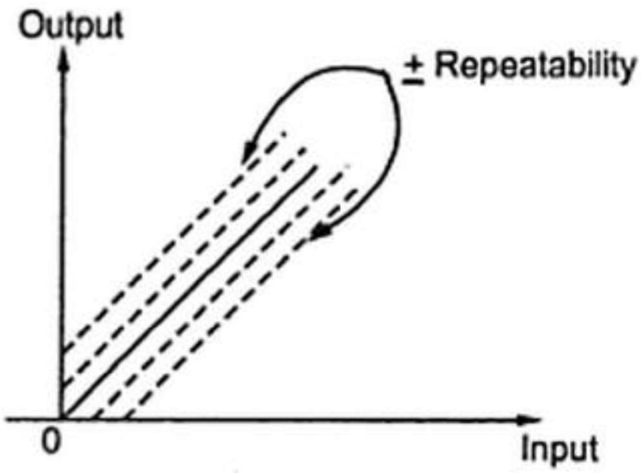
$$\text{Deflection factor} = \frac{1}{sensitivity} = \frac{\Delta qi}{\Delta qo}$$

6. Threshold

If the input quantity is slowly varied from zero onwards, the output does not change until some minimum value of the input is exceeded. This minimum value of the input is called threshold.

7. Reproducibility

It is the degree of closeness with which a given value may be repeatedly measured. It may be specified in terms of units for a given period of time. The repeatability is defined as variation of scale reading and is random in nature.



Reproducibility

8. Zero drift

The zero drift is defined as the deviation in the instrument output with time, from its zero value when the variable measured to be constant.

The environmental factors which affect the drift are stray electric field, stray magnetic field, temperature changes, contamination of metal, changes in atomic structure. Mechanical vibration wear and tear, corrosion etc,

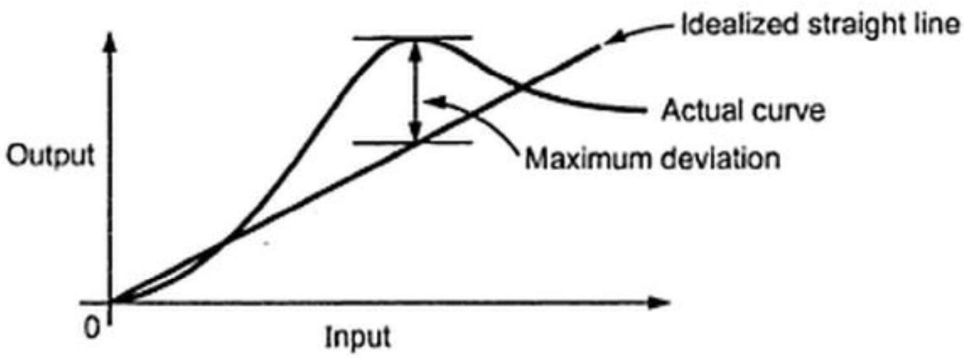
9. Stability

The ability of an instrument to retain its performance throughout its specified operating life and the storage life is defined as stability.

10. Linearity

The linearity is defined as the ability to reproduce the input characteristics symmetrically and linearly ie, the output values linearly according to the input. If the system is linear it is represented by a straight line.

The linearity is defined as the maximum deviation of the actual calibration curve from the idealized straight line.



Linearity

$$\% \text{ Linearity} = \frac{\text{Maximum deviation of output from idealized straight line}}{\text{Full scale deflection}} \text{X}100$$

4.8.2. Dynamic Characteristics

When the quantity under measurement changes rapidly with time. It is called dynamic characteristics. The dynamic behavior of the measuring system is expressed mathematically by the differential equations.

The various dynamic characteristics of an instrument are.

1. Speed of Response
2. Fidelity
3. Lag
4. Dynamic Error

1. Speed of Response

It is the rapidity with which the system responds to the change in the quantity to be measured. It indicates the activeness of the system.

2. Fidelity

It is defined as the degree to which an instrument indicates the change in the measured variable without any dynamic error.

3. Lag

Every system takes some time, to respond to the changes in the measured variable. This retardation or delay in the response of a system is called lag or measuring lag

The lags are of two types.

a. Retardation lag

b. Time delay

a. Retardation lag

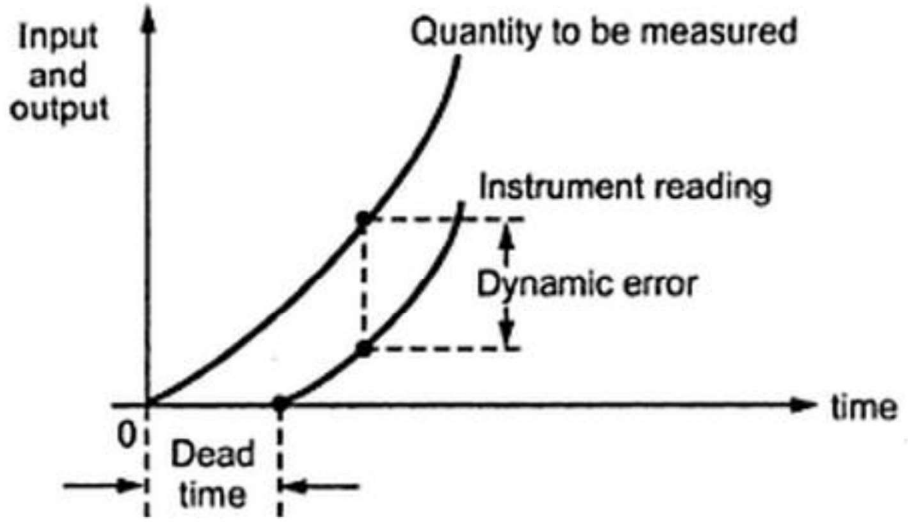
The response of the system begins immediately after a change in the variable has occurred.

b. Time delay

Response begins after sometime called dead time, after the application of input.

4. Dynamic Error

It is the difference between the true value of the variable to be measured, changing with time and the value indicated by the measurement system.



Dynamic Characteristics.

4.9 STANDARD INPUTS USED IN ANALYZING DYNAMIC RESPONSE

In the time domain analysis the following standard test inputs are used.

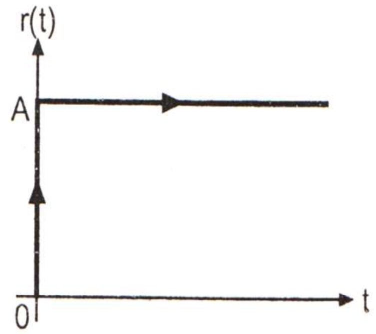
i) Step Input (Position function)

It is the sudden application of the input at a specified time as shown in the Fig.

Mathematically it can be described as,

$r(t) = A \quad \text{for } t \geq 0$

$= 0 \quad \text{for } t < 0$

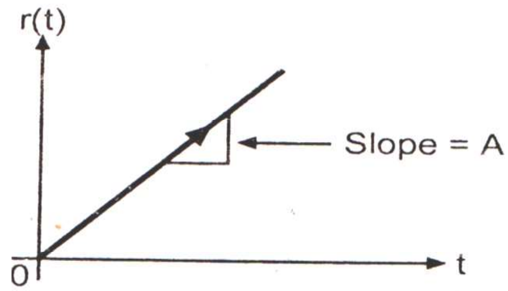


If A = 1, then it is called unit step function and denoted by u(t)

Laplace transform of such input is $\frac{A}{s}$

ii) Ramp Input (Velocity function)

It is constant rate of change in input I,e. gradual application of input as shown in the Fig.



Magnitude of Ramp input is nothing but its slope. Mathematically it is defined as,

r(t) = A for t $\geq$ 0

= 0 for t < 0

If A = 1, it is called Unit Ramp input. It is denoted as r(t). Its Laplace transform is $\frac{A}{s^2}$

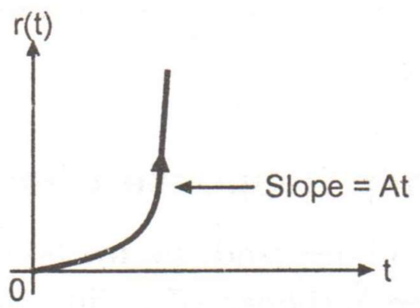
iii) Parabolic Input (Acceleration function)

This is the input which is one degree faster than a ramp type of input as shown in the Fig.

Mathematically this function is described as,

$$r(t) = \frac{A}{2}t^2 \quad \text{for } t \geq 0$$

$$= 0 \quad \text{for } t < 0$$



Where A is called magnitude of the parabolic input

Key point : parabolic function is expressed as $\frac{A}{2}t^2$ so that in Laplace transforms of different standard inputs, similarity will get maintained.

If A = 1, i.e $r(t) = \frac{t^2}{2}$ it is called unit parabolic input. Its Laplace transform is $\frac{A}{s^3}$

iv) Impulse Input

It is the input applied instantaneously (for short duration of time) of very high amplitude as shown in the Fig.

It is the pulse whose magnitude is infinite while its width tends to zero i.e t 0, applied momentarily

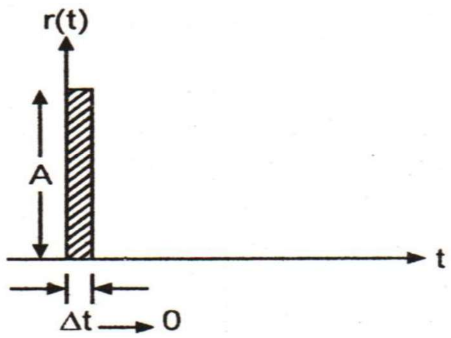
Area of the impulse is nothing but its magnitude. If its area is unity it is called unit Impulse Input, denoted as $\delta(t)$.

Mathematically it can be expressed as,

$r(t) = A$, for $t = 0$

$= 0$, for $t \# 0$

Its Laplace transform is always 1 if A = 1. i.e. for unit impulse response. The unit impulse is denoted as δ(t).



4.10 STANDARDS

A standard is a physical representation of a unit of measurement. A standard means known accurate measure of physical quantity. All the standards are preserved at the international Bureau of Weights and measures at severes, near paris.

The different types of standards of measurement are classified as,

1. International Standards
2. Primary Standards
3. Secondary Standards
4. Working Standards

1. International Standards

International Standards are defined on the basis of international agreement. These standards are maintained at the International Bureau of Weights and measures. It is periodically evaluated and checked by absolute measurements. The international standards are not available to the ordinary users. For improvements it can be replaced by the absolute units in 1948.

2. Primary Standards

Primary Standards are absolute standards of such high accuracy they can be used as the ultimate reference standards. These standards are maintained at National Standard Laboratories in different countries. The fundamental units are calibrated independently by absolute measurements at each of the national laboratories. These are not available for use, outside the national laboratories.

The main function of the primary standards is the calibration and verification of secondary standards.

3. Secondary Standards

Secondary Standards are the basic reference standards used in industrial measurement laboratories. The responsibility of maintance and calibration of these standards lies in particular industry. Secondary Standards are periodically send to the National Standard Laboratories for calibration. The National Laboratories sent them back to the industries with the certification comparing them with the primary standards. Each industry has its own standard.

4. Working Standards

The Working Standards are the major tools of a measurement laboratory. These standards are used to check and calibrate laboratory instruments for their accuracy and measurement.

For example the resistor manufacturing industry maintains a standard resistor in the laboratory for checking the values of the manufactured resistors. The manufacturers verifies that the value of manufactured resistor are well within the specified accuracy limits.

4.11 ERRORS IN MEASUREMENT SYSTEM

Error is defined as the difference between the true value of the variable and the value indicated by the instrument.

The different types of error are,

1. Gross errors

2. Systematic errors

3. Random errors

1. Gross errors

The gross error mainly occur due to the carelessness or lack of experience of the human being. These errors also occurs due to incorrect adjustments of instruments. These errors cannot be treated mathematically. These errors are also called personal errors.

Gross errors can be avoided by two means,

- Great care should be taken in reading and recording the data.
- Two, three or even more readings should be taken for the quantity under measurement.

2. Systematic errors

A constant uniform deviation of the operation of an instrument is known as systematic error.

The three types of systematic errors are,

a. Instrumental error

b. Environmental error

c. Observational error

a. Instrumental error

(i) Shortcoming of Instrument

These are because of the mechanical structure of the instrument. Example friction in bearings, irregular spring tensions, hysteresis, blacklash, stretching of spring, variation of air gap etc.

The errors can be avoided by,

- Selecting a proper instrument and planning the proper procedure for the measurement.
- Recognizing the effect of such errors and applying proper correction factors.
- Calibrating the instrument carefully against a standard.

(ii) Misuse of Instrument

Poor initial adjustments, improper zero setting, using leads of high resistance etc are examples of misusing of instrument. Such things cause serious errors.

(iii) Loadings effects

Loadings effects is due to improper way of using the instrument . The best example of loadings effect error is connecting a well calibrated voltmeter across the two points of high resistance circuit. This error can be avoided by using an instrument intelligently and correctly.

b. Environmental error

These errors are due to the external conditions. The various factors resulting these errors are temperature changes, pressure changes, thermal e.m.f, stray capacitance and frequency sensitivity of an instrument.

The various methods to reduce these errors are,

- Using proper correction factor
- Reducing the effect of dust, humidity on the components
- The effects of external fields can be minimized by using magnetic or electrostatic shields.

c. Observational error

These errors are introduced by the observer. Some of the sources of observational errors are parallax errors while reading a meter, wrong scale selection.

To eliminate such errors one should use the instrument with mirrors, knife edged pointers.

3. Random errors

The causes of some errors are unknown and hence the errors are called random errors. These errors cannot be determined in the ordinary process of taking measurements.

These errors are due to the accumulation of large number of small effects.

These errors can be reduced by increasing the number of observations and using statistical methods to obtain the best approximation of the reading.

4.12 STATISTICAL EVALUATION OF MEASUREMENT DATA

Statistical Analysis

The mathematical analysis of the various measurements is called Statistical analysis of the data. For Statistical analysis, the same reading is taken number of times using different observers.

i. **Arithmetic mean and median**

The most probable value of measured variable is the arithmetic mean of the member of reading taken.

The arithmetic mean is given by,

$$X = \frac{X_{1`} + X_2 + X_3 + ... + X_n}{n} = \frac{\sum_{n=1}^{n} X_n}{n}$$

$\overline{X}$ =arithmetic mean

X_n =n[th] reading taken

n = total number of reading

for a set of n measurements $X_{1`} + X_2 + X_3 + ... + X$ n the media value is given by

X median = X(n+1)/2

For even number of data values, the median value is midway between the center two values.

ii. **Deviation from mean**

Deviation from mean is given by

di = Xi - $\overline{X}$

di = deviation of i[th] reading

Xi = value of i[th] reading

$\overline{X}$ = arithmetic mean

iii. Average deviation

Average deviation is defined as tha sum of the absolute values of deviations divided by the number of readings. This is also called mean deviation.

$$\overline{D} = \sum \frac{|di|}{n}$$

iv. Standard deviation

The standard deviation is defined as the square root of the sum of the individual deviation squared, divided by the number of reading. It is denoted as σ .

$$\text{S.D} = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots\ldots + d_n^2}{n}} = \sqrt{\frac{\sum d^2}{n}}$$

If the number of observations is less than 20.

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots\ldots + d_n^2}{n-1}}, n < 20$$

v. Variance

The variance is the square of the standard deviation. It is denoted as V.

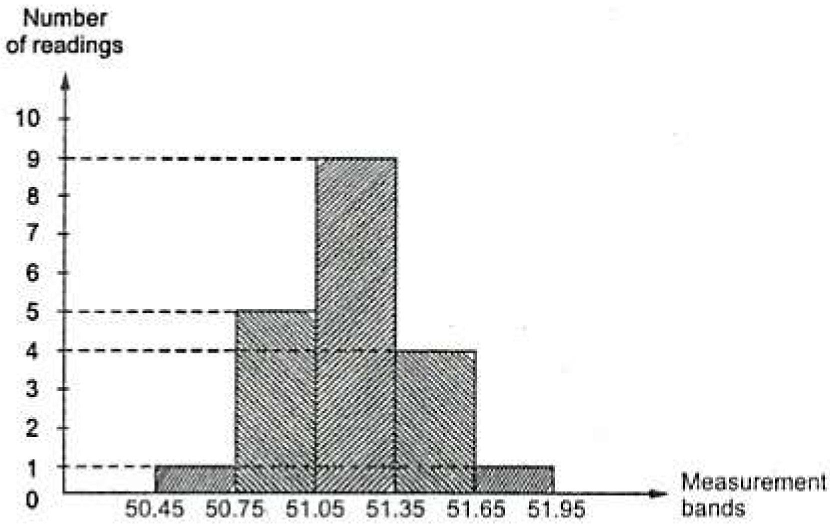
$$\text{V} = \sigma^2 = \frac{d_1^2 + d_2^2 + \ldots\ldots + d_n^2}{n}$$

$$\text{V} = \sigma^2 = \frac{d_1^2 + d_2^2 + \ldots\ldots + d_n^2}{n-1}, n < 20$$

4.13 FREQUENCY DISTRIBUTIONS

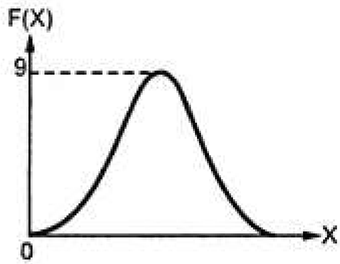
Histogram

When a number of multi sample observations are taken experimentally there is a scatter of data about the central value. One graphical method to present such test results is **histogram**.



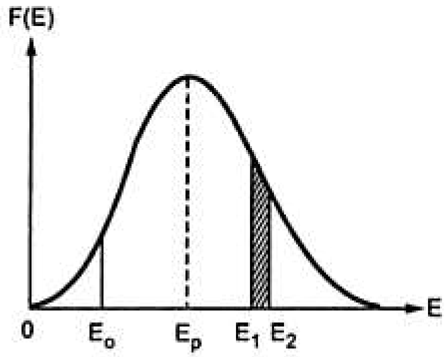
Histogram

A histogram is also called a **frequency distribution curve**.



Frequency Distribution Curve

If the height of frequency distribution curve of errors is normalized such that the area under it is unity, then the curve is called **probability curve** and the height F(E) at any particular error magnitude E is known as **probability density function**.



Frequency Distribution Curve of Errors

$$\int_{\infty}^{\infty} F(E)dE = 1$$

$$P(E_1 \leq E \leq E_2) = \int_{E_1}^{E_2} F(E)dE$$

This is called error function.

Gaussian distribution

The frequency distribution curve is called Gaussian distribution curve when the frequency and magnitude of quantities are related by mathematical expression.

$$F(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{(-(x-m)^2/2\sigma^2)}$$

Where,

 =Standard deviation

x=Value of reading

m=Mean value

The Gaussian distribution is also called **normal or Bell shaped distribution.**

Limiting error

The components like the resistor, inductor, capacitor are guaranteed to be within a certain percentage of rated value. This percentage indicates the deviations from the normal or specified value of the particular quantity. These deviations from the specified value are called limiting errors. These are also called Guarantee Error.

$A_a = A_s \pm \delta A$

A_a =Actual values

A_s =Specified value

δA =Limiting error

Relative Limiting Error

This is also called fractional error. It is the ratio of the error to the specified magnitude of a quantity.

Thus $$e = \frac{\delta A}{A_s}$$

Where e =relative timing error

$\delta_A = e.As$

and $A_a = A_s \pm \delta A$

$= A_s \pm e\, A_s$

$A_a = A_s [1 \pm e]$

The percentage relative limiting error is expressed as

$\% e = e \times 100$

The relative limiting error can be also be expressed as,

$$e = \frac{Actualvalue(A_a) - Specifiedvalue(A_s)}{Specifiedvalue(A_s)}$$

PROBLEMS

Example 1

The table shows the set of 5 measurements recorded in a laboratory. Calculate the precision of the 3rd measurement.

Measurement Number	Value of Measurement
1	49
2	51
3	52
4	50
5	49

Solution : The average value for the set of measurements is,

$$\bar{X}_n = \frac{\text{Sum of the readings}}{\text{Number of readings}} = \frac{251}{5} = 50.2$$

The value of 3rd measurement is

$\therefore$ $$P = 1 - \left|\frac{X_n - \bar{X}_n}{\bar{X}_n}\right| = 1 - \left|\frac{52 - 50.2}{50.2}\right| = 0.964 \text{ ie.} 96.4\%$$

This is the precision of the 3rd measurement.

Example 2

The expected value of the voltage to be measured is 150V. However, the measurement gives a value of 149 V. Calculate i) Absolute error; ii) percentage error; iii) Relative accuracy; iv) Percentage accuracy and v) Error expressed as percentage of full scale reading, if the scale range is 0-200 V.

Solution : The expected value means true value,

$\therefore$ $A_t = 150$ V

The measured value is given as 149 V

∴ A_m = 149 V

i)e= absolute error

= A_t-A_m= 150 – 149 = 1V

ii)% $e_r = \frac{A_t - A_m}{A_t}$ X100 = $\frac{1}{150}$x100

= 0.66%

iii) A = relative accuracy

= 1 - $\left|\frac{A_t - A_m}{A_t}\right| = 1 - \left[\frac{1}{150}\right] = 1 - \left[\frac{1}{150}\right]$=0.9933

iv) % a = A x 100 = 0.9933 x 100

= 9.33%

v) % error expressed as percentage of full scale reading is,

= $\frac{A_t - A_m}{f.s.d}$ X100

= $\frac{1}{200}$x100 as f.s.d is 200 V

= 0.5%

Example 3

A particular ammeter requires a change of 2 A in its coil to produce a change in deflection fo the pointer by 5mm. Determine its sensitivity and deflection factor.

Solution : The input is current while output is deflection.

Sensitivity = $\frac{\text{Change in output}}{\text{change in input}}$

$= \frac{5mm}{2A} = 2.5mm/A$

Example 4

A 30 cm scale has 30 uniform divisions. 1/20th of a scale deivison can be estimated with a fair degree of certainty. Determine the resolution of the scale in mm.

Solution :

Scale division = $\frac{\text{full scale deflection}}{\text{number of divisions}}$

$= \frac{30cm}{30}$ = 1cm = 10mm

Resolution = $\frac{1}{20} x(10mm) = \frac{1}{2} mm$

= 0.5mm

Example 5

The r.m.s current passing through a resistor of 120 $\pm$ 0.5 ohms is 2 $\pm$0.02A. Calculate the limiting error in the value of power dissipation.

Solution

$P = I^2R$

δa_1 = limiting error in current = 0.02

δa_2 = limiting error in resistor = 0.5

$\therefore$ $e_1 = \frac{\delta a_1}{A_1} = \frac{0.02}{2} = 0.01$

and $e_2 = \frac{\delta a_2}{A_2} = \frac{0.5}{120} = 4.167 \times 10^{-3}$

The current term in power appears as I^2 so it is second power of I.

Hence, the contribution by I^2 to the resultant error is ne_1

Where n = power = 2

e_1 = Limiting error

while the limiting error due to resistance is e_2.

As power is the product of I^2 and R, the resultant error is the sum of the contributions by I^2 and R.

$\therefore$ $e_T = ne_1 + e_2$ Where n=2

Hence, the limiting error in the power calculation is

$$e_T = \pm(2xe_1 + e_2)$$

$= \pm$ [2 x 0.01 + 1.167 x 10^{-3}]

$= \pm$ 0.02417 i.e. $\pm$ [2 2.417%

Example 6

In a survey of 15 owners of a certain model of car, the following figures are for average petrol consumption were reported.

25.5	30.3	31.1	29.6	32.4	39.4	28.9	30.0	33.3	31.4
29.5	30.5	31.7	33.0	29.2					

Calculate mean value, median value, standard deviation and the variance.

Solution : The total readings n = 15

Mean = $\bar{x} = \frac{x_1+x_2+\cdots+x_{15}}{n} = \frac{465.8}{15}$

= 31.0533

The median can be obtained as,

$X_{median} = X_{(n+1)/2} = X_{(15+1)/2} = x_8$ when readings are arranged in ascending order.

∴ $X_{median} = 30.5$

Calculate the diviation of each reading as $d_i = x_i - \bar{x}$

∴ d_1 = -5.55 d_2 = -0.753

d_3 = +0.0466 d_4 = -1.4533

d_5 = +1.3467 d_6 = +8.3466

d_7 = -2.153 d_8 = -1.053

d_9 = +2.246 d_{10} = +0.346

d_{11} = -1.5533 d_{12} = -0.553

d_{13} = +0.6467 d_{14} = +1.9467

d_{15} = -1.8533

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}} \quad \text{as } n < 20$$

$$= \sqrt{\frac{126.2326}{14}} = 3.00$$

$V = \sigma^2 = 9.00$

Example 7

The following values were obtained from the measurements of the valves of 147.2, 147.4, 147.9, 148.1, 147.7, 147.5, 147.6, 147.4, 147.6 and 147.5

Calculate

a) The arithmetic mean
b) The standard deviation
c) The probable error of average of the ten readings

Solution : The values obtained from the measurements are tabulated as follows.

No.(n)	X	$D_i = x_1 - \bar{x}$	d_i^2
1	147.2	-0.39	0.1521
2	147.4	-0.19	0.0361
3	147.9	+0.31	0.0961
4	148.1	+0.51	0.2601
5	147.7	+0.11	0.0121
6	147.5	-0.09	0.0081
7	147.6	+0.01	0.0001
8	147.4	-0.19	0.0361
9	147.6	+0.01	0.0001
10	147.5	-0.09	0.0081
n = 10	Σx=1475.9	$\Sigma\lvert d_i^2\rvert$=1.9	Σd_i^2 = 0.609

i) Arithmetic mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{1475.9}{10} = 147.59$

ii) Standard deviation, $\sigma = \sqrt{\frac{\Sigma d_i^2}{n-1}}$ as $n < 20$

$= \sqrt{\frac{0.609}{9}}$

= 0.2601

iii) The probable error of average of the ten readings,

$e_m = 0.6745 \frac{\sigma}{\sqrt{n-1}} = 0.6745 \frac{0.2601}{\sqrt{10-1}}$

= 0.058479

Example 8

The set of independent measurement of voltages are recorded as 101.2, 101.4, 101.7, 101.3, 101.3, 101.2, 101.0, 101.3, 101.5 and 101.1

Calculate :

i) Arithmetic mean
ii) Deviation from mean

iii) Standard deviation and

iv) Probable error

Solution

The result is tabulated as shown where di is the deviation from mean.

No.(n)	x	$D_i = x_I - \bar{x}$	d_i^2
1	101.2	-0.1	0.01
2	101.4	0.1	0.01
3	101.7	0.4	0.16
4	101.3	0	0
5	101.3	0	0
6	101.2	-0.1	0.01
7	101.0	-0.3	0.09
8	101.3	0	0
9	101.5	0.2	0.04
10	101.1	-0.2	0.04
n = 10	Σx=1013	$\Sigma\|d_i\|$=1.4	Σd_i^2 = 0.36

i) Arithmetic mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{1013}{10} = 101.3$

ii) Deviation from mean = Average deviation

$$= \frac{\Sigma|d_i|}{n} = \frac{1.4}{10} = 0.14$$

iii) Standard deviation, $\sigma = \sqrt{\frac{\Sigma d_i^2}{n-1}} = \sqrt{\frac{0.36}{9}} = 0.2$ V

iv) The probable error of average of the ten readings = 0.6745σ

= 0.6745 x 0.2 = 0.1349V

e_m = Probable error of mean = $\frac{0.6745}{\sqrt{n-1}}$

$= \frac{0.1349}{\sqrt{10-1}} = 0.0449$

Example 9

The table given below lists a sample of experimental data.

Value	3	4	5	6	7	8	9	10	11
Frequency of occurrence	1	2	3	6	7	6	4	2	1

Calculate : a) Mean b) Mean deviation c) Standard Deviation

d) Variance e) Probable error of one reading f) Probable error of mean.

Solution : Let us tabulate the given data in different manner,

Value X	**Frequency F**	$\mathbf{x \times f}$	**Deviation** $\mathbf{d_i = x_i - \bar{x}}$	$\mathbf{f \times d_i}$	$\mathbf{d_i^2}$	$\mathbf{f \times d_i^2}$
3	1	3	-4.0625	-4.0625	16.503	16.503
4	2	8	-3.0625	-6.125	9.378	18.7578
5	3	15	-2.0625	-6.1875	4.2539	12.7617

6	6	36	-1.0625	-6.375	1.1289	6.7734
7	7	49	-0.0625	-0.4375	3.9 x 10^{-3}	0.0273
8	6	48	+0.9375	+5.625	0.8789	5.2734
9	4	36	+1.9375	+7.75	3.7589	15.0156
10	2	20	+2.9375	+5.875	8.6289	17.2578
11	1	11	+ 3.9375	+3.9375	15.503	15.5039
	n = 32	$\Sigma x \times f =$ = 226		$\Sigma\|fxd_i\|$ = 46.375		$\Sigma fd_i^2 = 107.87$

a) Mean, $\bar{x} = \frac{\Sigma x \times f}{n} = \frac{226}{32} = 7.0625$

b) Mean deviation $= \frac{\Sigma|f \times d_i|}{n} = \frac{46.375}{32} = 1.45$

c) $\sigma = \sqrt{\frac{\Sigma fd_i^2}{n}} = \sqrt{\frac{107.875}{32}} = 1.836$

d) $V = \sigma^2 = (1.836)^2 = 3.3708$

e) e = Probable error of one reading

$= 0.6745 \times \sigma = 0.6745 \times 1.836$

$= 1.238$

f) $e_m = \dfrac{0.6745\sigma}{\sqrt{n}}$ as n > 20

$= \dfrac{1.238}{\sqrt{32}} = 0.22$

Example 10

In a survey of 12 owners of certain model of car, the following figures for average petrol consumption were reported.

29.6, 32.4, 39.4, 28.9, 30.0, 33.3, 31.4, 29.5, 30.5, 31.7, 33.0, 29.2

Calculate : i) Mean value ii) Median value and iii) standard deviation

Solution : The results are tabulated as shown,

No.(n)	x	$d_i = x - \bar{x}$	d_i^2
1	29.6	-1.975	3.9006
2	32.4	0.825	0.6806
3	39.4	7.825	61.2306
4	28.9	-2.675	7.1556
5	30.0	-1.575	2.4806
6	33.3	1.725	2.9756
7	31.4	-0.175	0.0306
8	29.5	-2.075	4.3056
9	30.5	-1.075	1.1556
10	31.7	0.125	0.0156

11	33.0	1.425	2.0306
12	29.2	-2.375	5.6406
n = 10	Σx=378.9	$\Sigma\|d_i^2\|$= 23.85	Σd_i^2 = 91.6022

i) Mean value = Arithmetic mean $= \bar{x} = \frac{\Sigma x}{n}$

$= \frac{378.9}{12} = 31.575$

ii) When values are arranged in ascending order then,

Median value = $x_{(n+1)/2}$ for odd values

= Midway between centre two values for even values

For given set in ascending order

$$x_1 = 28.9, x_2 = 29.2, x_3 = 29.5, x_4 = 29.6, x_5 = 30.0, x_6 = 30.5, x_7 = 31.4$$

$$x_8 = 31.7, x_9 = 32.4, x_{10} = 33.0, x_{11} = 33.3, x_{12} = 39.4$$

As n = 12 is even, centre two values are x_6 and x_7

Example 11

The limiting errors for a four dial resistance box are :

Units = ±0.15%, Tens = ±0.1%, Hundreds = , Thousands = ±0.025%. If the resistance value is set at 4352Ω calculate the limiting error in the resistance value.

Solution

The thousands place is 4 and its value is 4000

∴ error $= \pm 4000 \frac{0.025}{100} = \pm 1\ \Omega$

The hundred place is 3 and its value is 300.

∴ error $= \pm 300 \frac{0.075}{100} = \pm 0.225\ \Omega$

The tens place is 5 and its value is 2

error $= \pm 50 \frac{0.1}{100} = \pm 0.05\ \Omega$

The units place is 2 and its value is 2

error $\pm 2 \times \frac{0.15}{100} = \pm 0.003\ \Omega$

Total error = ± [1 + 0.225 + 0.05 + 0.003]

$= \pm 1.278\ \Omega$

% limiting error $= \pm \frac{\pm 1.278}{4352} \times 100 = \pm 0.0293\%$

Example 12

If the r.m.s. value of reading in volts are observed on a digital CRO were 3.5, 3.452, 3.620, 3.523

Determine 1) Arithmetic mean 2) Average deviation 3) Standard deviation

Solution : The result is tabulated as shown below where d_i is the deviation of i^{th} reading from the mean.

Number (n)	**x**	$d_i = x - \bar{x}$	d_i^2
1	3.5	-0.02375	5.6406×10^{-4}

2	3.452	-0.07175	5.14806×10^{-3}
3	3.620	0.09625	9.26406×10^{-3}
4	3.523	-7.5×10^{-4}	5.625×10^{-7}
n = 4	$\Sigma x = 14.095$	$\Sigma\|d_j\| = 0.1925$	$\Sigma d_i^2 = 0.0149767$

1. Arithmetic mean = $\bar{x} = \frac{Ex}{n} = \frac{14.095}{4} = 3.52375$

2. Average deviation = $\bar{D} = \frac{\Sigma|d_i|}{n} = \frac{0.1925}{4} = 0.048125$

3. Standard deviation = $\sigma = \sqrt{\frac{0.0149767}{4-1}}$ for n < 20

Example 13 : If a set of six observation are as follows :

1.5 V, 3 V, 1 V, 5 V, 2 V, 4 V

Calculate the arithmetic mean, average deviation and standard deviation.

Solution : The result is tabulated as shown below where d_i is the deviation fo i^{th} reading from the mean,

Number (n)	x	$d_i = x - \bar{x}$	d_i^2
1	1.5	-1.25	1.5625
2	3	0.25	0.0625
3	1	-1.75	3.0625
4	5	2.25	5.0625
5	2	-0.75	0.5625
6	4	1.25	1.5625

n = 4	Σx = 16.5	$\Sigma\lvert d_i \rvert$ = 7.5	$\Sigma d_i^2 = 11.875$

1. Arithmetic mean $\bar{x} = \frac{Ex}{n} = \frac{16.5}{6} = 2.75V$

2. Average deviation $\bar{D} = \frac{\Sigma\lvert d_i \rvert}{n} = \frac{7.5}{6} = 1.25$

3. Standard deviation $\sigma = \sqrt{\frac{\Sigma d^2}{n-1}}$ for $n < 20$

$$\sigma = \sqrt{\frac{11.875}{(6-1)}} = 1.5411$$

4.14 TRANSDUCER

A device which converts a physical quantity into the proportional electrical signal is called a transducer.

4.15 ADVANTAGES OF TRANSDUCERS

1. Power requirement of the transducer is very small

2. Reduced effects of friction and other mechanical nonlinearities.

3. Less Weight and portable

4. The output of the transducer may be easily used transmitted and processed for the purpose of measurement.

4.16 CHARACTERISTICS OF TRANSDUCERS

1. Accuracy

2. Ruggedness

3. Linearity

4. Repeatability

5. High Stability and Reliability

6. Sensitivity

7. Dynamic Range

8. High output

9. size

10. Speed of response

4.17 CLASSIFICATION OF TRANSDUCERS

Basically there are two types of transducers.

1. Electrical Transducer

2. Mechanical Transducer

Electrical Transducers are classified into

(i) Active and Passive transducers

(ii) On the basis of Transduction principle used

(iii) Analog and Digital transducer

(iv) Primary and Secondary transducer

(v) Transducer and Inverse transducer

Active and Passive transducers

Active transducers

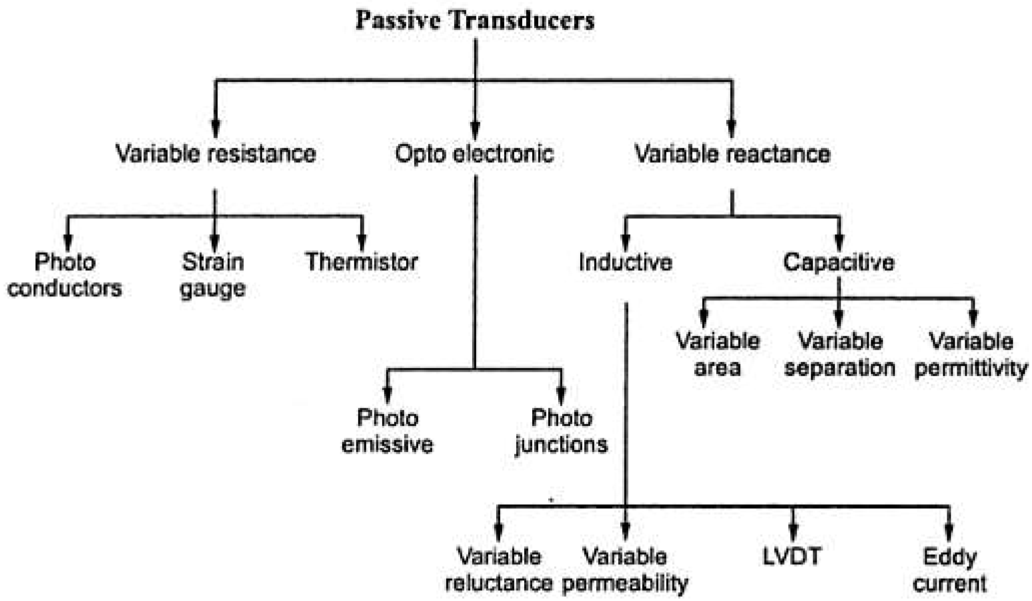
Active transducers are self - generating type of transducers. These transducers develop a electrical parameter proportional to the quantity under measurement. They do not require any external source.

Active transducers are classified into

- Photovoltaic transducer
- Thermo electric transducer
- Piezo electric transducer
- Electromagnetic transducer

Passive Transducer

Passive Transducer do not generate any electrical signals by themselves. External source of power is essential. It is also known as externally power driven transducers. They can be subdivided into the following commonly used types.



On the basis of Transduction principle used

(i) Capacitive Transduction

(ii) Electromagnetic transduction

(iii) Inductive Transduction

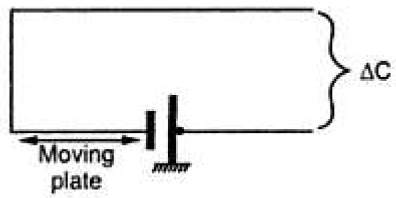
(iv) Piezo electric Transduction

(v) Photovoltaic Transduction

(vi) Photo conductive Transduction

(i) Capacitive Transduction

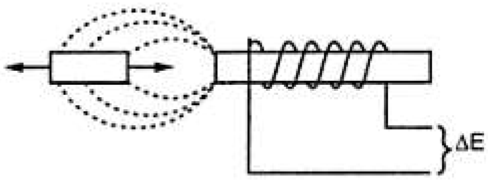
Measurand is converted into change in capacitance. A change in capacitance occurs by changing the distance between them or change in dielectric.



Capacitive Transduction

(ii) Electromagnetic transduction

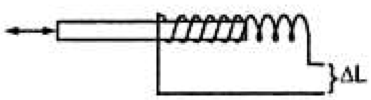
Measurand is converted into an electromotive force(voltage).These are self- generating type transducer.



Electromagnetic transduction

(iii) Inductive Transduction

Measurand is converted into change in the self-inductance of the single coil



Inductive Transduction

(iv) Piezo electric Transduction

Measurand is converted into change in electrostatic charge or voltage generated by crystals when mechanically stressed.

(v) Photovoltaic Transduction

Measurand is converted into change in voltage generated. when a junction between dissimilar materials are illuminated.

(vi)Photo conductive Transduction

Measurand is converted into change in resistance of a semiconductor material by a change in amount of incident light.

Analog and Digital transducer

Analog transducer

These transducers converts the input quantity into an analog output which is a continuous function of time.

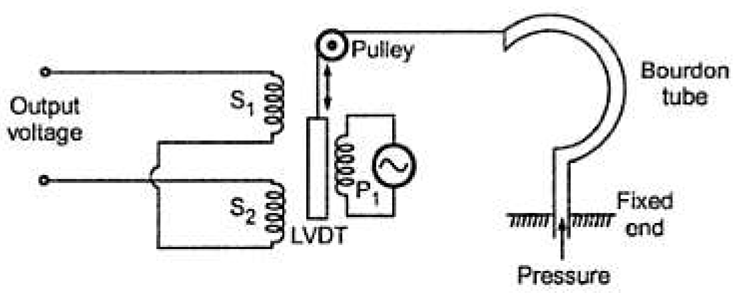
Examples are strain guage , LVDT, Thermocouple or thermistor

Digital transducer

It produces an electrical signal in the form of pulses.

Primary and Secondary transducer

It consists of a mechanical device along with electrical device. mechanical device acts as primary transducer which converts physical quantity into mechanical signal. Electrical device converts mechanical signal into electrical signal. It acts as a Secondary transducer.



Example in pressure measurement Bourdon's tube acts primary transducer which converts a pressure into displacement and LVDT acts Secondary transducer which converts this displacement into an electrical signal.

Transducer and Inverse transducer

Transducer converts non electrical quantity into electrical quantity. Inverse transducer converts electrical quantity into non electrical quantity.

For example micro phone converts sound signal into an electrical signal

Loudspeaker converts an electrical signal into sound signal.

4.18 TRANSDUCER SELECTION FACTORS

1. Nature of Measurement

It depends upon the nature of quantity to be measured. For example, for temperature measurement temperature sensors are used.

2. Loading Effect

Transducer is selected to have minimum loading effect to keep the errors minimum.

3. Environmental Considerations

Based on the environmental condition such as temperature changes, shock and vibration and electromagnetic interference.

To minimize the errors due to temperature changes some transducers are temperature compensated.

Other Environmental Considerations are

- Simplicity of mounting and cable installation
- Convenient size, shape and weight
- Resistance of Corrosion

4. Measuring System Compatibility

The transducers selected and the electrical system used for measurement should be compatible. The output impedance of the transducer and the impedance imposed by the measuring should be such that one does not adversely affect each other.

5. Cost and Availability

Low cost, basic simplicity, reliability and low maintance. Transducers are selected which do not require excessive repair or continuous calibration checking.

4.19 PASSIVE TRANSDUCER

(i) Resistive Transducer

(ii) Inductive Transducer

(iii) Capacitive Transducer

4.19.1 RESISTIVE TRANSDUCER

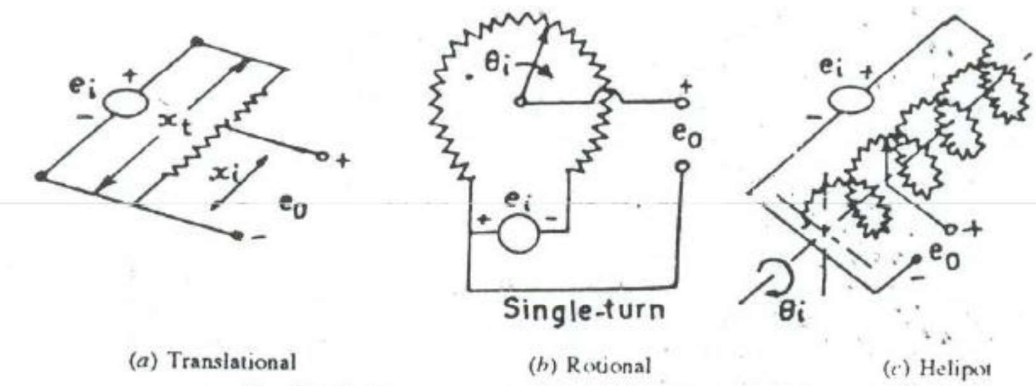
Resistive Transducer can be used either as primary transducer or secondary transducer. The methods based on measurement of the resistance change are most widely used in industrial applications.

POTENTIOMETRIC RESISTANCE TRANSDUCER

Potentiometric Resistance Transducer is generally used to measure linear or angular displacement.

Construction

A Resistance Transducer consists of a wire wound element along with a sliding contact which is called as wiper. A wire is made up of platinum or nickel alloy with diameter as small as 0.01mm. The wire is wound on an insulating former.



Resistance potentiometers

Operation

Using Potentiometric Resistance Transducer mechanical displacement is converted into electrical output. Linear or angular displacement is given to the sliding contact and the corresponding change in resistance is converted into voltage or current. To measure the combination of linear and angular motion helipots are used.

Advantages

(i) Simple in construction and operation

(ii) High electrical efficiency

(iii) Inexpensive

(iv) Useful for displacement measurement for large amplitude.

Disadvantages

(i) Suffer from Mechanical wear and misalignment of wiper

(ii) Limited resolution and high electronic noise in output.

STRAIN GAUGE

Strain gauge is a passive transducer. It converts mechanical displacement into change in resistance. The basic principle of operation is that the resistance of the wire changes as a function of time. The change in resistance is measured by using Wheatstone bridge.

The common material used for wire strain gauge are constantan alloys containing 45% nickel and 55% copper. Bonding cement are adhesives used to fix the strain gauge into the test specimen. The proper functioning of strain gauge depends on the quality of bonding. Improper bonding of gauge may cause serious errors.

Derivation of gauge factor

Gauge factor is defined as the unit change in resistance per unit change in length. It is denoted by K or S. It is also called as sensitivity of strain gauge.

$$s = \frac{\Delta R / R}{\Delta l / l}$$

where S = Gauge factor or Sensitivity

R = Gauge wire resistance

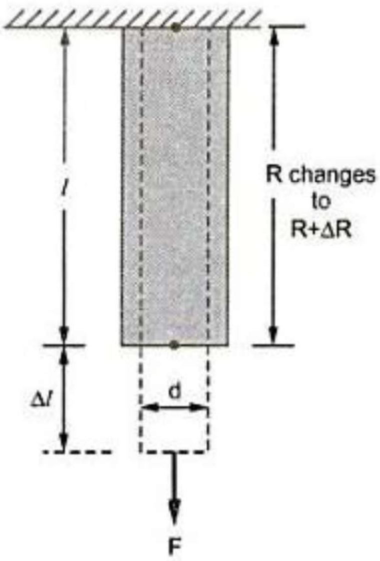
ΔR = Change in wire resistance

l = Length of the gauge wire in unstressed condition.

Δl = Change in length in stressed condition.

Derivation

Consider the resistance wire is under tensile stress and is defined by Δl



Deformed resistance wire

ρ = Specific resistivity of the wire material in Ω-m.

l = Length of the wire in m.

A = Area of cross section of the wire in m^2.

Δl/l = per unit change in length.

ΔA/A = per unit change in Area.

Δρ/ρ = per unit change in resistance.

$$R = \frac{\rho l}{A}$$

$$\frac{dR}{d\sigma} = \frac{d(\frac{\rho l}{A})}{d\sigma}$$

$$= \frac{\rho}{A}\frac{\partial l}{\partial \sigma} - \frac{\rho l}{A}\frac{\partial A}{\partial \sigma} + \frac{1}{A}\frac{\partial \rho}{\partial \sigma}$$

$$\frac{\partial}{\partial \sigma}\frac{1}{A} = -\frac{1}{A}\frac{\partial A}{\partial \sigma}$$

Multiply both sides by $\frac{1}{R}$

$$\frac{1}{R}\frac{dR}{d\sigma} = \frac{\rho}{RA}\frac{\partial l}{\partial \sigma} - \frac{\rho l}{RA}\frac{\partial A}{\partial \sigma} + \frac{1}{RA}\frac{\partial \rho}{\partial \sigma}$$

Put $R = \frac{\rho l}{A}$ on the right hand side

$$\frac{1}{R}\frac{dR}{d\sigma} = \frac{1}{l}\frac{\partial l}{\partial \sigma} - \frac{1}{A}\frac{\partial A}{\partial \sigma} + \frac{1}{\rho}\frac{\partial \rho}{\partial \sigma}$$

Cancelling $\partial \sigma$ from both sides

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

For a circular wire

$$A = \frac{\pi}{4}d^2$$

$$\frac{\partial A}{\partial s} = \frac{\pi}{4}(2d)\frac{\partial d}{\partial s}$$

$$\frac{1}{A}\frac{\partial A}{\partial s} = \frac{1}{A}\frac{\pi}{4}(2d)\frac{\partial d}{\partial s}$$

$$\frac{1}{A}\frac{\partial A}{\partial s} = \frac{1}{d2}\,(2d)\frac{\partial d}{\partial s}$$

$$\frac{\partial A}{A} = \frac{2\partial d}{d}$$

$$\frac{\Delta A}{A} = \frac{2\Delta d}{d}$$

Poisson's ratio μ for the wire is defined as the ratio of strain in lateral direction to the strain in the axial direction.

$$\mu = -\frac{\Delta d/d}{\Delta l/l}$$

$$\frac{\Delta d}{d} = -\mu\frac{\Delta l}{l}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{2\Delta d}{d} + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - 2[-\mu\frac{\Delta l}{l}] + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l}[1+2\mu] + \frac{\Delta \rho}{\rho}$$

Neglecting piezoelectric effect, $\frac{\Delta \rho}{\rho}$ can be neglected.

$$\frac{\Delta R}{R} = \frac{\Delta l}{l}[1+2\mu]$$

$$S = \text{gauge factor} = \frac{\Delta R/R}{\Delta l/l} = [1+2\mu]$$

This is also defined as G

In general $\quad S = G = [1+2\mu] + \frac{\Delta \rho/\rho}{\Delta l/l}$

TYPES OF STRAIN GAUGE

(i) Mechanical gauge

(ii)Optical gauge

(iii) Electrical gauge

RESISTANCE TEMPERATURE DETECTOR (RTD)

It is a primary electrical transducer which is used to measure the change in the temperature. It is based on the principle that the resistance of the conductor changes when the temperature changes.

The relationship between the temperature and resistance of the conductor is given by

$R_t = R_{ref}\,[1+\alpha\Delta t]$

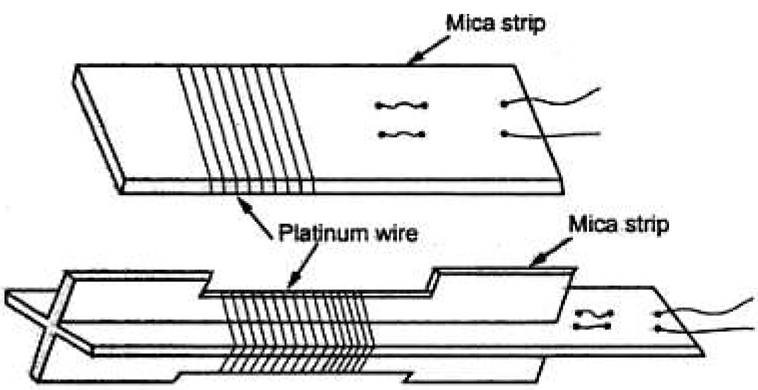
R_t =Resistance of the conductor at temperature T

R_{ref} = Resistance of the conductor at reference temperature

α = Temperature Coefficient

Δt = Difference in temperature

Construction of RTD



Resistance Temperature Detector

The wire resistance thermometer usually consists of a coil wound on a mica or ceramic former. The coil is wound in bifilar form. To avoid the corrosion of resistive element are enclosed in a protective tube of pyrex glass and porcelain. The tube is evacuated and sealed or filled with air or any other inert gas.

Advantages

1. Measurement is accurate
2. Measurement of differential temperature is possible
3. Smaller in size
4. Suitable for remote indication

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Disadvantages

1. A bridge circuit with external power supply is needed
2. They are comparatively costly
3. There is a possibility of self -heating.

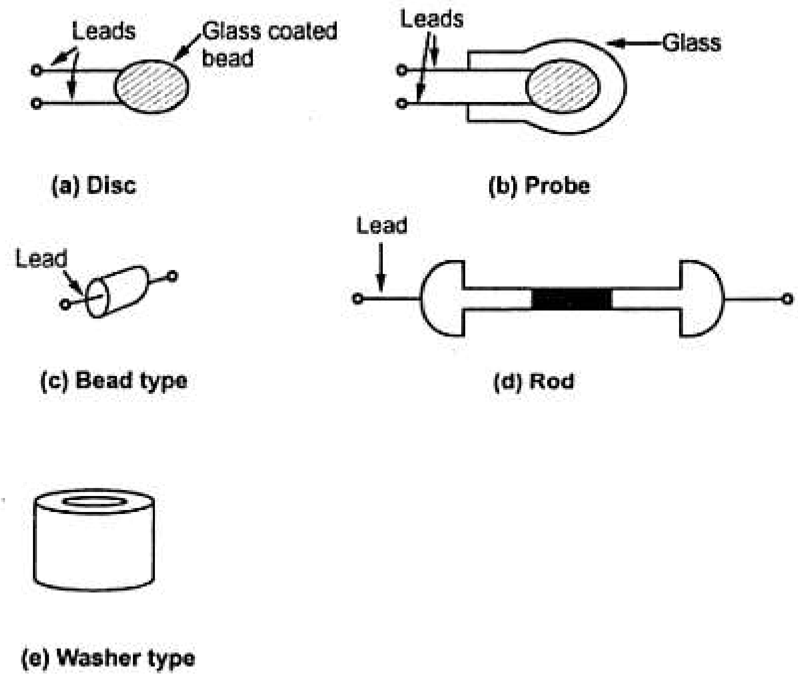
THERMISTOR

Thermistor is derived from thermal resistor. These resistors have negative temperature coefficient. The resistance of the thermistor decreases as temperature increases. The thermistors are very sensitive and can detect very small change in temperature.

Construction of Thermistor

Thermistors are composed of a sintered mixture of metallic oxides, such as manganese, nickel, cobalt, copper, iron, and uranium. Their resistances at ambient temperature may range from 100 ohm to 100 kilo ohm. Thermistors are available in a wide variety of shapes and sizes smallest in size are the beads with a diameter of 0.15 mm to 1.25 mm. Beads may sealed in the tips of solid glass rods to form probes. Disks and washers are made by pressing thermistor material under high pressure into flat cylindrical shapes. Washers can be placed in series or in parallel to increase power dissipation rating.

Thermistors are well suited for precision temperature measurement, temperature control, and temperature compensation because of their very large change in resistance with temperature. They are widely used for measurements in the temperature range -100^0C to $+200^0$C. The measurement of the change in resistance with temperature is carried out with a Wheatstone bridge.



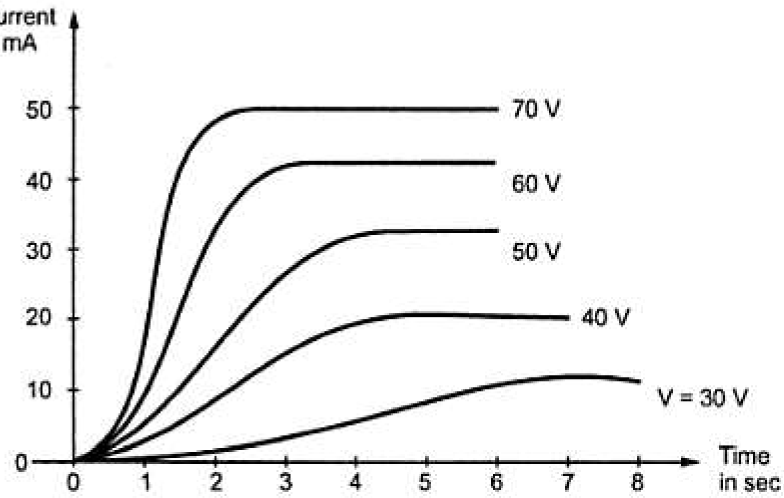
Different forms of construction of thermistors

Resistance Temperature Characteristics

The mathematical relationship according to which the resistance of thermistor behaves as temperature is given by,

$$R_{T1} = R_{T2}\, e^{\left[\beta\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right]}$$

Current Time Characteristics



Current Time Characteristics

Advantage

1. Small in size

2. Low in cost

3. Large change in resistance for small change in temperature

4. Fast response

Limitations

1. The resistance versus temperature characteristics is highly non-linear.

2. Not suitable over a wide temperature range

Applications

1. Useful for temperature transducers.

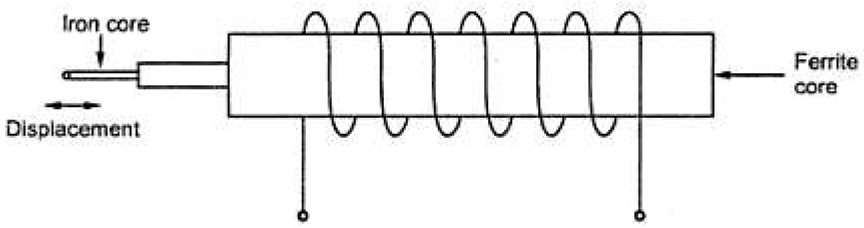
2. Measurement of power level measurements.

3. For pressure, flow liquid level measurements.

4. For measurements of composition of gases.

4.19.2 INDUCTIVE TRANSDUCER

Inductive transducer is a simple and most popular type of displacement transducer in which variation of inductance as a function of displacement is achieved by variation in self-inductance or mutual inductance.



Inductive Transducer

The value of self- inductance of an inductor is given as,

$$L=\frac{N^2}{S}$$

Where, N = Number of turns of the coil

S = Reluctance of the coil (A/wb)

Thus the variation in the self- inductance may be due to

1. Change in number of turns.

2. Change in reluctance

3. Change in permeability

LINEAR VARIABLE DIFFERENTIAL TRANSDUCER (LVDT)

Displacement is a vector quantity representing a change in position of a body or a point with respect to a reference. It can be linear or angular (rotation) motion, with the help of displacement transducer, many other quantities, such as force, stress, pressure, velocity, and acceleration can be found.

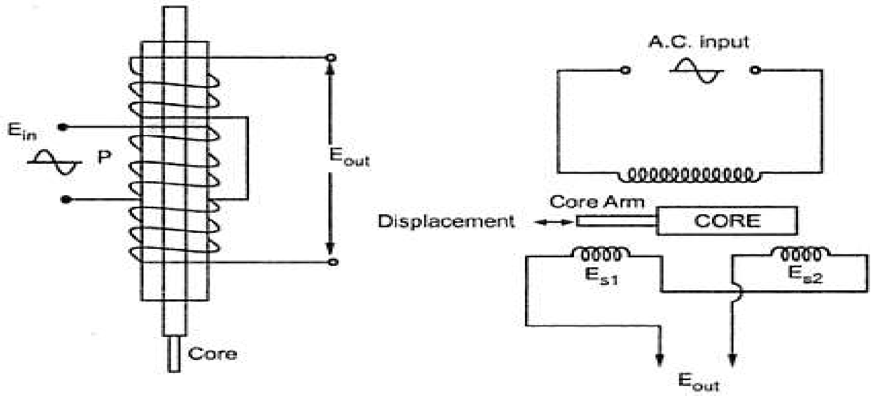
The main electrical displacement transducers work on the principle of:

1. **Variable resistance** : transducer is strain gauge.

2. **Variable inductance:** transducers is linear variable differential transformer

3. **Variable capacitance** : transducers is parallel plate capacitor with variable gap

Construction and Working of LVDT

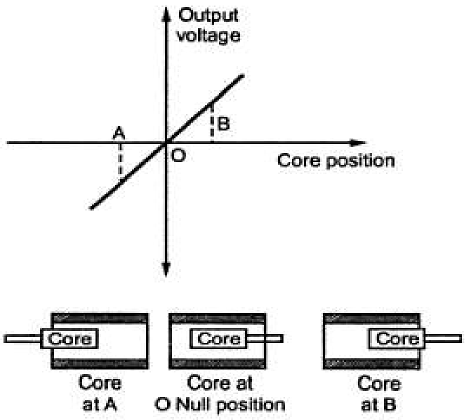


Linear Variable Differential Transducer

The linear variable differential transformer consists of a single primary winding P_1 and two secondary windings S_1 and S_2 wound on a hollow cylindrical former. The secondaries have a equal number of turns but they are connected in series opposition so that the emfs induced in the coils oppose each other. The primary winding is connected to an ac source, whose frequency may range from 50 Hz to 20 kHz. A movable soft iron core slide inside the hollow former. The position of the movable core determines the flux linkage between the ac excited primary winding and each of the two secondary windings. The core made up of nickel-iron alloy is slotted longitudinally to reduce eddy current losses. The displacement to the measured is applied to an arm attached to the core. With the core in the center, or reference, position, the induced emfs in the secondaries are equal and since they oppose each other, the output voltage will be zero volt.

When an externally applied force moves the core to the left-hand position, more magnetic flux links the lift-hand coil than the right-hand coil. The emf induced in the left-hand coil E_{s1} is therefore larger than the induced emf of the right-hand coil E_{s2} The magnitude of the output voltage is then equal to the difference between the two secondary voltage and it is in phase with the voltage of the left-hand coil.

Similarly, when the core is forced to move to the right, more flux links the right-hand coil than the left-hand coil and the resulting output voltage, which is the difference between E_{s2} and $E_{s1, is}$ now in phase with the emf of the right-hand coil.



Thus the LVDT output voltage is a function of the core position. The amount of a voltage change in either secondary winding is proportional to the amount or movement of the core. By noting which output is increasing decreasing, the direction of motion can be determined. The output ac voltage inverts in phase as the core passes through the central null position. Further as the core moves from the center, the greater is the difference in value between E_{s1} and E_{s2} and consequently the greater the output voltage. Therefore the amplitude of the output voltage is a function of the distance the core moves, while the polarity or phase indicates the direction or the motion.

The amount of output voltage of an LVDT is a linear function of the core displacement within a limited range of motion.

Advantages

1. Linearity

2. Infinite Resolution

3. High Output

4. High sensitivity

5. Ruggedness:

6. Less Friction

7. Low hysteresis

8. Low power consumption'

9. The LVDT transducers are small, simple, and light in weight. The are stable and easy to align and maintain.

Disadvantages of LVDT

1. Comparatively large displacements are necessary for appreciable differential output.

2. They are sensitive to stray magnetic fields

3. The dynamic response is limited by the mass of the core

4. Temperature affects the transducer.

Applications

1. Displacement ranging from fraction of few mm to a few cm have to be measured.

2. It is used to measure force, weight, pressure etc.

4.19.3 CAPACITIVE TRANSDUCER

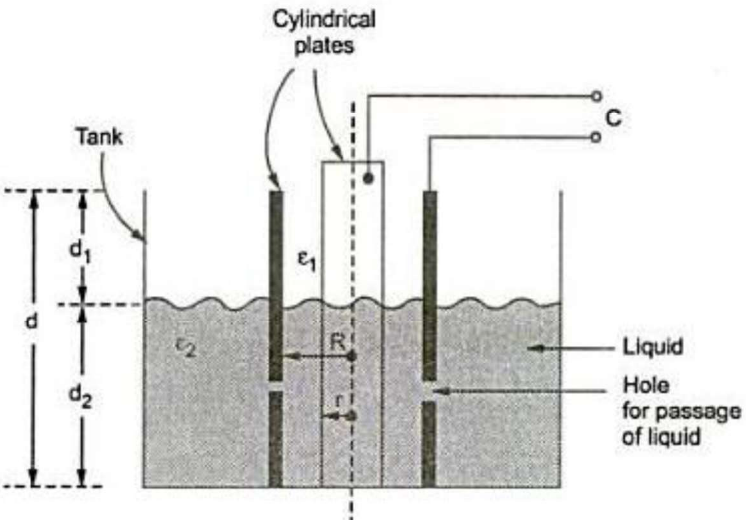
In Capacitive transducer the measurand is converted into change in capacitance.

The variation in capacitance can be achieved by

- Change in distance
- Change in common plate area
- Change in dielectric
- Using Quartz diaphragms

Capacitance type level meter

Capacitive transducer using the method of change in dielectric is used for the measurement of the liquid level. It uses concentric cylindrical capacitor. The two plates are cylindrical using the dielectric material with a permittivity ε_1. The dielectric is an air with $\varepsilon_r = 1$ and $\varepsilon = \varepsilon_o$. The outer cylindrical plates have holes at the bottom through which passage of liquid is possible between the plates.



Capacitance type level meter

Let r = Outer radius of inner cylinder

R = Inner radius of outer cylinder

D = Height of the tank

E_2 = Permittivity of the liquid

As the liquid level d_2 changes, the composite capacitor formed experiences change in its value. The value of the capacitance is given by,

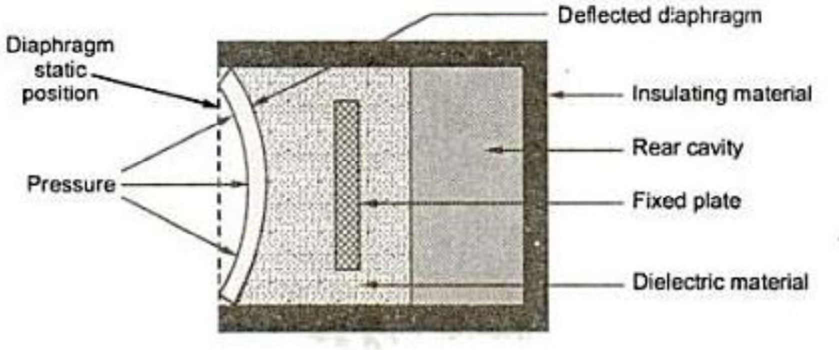
$$C = 2\pi\varepsilon_o[\varepsilon_1 d_1 + \varepsilon_2 d_2]/\ln(R/r)$$

Thus, change in liquid level causes the change in the capacitance measured between the cylinders.

This change in capacitance is detected by some other circuit with which the electrical signal is proportional to the liquid level can be obtained.

Capacitive Pressure Transducer

The capacitive pressure transducer is based on the principle that when the distance between the two parallel plates changes, capacitive of the parallel plate capacitor changes.



Capacitive Pressure Transducer

In capacitive pressure transducer diaphragm acts as one of the plates of a two plate capacitor while other plate is fixed. The fixed plate and the diaphragm are separated by a dielectric material. When the force is applied to the diaphragm, it changes its position from initial static position obtained with no force applied. Due to this, the distance of separation between the fixed plate and the diaphragm changes, hence the capacitance also changes. The changes in the capacitance can be measured by using any simple a.c. bridge. But practically the change in capacitance is measured using an oscillator circuit change where capacitive transducer is part of that circuit. Hence, when capacitance changes, the oscillator frequency changes accordingly. In this way, by using capacitive transducer, applied force can be measured in terms of change in the capacitance.

Advantages

I] The force requirement is very small.

Ii] They are highly sensitive.

Iii] They have good frequency response and very high input impedance, so loading effects are minimum.

Iv]They are useful in the application where stray magnetic fields affect performance of the inductive transducer.

Disadvantages

1. Proper insulation is required between the metallic parts
2. The stray capacitance affect the performance of the transducer

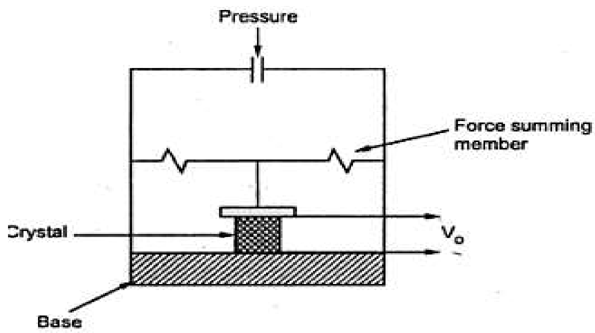
4.20 ACTIVE TRANSDUCERS

The transducers which generate an electrical signal directly in response to the physical parameter without requiring external power for the operation are called active transducers. Active transducers are also known as self- generating transducers

4.21 PIEZOELECTRIC TRANSDUCER

Piezoelectric Transducer works on the principle of Piezoelectric effect. When two opposite face of a thin slice of certain crystals are subjected to a mechanical force, then opposite charges are developed on the two faces of the slice. The magnitude of the electric potential between the two faces is proportional to the deformation produced.

The main substances exhibiting piezoelectric effect are quartz, Rochelle salts and tourmaline.

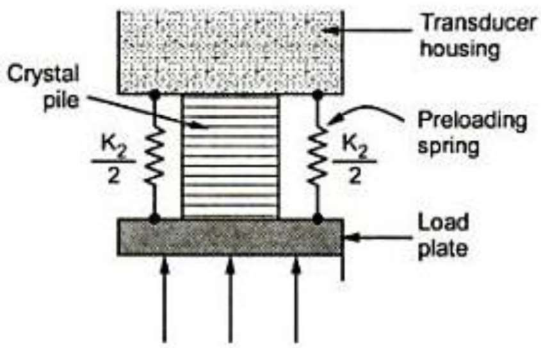


A crystal is placed between the solid base and force summing member. Metal electrodes plated onto the faces of Piezoelectric crystal are taken out to measure the output. The electrode become plates of parallel plate capacitor. The output voltage is given by

$$V_o = Q/C$$

The output is high about 1 to 30mV. No external power supply is required. It is small in size and construction is simple.

Piezoelectric pressure Transducer is in the form of pile of pairs of quartz disc. These disc are kept in such a way that optically flat faces are between a flat metal plate called load plate on one side and transducer housing on the other side. For preloading, spring of stiffness K_2 is used. The stiffness of the crystal is denoted by K_1.



Crystal pile in a Piezoelectric Pressure Transducer

Let P be the force produced by external pressure. This gets splits into two parts. Force P_1 in the crystal pile and force P_2 in the preloading thin walled tube or diaphragm, The pile is compressed by Δx

$$P = P_1 + P_2 = K_1\Delta x + K_2\,\Delta x$$

$$= (K_1 + K_2)\,\Delta x$$

$$P_1/P = K_1\Delta x/(K_1 + K_2)\,\Delta x$$

$$= K_1/(K_1 + K_2) = 1/(1+K_2/K_1)$$

The ratio K_2/K_1 decreases, the sensitivity P_1/P increases. If K_2/K_1 is constant over the power range to be measured then linear operation is possible. The output voltage is available across the output connections proportional to the pressure to be measured.

Advantages

1. Rugged construction and small size.
2. Excellent frequency response.
3. High output with negligible phase shift.

Limitations

1. Piezoelectric crystals are water soluble.
2. Temperature sensitive.
3. It can be used for dynamic response only.

Applications

1. It can be used for measurement for non- electrical quantities such as acceleration, vibration, sound intensity and dynamic pressure.

2. It is widely used in aero dynamics, supersonic wind tunnels, bomb blast etc.

3. It is used in ultrasonic, non- destructive test, ultrasonic flow meters etc.

4. It is used in spark ignition engine and electrostatic dust filters.

TWO MARKS

1. What is meant by measurement?

Measurement is an act or the result of comparison between the quantity and a predefined standard.

2. Mention the basic requirements of measurement.

· The standard used for comparison purpose must be accurately defined and should be commonly accepted.

· The apparatus used and the method adopted must be provable.

3. What are the 2 methods for measurement?

· Direct method and

· Indirect method.

4. Explain the function of measurement system.

The measurement system consists of a transducing element which converts the quantity to be measured in an analogous form. The analogous signal is then processed by some intermediate means and is then fed to the end device which presents the results of the measurement.

5. Define Instrument.

Instrument is defined as a device for determining the value or magnitude of a quantity or variable.

6. List the types of instruments.

· The 3 types of instruments are