UNIT I

CLASSIFICATION OF SIGNALS AND SYSTEMS

1.1Definition for signal

Any physical phenomenon that conveys or carries some information can be called a signal. The music, speech, motion pictures, still photos, heart beat, etc., are examples of signals that we normally encounter in day to day life.

Mathematically, any signal can be represented as a function of one or more independent variables.Therefore, ***a signal** is defined as any physical quantity that varies with one or more independent variables.*

For example, the functions $x_1(t)$ and $x_2(t)$ as defined by the equations (1.1) and (1.2) represents two signals: one that varies linearly with time "t" and the other varies quadratically with time "t". The equation (1.3) represents a signal which is a function of two independent variables "p" and "q".

$$x_1(t) = 0.7t \quad .....(1.1)$$

$$x_2(t) = 1.8t^2 \quad .....(1.2)$$

$$x(p,q) = 0.6p + 0.5q + 1.1q^2 \quad .....(1.3)$$

1.2 CLASSIFICATION OF SIGNALS

The signals can be classified in number of ways. Some way of classifying the signals are,

I. Depending on the number of sources for the signals.

1.One-channel signals

2. Multichannel signals

II. Depending on the number of dependent variables.

1.One-dimensional signals

2. Multidimensional signals

III. Depending on whether the dependent variable is continuous or discrete.

1.Analog or Continuous signals

2. Discrete signals

One-channel signals

Signals that are generated by a single source or sensor are called one-channel signals.

The record of room temperature with respect to time, the audio output of a mono speaker, etc., are examples of one-channel signals.

Multichannel signals

Signals that are generated by multiple sources or sensors are called multichannel signals.

The audio output of two stereo speakers is an example of two-channel signal.The record of ECG (Electro Cardio Graph) at eight different places in a human body is an example of eight-channel signal.

One-dimensional signals

A signal which is a function of single independent variable is called one-dimensional signal.

The signals represented by equation (1.1) and (1.2) are examples of one-dimensional signals.

The music, speech, heart beat, etc., are examples of one-dimensional signals where the single independent variable is time.

Multidimensional signals

A signal which is a function of two or more independent variables is called multidimensional signal.

The equation (1.3) represents a two dimensional signal.

A photograph is an example of a two-dimensional signal. The intensity or brightness at each point of a photograph is a function of two spatial coordinates "x" and "y", (and so the spatial coordinates are independent variables). Hence, the intensity or brightness of a photograph can be denoted by b(x, y).

The motion picture of a black and white TV is an example of a three-dimensional signal. The intensity or brightness at each point of a black and white motion picture is a function of two spatial coordinates "x" and "y", and time "t". Hence, the intensity or brightness of a black and white motion picture can be denoted by b(x, y, t).

Analog or Continuous signals

When a signal is defined continuously for any value of independent variable, it is called analog or continuous signal. Most of the signals encountered in science and engineering are analog in nature. When the dependent variable of an analog signal is time, it is called continuous time signal.

Discrete signals

When a signal is defined for discrete intervals of independent variable, it is called discrete signal . When the dependent variable of a discrete signal is time, it is called discrete time signal. Most of the discrete signals are either sampled version of analog signals for processing by digital systems or output of digital systems.

1.3 CONTINUOUS TIME SIGNALS

In a signal with time as independent variable, if the signal is defined continuously for any value of the independent variable time "t", then the signal is called ***continuous time signal***. The continuous time signal is denoted as "x(t)".

The continuous time signal is defined for every instant of the independent variable time and so the magnitude (or the value) of continuous time signal is continuous in the specified range of time. Here both the magnitude of the signal and the independent variable are continuous.

1.4 DISCREATE TIME SIGNALS

In a signal with time as independent variable, if the signal is defined only for discrete instants of the independent variable time, then the signal is called ***discrete time signal***.

In discrete time signal the independent variable time "t" is uniformely divided into discrete intervals of time and each interval of time is denoted by an integer "n", where "n" stands for discrete interval of time and "n" can take any integer value in the range $-\infty$ to $+\infty$. Therefore, for a discrete time signal the independent variable is "n" and the magnitude of the discrete time signal is defined only for integer values of independent variable "n". The discrete time signal is denoted by "x(n)".

1.5 SYSTEMS

Any process that exhibits cause and effect relation can be called a ***system***. A system will have an input signal and an output signal. The output signal will be a processed version of the input signal. A system is either interconnection of hardware devices or software / algorithm.

A system is denoted by letter $\mathcal{H}$. The diagrammatic representation of a system is shown in fig 1.1.

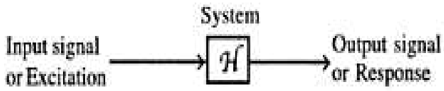


Fig 1.1 : Representation of a system.

The operation performed by a system on input signal to produce output signal can be expressed as,

$$\text{Output} = \mathcal{H}\{\text{Input}\}$$

where $\mathcal{H}$ denotes the system operation (also called ***system operator***).

The systems can be classified in many ways.

Depending on type of energy used to operate the systems, the systems can be classified into Electrical systems, Mechanical systems, Thermal systems, Hydraulic systems, etc.

Depending on the type of input and output signals, the systems can be classified into Continuous time systems and Discrete time systems.

1.6 STANDARD CONTINUOUS TIME SIGNALS

Impulse signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty;\ t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = A$$
$$= 0;\ t \neq 0$$

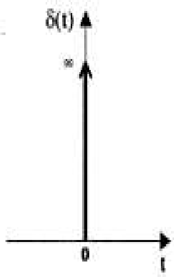


Fig 2.1 : Impulse signal (or Unit Impulse signal).

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty;\ t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0;\ t \neq 0$$

Step signal

The step signal is defined as,

$$x(t) = A \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit step signal is defined as,

$$x(t) = u(t) = 1 \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

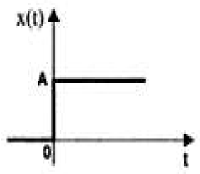


Fig 2.2 : Step signal.

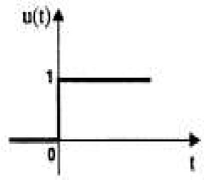


Fig 2.3 : Unit step signal.

Ramp signal

The ramp signal is defined as,

$$x(t) = At \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit ramp signal is defined as,

$$x(t) = t \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

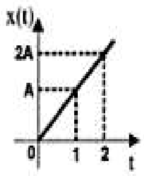


Fig 2.4 : Ramp signal.

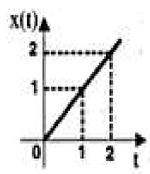


Fig 2.5 : Unit ramp signal.

Parabolic signal

The parabolic signal is defined as,

$$x(t) = \frac{At^2}{2} \; ; \; \text{for } t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit parabolic signal is defined as,

$$x(t) = \frac{t^2}{2} \; ; \; \text{for } t \geq 0$$
$$= 0 \; ; \; t < 0$$

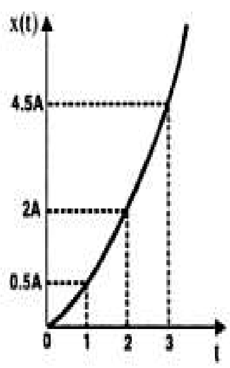


Fig 2.6 : Parabolic signal.

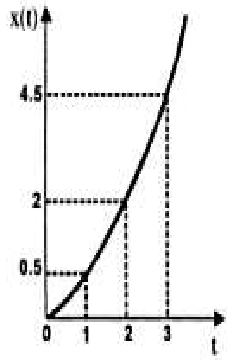


Fig 2.7 :Unit parabolic signal.

Unit pulse signal

The unit pulse signal is defined as,

$$x(t) = \Pi(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$$

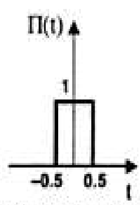


Fig 2.8 : Unit pulse signal.

Sinusoidal signal

Case i :** **Cosinusoidal signal

The cosinusoidal signal is defined as,

$$x(t) = A\cos(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $x(t) = A\cos\Omega_0 t$

When ϕ = Positive, $x(t) = A\cos(\Omega_0 t + \phi)$

When ϕ = Negative, $x(t) = A\cos(\Omega_0 t - \phi)$

Case ii :** **Sinusoidal signal

The sinusoidal signal is defined as,

$$x(t) = A\sin(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $x(t) = A\sin\Omega_0 t$

When ϕ = Positive, $x(t) = A\sin(\Omega_0 t + \phi)$

When ϕ = Negative, $x(t) = A\sin(\Omega_0 t - \phi)$

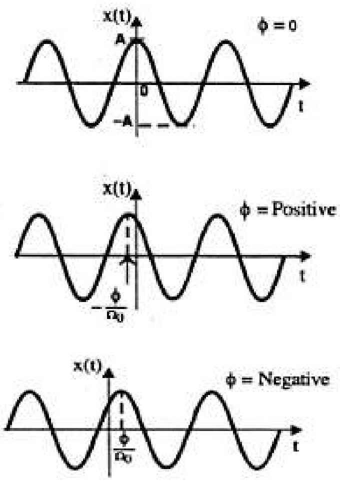


Fig 2.9 : Cosinusoidal signal.

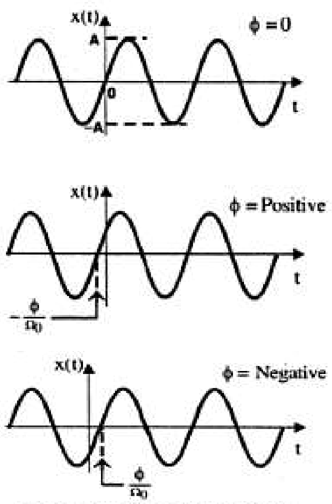


Fig 2.10 : Sinusoidal signal.

Exponential Signals

Case i : Real exponential signal

The real exponential signal is defined as,

$$x(t) = A\,e^{bt}$$

where, A and b are real

Here, when b is positive, the signal x(t) will be an exponentially rising signal; and when b is negative the signal x(t) will be an exponentially decaying signal.

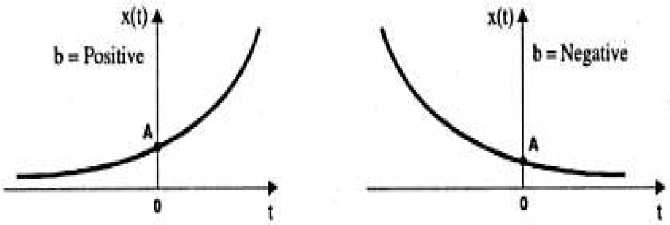


Fig 2.11 : Real exponential signal.

Case ii : Complex exponential signal

The complex exponential signal is defined as,

$$x(t) = A\,e^{j\Omega_0 t}$$

where, $\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

The complex exponential signal can be represented in a complex plane by a rotating vector, which rotates with a constant angular velocity of Ω_0 rad/sec.

The complex exponential signal can be resolved into real and imaginary parts as shown below,

$$x(t) = A\,e^{j\Omega_0 t} = A\,(\cos\Omega_0 t + j\sin\Omega_0 t)$$

$$= A\cos\Omega_0 t + jA\sin\Omega_0 t$$

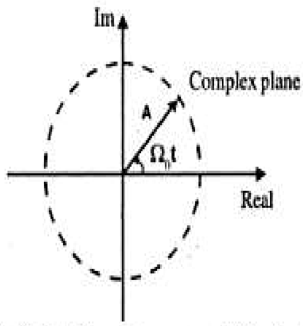


Fig 2.12 : Complex exponential signal.

$\therefore$ $A\cos\Omega_0 t$ = Real part of x(t)

$A\sin\Omega_0 t$ = Imaginary part of x(t)

From the above equation, we can say that a complex exponential signal is the vector sum of two sinusoidal signals of the form $\cos\Omega_0 t$ and $\sin\Omega_0 t$.

Exponentially rising/decaying sinusoidal signal

The exponential rising/decaying sinusoidal signal is defined as,

$$x(t) = A\,e^{bt}\,\sin\Omega_0 t$$

where, $\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

Here, A and b are real constants. When b is positive, the signal x(t) will be an exponentially rising sinusoidal signal; and when b is negative, the signal x(t) will be an exponentially decaying sinusoidal signal.

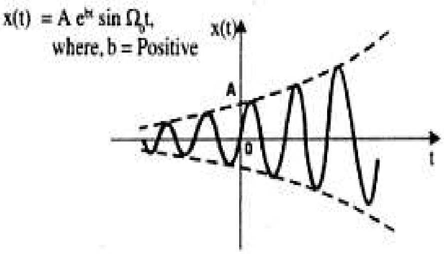


Fig 2.13 : Exponentially rising sinusoid.

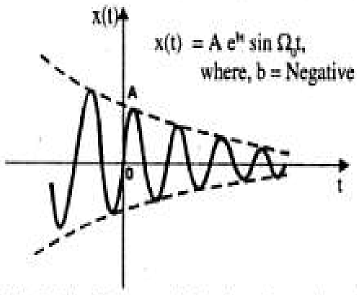


Fig 2.14 : Exponentially decaying sinusoid.

Triangular pulse signal

The Triangular pulse signal is defined as

$$x(t) = \Delta_a(t) = 1 - \frac{|t|}{a} \quad ; \quad |t| \le a$$
$$= 0 \quad ; \quad |t| > a$$

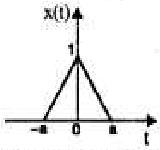


Fig 2.15 : Triangular pulse signal.

Signum signal

The Signum signal is defined as the sign of the independent variable t. Therefore, the Signum signal is expressed as,

$$x(t) = \text{sgn}(t) = 1 \quad ; \quad t > 0$$
$$= 0 \quad ; \quad t = 0$$
$$= -1 \quad ; \quad t < 0$$

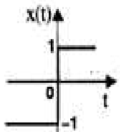


Fig 2.16 : Signum signal.

Sinc signal

The Sinc signal is defined as,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \quad ; \quad -\infty < t < \infty$$

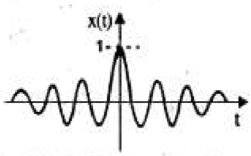


Fig 2.17 : Sinc signal.

Gaussian signal

The Gaussian signal is defined as,

$$x(t) = g_a(t) = e^{-a^2t^2} \; ; \; -\infty < t < \infty$$

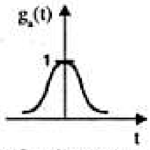


Fig 2.18 : Gaussian signal.

1.7 CLASSIFICATION OF CONTINUOUS TIME SIGNALS

The continuous time signals are classified depending on their characteristics. Some ways of classifying continuous time signals are,

1. Deterministic and Nondeterministic signals
2. Periodic and Nonperiodic signals
3. Symmetric and Antisymmetric signals (Even and Odd signals)
4. Energy and Power signals
5. Causal and Noncausal signals

The signal that can be completely specified by a mathematical equation is called a ***deterministic signal***. The step, ramp, exponential and sinusoidal signals are examples of deterministic signals.

Examples of deterministic signals:	$x_1(t) = At$
	$x_2(t) = X_m \sin\Omega_0 t$

The signal whose characteristics are random in nature is called a ***nondeterministic signal***. The noise signals from various sources like electronic amplifiers, oscillators, radio receivers, etc., are best examples of nondeterministic signals.

1.7.1 Periodic and Nonperiodic Signals

A periodic signal will have a definite pattern that repeats again and again over a certain period of time. Therefore the signal which satisfies the condition,

$\boxed{x(t+T) = x(t)}$ is called a ***periodic signal***.

A signal which does not satisfy the condition, $x(t + T) = x(t)$ is called an ***aperiodic or nonperiodic signal***. In periodic signals, the term T is called the ***fundamental time period*** of the signal. Hence, inverse of T is called the ***fundamental frequency***, F_0 in cycles/sec or Hz, and $2\pi F_0 = \Omega_0$ is called the ***fundamental angular frequency*** in rad/sec.

The sinusoidal signals and complex exponential signals are always periodic with a periodicity of T, where, $T = \frac{1}{F_0} = \frac{2\pi}{\Omega_0}$. The proof of this concept is given below.

Proof :

a) Cosinusoidal signal

Let, $x(t) = A\cos\Omega_0 t$

$$\therefore x(t+T) = A\cos\Omega_0(t+T) = A\cos(\Omega_0 t + \Omega_0 T)$$

$$= A\cos\left(\Omega_0 t + \frac{2\pi}{T}T\right) \qquad \boxed{\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}}$$

$$= A\cos(\Omega_0 t + 2\pi) = A\cos\Omega_0 t = x(t) \qquad \boxed{\cos(\theta + 2\pi) = \cos\theta}$$

b) Sinusoidal signal

Let, $x(t) = A\sin\Omega_0 t$

$\therefore x(t+T) = A\sin\Omega_0(t+T) = A\sin(\Omega_0 t + \Omega_0 T)$

$= A\sin\left(\Omega_0 t + \frac{2\pi}{T}T\right)$

$= A\sin(\Omega_0 t + 2\pi) = A\sin\Omega_0 t = x(t)$

$\boxed{\Omega_0 = 2\pi f_0 = \frac{2\pi}{T}}$

$\boxed{\sin(\theta + 2\pi) = \sin\theta}$

c) Complex exponential signal

Let, $x(t) = Ae^{j\Omega_0 t}$

$\therefore x(t+T) = Ae^{j\Omega_0(t+T)} = Ae^{j\Omega_0 t}e^{j\Omega_0 T} = Ae^{j\Omega_0 t}e^{j\frac{2\pi}{T}T} = Ae^{j\Omega_0 t}e^{j2\pi}$

$= Ae^{j\Omega_0 t}(\cos 2\pi + j\sin 2\pi) = Ae^{j\Omega_0 t}(1 + j0) = x(t)$

$\boxed{\cos 2\pi = 1,\ \sin 2\pi = 0}$

Verify whether the following continuous time signals are periodic. If periodic, find the fundamental period.

a) $x(t) = 2\cos\frac{t}{4}$ b) $x(t) = e^{\alpha t}\ ;\ \alpha > 1$ c) $x(t) = e^{\frac{-j2\pi t}{7}}$ d) $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$ e) $x(t) = \cos^2\left(2t - \frac{\pi}{4}\right)$

Solution

a) Given that, $x(t) = 2\cos\frac{t}{4}$

The given signal is a cosinusoidal signal, which is always periodic.

On comparing x(t) with the standard form "$A\cos 2\pi F_0 t$" we get,

$2\pi F_0 = \frac{1}{4} \Rightarrow F_0 = \frac{1}{8\pi}$

Period, $T = \frac{1}{F_0} = 8\pi$

$\therefore$ x(t) is periodic with period, $T = 8\pi$.

b) Given that, $x(t) = e^{\alpha t}\ ;\ \alpha > 1$

$\therefore x(t+T) = e^{\alpha(t+T)}$

$= e^{\alpha t}e^{\alpha T}$

For any value of α, $e^{\alpha T} \neq 1$ and so $x(t+T) \neq x(t)$

Since $x(t+T) \neq x(t)$, the signal x(t) is non-periodic.

c) Given that, $x(t) = e^{\frac{-j2\pi t}{7}}$

The given signal is a complex exponential signal, which is always periodic.

On comparing x(t) with the standard form "$Ae^{-j2\pi F_0 t}$"

We get, $F_0 = \frac{1}{7}$

$\therefore$ Peroid, $T = \frac{1}{F_0} = 7$

$\therefore$ x(t) is periodic with period, $T = 7$.

d) Given that, $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$

The given signal is a cosinusoidal signal, which is always periodic.

$$\therefore\ x(t+T) = 3\cos\left(5(t+T) + \frac{\pi}{6}\right) = 3\cos\left(5t + 5T + \frac{\pi}{6}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5T\right)$$

Let $5T = 2\pi$, $\therefore\ T = \frac{2\pi}{5}$

$$\therefore\ x(t+T) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5 \times \frac{2\pi}{5}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 2\pi\right)$$

$$= 3\cos\left(5t + \frac{\pi}{6}\right) = x(t)$$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

Since $x(t+T) = x(t)$, the signal $x(t)$ is periodic with period, $T = \frac{2\pi}{5}$

e) Given that, $x(t) = \cos^2\left(2t - \frac{\pi}{3}\right)$

$$x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$\therefore\ x(t+T) = \frac{1 + \cos\left(4(t+T) - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t + 4T - \frac{2\pi}{3}\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4T\right)}{2}$$

Let $4T = 2\pi$, $\therefore\ T = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore\ x(t+T) = \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4 \times \frac{\pi}{2}\right)}{2} = \frac{1 + \cos\left(\left(4t - \frac{2\pi}{3}\right) + 2\pi\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \cos^2\left(2t - \frac{\pi}{3}\right) = x(t)$$

Since $x(t+T) = x(t)$, the signal $x(t)$ is periodic with period, $T = \frac{\pi}{2}$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

EXAMPLE:2

(b) Given that, $x(t) = 2\cos 3t + 3\sin 7t$

Let, $x_1(t) = 2\cos 3t$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A\cos 2\pi F_{01}t$", we get,

$$F_{01} = \frac{3}{2\pi}; \qquad \therefore\ \text{Period, } T_1 = \frac{1}{F_{01}} = \frac{2\pi}{3}$$

Let, $x_2(t) = 3\sin 7t$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A\sin 2\pi F_{02}t$", we get,

$$F_{02} = \frac{7}{2\pi}; \qquad \therefore\ \text{Period, } T_2 = \frac{1}{F_{02}} = \frac{2\pi}{7}$$

Now, $\frac{T_1}{T_2} = T_1 \times \frac{1}{T_2} = \frac{2\pi}{3} \times \frac{7}{2\pi} = \frac{7}{3}$

Determine the periodicity of the following continuous time signals.

(a) $x(t) = 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7}$ (b) $x(t) = 2\cos 3t + 3\sin 7t$ (c) $x(t) = 5\cos 4\pi t + 3\sin 8\pi t$

Solution

(a) Given that, $x(t) = 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7}$

Let, $x_1(t) = 2\cos\frac{2\pi t}{3}$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A\cos 2\pi F_{01}t$", we get,

$$F_{01} = \frac{1}{3}\ ; \qquad \therefore \text{Period, } T_1 = \frac{1}{F_{01}} = 3$$

Let, $x_2(t) = 3\cos\frac{2\pi t}{7}$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A\cos 2\pi F_{02}t$", we get,

$$F_{02} = \frac{1}{7}\ ; \qquad \therefore \text{Period, } T_2 = \frac{1}{F_{02}} = 7$$

Now, $\frac{T_1}{T_2} = \frac{3}{7}$

Since $x_1(t)$ and $x_2(t)$ are periodic, and the ratio of T_1 and T_2 is a rational number, the signal $x(t)$ is also periodic. Let T be the periodicity of $x(t)$. Now the periodicity of $x(t)$ is the LCM (Least Common Multiple) of T_1 and T_2, i.e., LCM of 3 and 7, which is 21.

$\therefore$ Period, T = 21

Proof : $x(t+T) = 2\cos\frac{2\pi(t+T)}{3} + 3\cos\frac{2\pi(t+T)}{7} = 2\cos\left(\frac{2\pi t}{3} + \frac{2\pi T}{3}\right) + 3\cos\left(\frac{2\pi t}{7} + \frac{2\pi T}{7}\right)$

$$= 2\cos\left(\frac{2\pi t}{3} + \frac{2\pi \times 21}{3}\right) + 3\cos\left(\frac{2\pi t}{7} + \frac{2\pi \times 21}{7}\right)$$

Put, T = 21

$$= 2\cos\left(\frac{2\pi t}{3} + 14\pi\right) + 3\cos\left(\frac{2\pi t}{7} + 6\pi\right)$$

$$= 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7} = x(t)$$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

Since $x_1(t)$ and $x_2(t)$ are periodic and the ratio of T_1 and T_2 is a rational number, the signal $x(t)$ is also periodic. Let T be the periodicity of $x(t)$. Now the periodicity of $x(t)$ is the LCM (Least Common Multiple) of T_1 and T_2, which is calculated as shown below.

$$T_1 = \frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{21}{2\pi} = 7$$

$$T_2 = \frac{2\pi}{7} = \frac{2\pi}{7} \times \frac{21}{2\pi} = 3$$

Note : *To find LCM, first convert T_1 and T_2 to integers by multiplying by a common number. Find LCM of integer values of T_1 and T_2. Then divide this LCM by the common number.*

Now LCM of 7 and 3 is 21

$$\therefore \text{Period, } T = 21 \div \frac{21}{2\pi} = 21 \times \frac{2\pi}{21} = 2\pi$$

Proof : $x(t+T) = 2\cos 3(t+T) + 3\sin 7(t+T)$

$$= 2\cos(3t + 3T) + 3\sin(7t + 7T)$$

$$= 2\cos(3t + 3 \times 2\pi) + 3\sin(7t + 7 \times 2\pi)$$

$$= 2\cos(3t + 6\pi) + 3\sin(7t + 14\pi)$$

$$= 2\cos 3t + 3\sin 7t = x(t)$$

Put, $T = 2\pi$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$, $\sin(\theta + 2\pi M) = \sin\theta$

(c) Given that, $x(t) = 5\cos 4\pi t + 3\sin 8\pi t$

Let, $x_1(t) = 5\cos 4\pi t$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A\cos 2\pi F_{01}t$", we get,

$$F_{01} = 2; \qquad \therefore \text{Period, } T_1 = \frac{1}{F_{01}} = \frac{1}{2}$$

Let, $x_2(t) = 3\sin 8\pi t$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A\sin 2\pi F_{02}t$", we get,

$$F_{02} = 4; \qquad \therefore \text{Period, } T_2 = \frac{1}{F_{02}} = \frac{1}{4}$$

Now, $\frac{T_1}{T_2} = T_1 \times \frac{1}{T_2} = \frac{1}{2} \times \frac{4}{1} = 2$

Since $x_1(t)$ and $x_2(t)$ are periodic and the ratio of T_1 and T_2 is a rational number, the signal x(t) is also periodic. Let T be the periodicity of x(t). Now, the periodicity of x(t) is the LCM (Least Common Multiple) of T_1 and T_2, which is calculated as shown below.

$$T_1 = \frac{1}{2} = \frac{1}{2} \times 4 = 2$$

$$T_2 = \frac{1}{4} = \frac{1}{4} \times 4 = 1$$

Note : To find LCM, first convert T_1 and T_2 to integers by multiplying by a common number. Find LCM of integer values of T_1 and T_2. Then divide this LCM by the common number.

Now LCM of 2 and 1 is 2.

$$\therefore \text{Period, } T = 2 \div 4 = 2 \times \frac{1}{4} = \frac{1}{2}$$

Proof : $x(t+T) = 5\cos 4\pi(t+T) + 3\sin 8\pi(t+T)$

$$= 5\cos(4\pi t + 4\pi T) + 3\sin(8\pi t + 8\pi T)$$

$$= 5\cos\left(4\pi t + 4\pi \times \frac{1}{2}\right) + 3\sin\left(8\pi t + 8\pi \times \frac{1}{2}\right)$$

$$= 5\cos(4\pi t + 2\pi) + 3\sin(8\pi t + 2\pi)$$

$$= 5\cos 4\pi t + 3\sin 8\pi t = x(t)$$

Put, $T = \frac{1}{2}$

For integer values of M,
$\cos(\theta + 2\pi M) = \cos\theta$
$\sin(\theta + 2\pi M) = \sin\theta$

1.7.2 Symmetric (Even) and Antisymmetric (Odd) Signals

The signals may exhibit symmetry or antisymmetry with respect to t = 0.

When a signal exhibits symmetry with respect to t = 0 then it is called an ***even signal***. Therefore, the even signal satisfies the condition, $\mathbf{x(-t) = x(t)}$.

When a signal exhibits antisymmetry with respect to t = 0, then it is called an ***odd signal***. Therefore, the odd signal satisfies the condition, $\mathbf{x(-t) = -x(t)}$.

Since $\cos(-\theta) = \cos\theta$, the cosinusoidal signals are even signals and since $\sin(-\theta) = -\sin\theta$, the sinusoidal signals are odd signals.

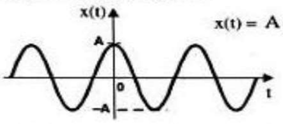


Fig 2.19a : Symmetric or Even signal.

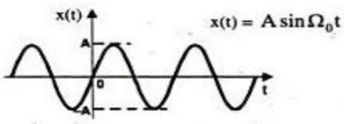


Fig 2.19b : Antisymmetric or Odd signal.

Fig 2.19 : Symmetric and antisymmetric continuous time signals.

A continuous time signal x(t) which is neither even nor odd can be expressed as a sum of even and odd signal.

Let, $x(t) = x_e(t) + x_o(t)$

where, $x_e(t)$ = Even part of x(t) and $x_o(t)$ = Odd part of x(t)

Now, it can be proved that,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Proof :

Let, $x(t) = x_e(t) + x_o(t)$

On replacing t by −t in equation (2.1) we get,

$$x(-t) = x_e(-t) + x_o(-t)$$

Since $x_e(t)$ is even, $x_e(-t) = x_e(t)$

Since $x_o(t)$ is odd, $x_o(-t) = -x_o(t)$

Hence the equation (2.2) can be written as,

$$x(-t) = x_e(t) - x_o(t)$$

On adding equations (2.1) & (2.3) we get,

$$x(t) + x(-t) = 2\,x_e(t)$$

$$\therefore\ x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

On subtracting equation (2.3) from equation (2.1) we get,

$$x(t) - x(-t) = 2\,x_o(t)$$

$$\therefore\ x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

The properties of signals with symmetry are given below without proof.

1. When a signal is even, then its odd part will be zero.
2. When a signal is odd, then its even part will be zero.
3. The product of two odd signals will be an even signal.
4. The product of two even signals will be an even signal.
5. The product of an even and odd signal will be an odd signal.

Example

Determine the even and odd part of the following continuous time signals.

a) $x(t) = e^t$ b) $x(t) = 3 + 2t + 5t^2$ c) $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

Solution

a) Given that, $x(t) = e^t$

$$\therefore\ x(-t) = e^{-t}$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^t + e^{-t}]$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^t - e^{-t}]$$

b) Given that, $x(t) = 3 + 2t + 5t^2$

$$\therefore\ x(-t) = 3 + 2(-t) + 5(-t)^2$$

$$= 3 - 2t + 5t^2$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 + 3 - 2t + 5t^2]$$

$$= \frac{1}{2}[6 + 10t^2] = 3 + 5t^2$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 - 3 + 2t - 5t^2]$$

$$= \frac{1}{2}[4t] = 2t$$

c) Given that, $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

$$\therefore x(-t) = \sin 2(-t) + \cos(-t) + \sin(-t)\cos 2(-t)$$

$$= -\sin 2t + \cos t - \sin t \cos 2t$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t - \sin 2t + \cos t - \sin t \cos 2t]$$

$$= \frac{1}{2}[2 \cos t] = \cos t$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t + \sin 2t - \cos t + \sin t \cos 2t]$$

$$= \frac{1}{2}[2 \sin 2t + 2 \sin t \cos 2t] = \sin 2t + \sin t \cos 2t$$

1.7.3 Energy and Power Signals

The signals which have finite energy are called ***energy signals***. The nonperiodic signals like exponential signals will have constant energy and so nonperiodic signals are energy signals.

The signals which have finite average power are called ***power signals***. The periodic signals like sinusoidal and complex exponential signals will have constant power and so periodic signals are power signals.

The ***energy*** E of a continuous time signal x(t) is defined as,

$$\text{Energy, } E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in joules}$$

The average ***power*** of a continuous time signal x(t) is defined as,

$$\text{Power, } P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in watts}$$

For periodic signals, the average power over one period will be same as average power over an infinite interval.

$$\therefore \text{For periodic signals, power, } P = \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt$$

For energy signals, the energy will be finite (or constant) and average power will be zero. For power signals the average power is finite (or constant) and energy will be infinite.

i.e., For energy signal, E is constant (i.e., $0 < E < \infty$) and $P = 0$.

For power signal, P is constant (i.e., $0 < P < \infty$) and $E = \infty$.

Proof:

The energy of a signal x(t) is defined as,

$$E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |x(t)|^2 \, dt$$

The power of a signal is defined as,

$$P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \underset{T \to \infty}{Lt} \frac{1}{2T} \underset{T \to \infty}{Lt} \int_{-T}^{T} |x(t)|^2 \, dt$$

Using equation (2.4), the equation (2.5) can be written as,

$$P = \underset{T \to \infty}{Lt} \frac{1}{2T} \times E$$

In equation (2.6), When E = constant,

$$P = E \times \underset{T \to \infty}{Lt} \frac{1}{2T}$$

$$= E \times \frac{1}{2 \times \infty} = E \times 0 = 0$$

From the above analysis, we can say that when a signal has finite energy the power will be zero. Also, from the above analysis we can say that the power is finite only when energy is infinite.

Example

Determine the power and energy for the following continuous time signals.

a) $x(t) = e^{-2t}\,u(t)$

b) $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$

c) $x(t) = 3\cos 5\Omega_0 t$

Solution

a) Given that, $x(t) = e^{-2t}\,u(t)$

Here, $x(t) = e^{-2t}\,u(t)$; for all t

$\therefore\ x(t) = e^{-2t}$; for $t \geq 0$

$$\therefore \int_{-T}^{T} |x(t)|^2\,dt = \int_{0}^{T} \left(\left|e^{-2t}\right|\right)^2 dt = \int_{0}^{T} \left(e^{-2t}\right)^2 dt = \int_{0}^{T} e^{-4t}\,dt = \left[\frac{e^{-4t}}{-4}\right]_0^T$$

$$= \left[\frac{e^{-4T}}{-4} - \frac{e^{0}}{-4}\right] = \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$\text{Energy, } E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2\,dt = \underset{T\to\infty}{Lt} \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{4} - \frac{e^{-\infty}}{4} = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} \text{ joules}$$

$$\text{Power, } P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt = \underset{T\to\infty}{Lt} \frac{1}{2T}\left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{\infty}\left[\frac{1}{4} - \frac{e^{-\infty}}{4}\right] = 0 \times \left[\frac{1}{4} - 0\right] = 0$$

Since energy is constant and power is zero, the given signal is an energy signal.

b) Given that, $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$

Here, $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)} = 1\angle\left(2t+\frac{\pi}{4}\right)$

$\therefore\ |x(t)| = 1$

$$\int_{-T}^{T} |x(t)|^2\,dt = \int_{-T}^{T} 1 \times dt = [t]_{-T}^{T} = T + T = 2T$$

$$\text{Energy, } E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2\,dt = \underset{T\to\infty}{Lt}\ 2T = \infty$$

$$\text{Power, } P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt = \underset{T\to\infty}{Lt} \frac{1}{2T} \times 2T = 1 \text{ watt}$$

Since power is constant and energy is infinite, the given signal is a power signal.

c) Given that, $x(t) = 3\cos 5\Omega_0 t$

$$\therefore \int_{-T}^{T} |x(t)|^2\, dt = \int_{-T}^{T} \left(|3\cos 5\Omega_0 t|\right)^2 dt = \int_{-T}^{T} \left|(3\cos 5\Omega_0 t)^2\right| dt = \int_{-T}^{T} (3\cos 5\Omega_0 t)^2\, dt$$

$$= \int_{-T}^{T} 9\cos^2 5\Omega_0 t\, dt = 9\int_{-T}^{T} \left(\frac{1+\cos 10\Omega_0 t}{2}\right) dt$$

$$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$= \frac{9}{2}\int_{-T}^{T} (1+\cos 10\Omega_0 t)\, dt = \frac{9}{2}\left[t + \frac{\sin 10\Omega_0 t}{10\Omega_0}\right]_{-T}^{T}$$

$$= \frac{9}{2}\left[T + \frac{\sin 10\Omega_0 T}{10\Omega_0} - \left(-T + \frac{\sin 10\Omega_0(-T)}{10\Omega_0}\right)\right]$$

$$= \frac{9}{2}\left[2T + 2\frac{\sin 10\Omega_0 T}{10\Omega_0}\right] = \frac{9}{2}\left[2T + 2\frac{\sin 10\frac{2\pi}{T}T}{10\frac{2\pi}{T}}\right]$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$

$$\boxed{\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}}$$

$$= \frac{9}{2}\left[2T + \frac{T}{10\pi}\sin 20\pi\right] = \frac{9}{2}\left[2T + \frac{T}{10\pi}\times 0\right] = 9T$$

$$\boxed{\text{For integer M, } \sin \pi M = 0}$$

$$\text{Energy, } E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt}\, 9T = \infty$$

$$\text{Power, } P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt} \frac{1}{2T}\times 9T = \underset{T\to\infty}{Lt} \frac{9}{2} = \frac{9}{2} = 4.5 \text{ watts}$$

Since energy is infinite and power is constant, the given signal is a power signal.

1.7.4 Causal, Noncausal and Anticausal Signals

A signal is said to be ***causal***, if it is defined for $t \geq 0$.

Therefore if x(t) is causal, then $x(t) = 0$, for $t < 0$.

A signal is said to be ***noncausal***, if it is defined for either $t \leq 0$, or for both $t \leq 0$ and $t > 0$.

Therefore if x(t) is noncausal, then $x(t) \neq 0$, for $t < 0$.

When a noncausal signal is defined only for $t \leq 0$, it is called ***anticausal signal***.

Examples of causal and noncausal signals

Step signal,	$x(t) = A \quad ; t \geq 0$	Causal signals
Unit step signal,	$x(t) = u(t) = 1 \quad ; t \geq 0$	Causal signals
Exponential signal,	$x(t) = A\, e^{bt}\, u(t)$	Causal signals
Complex exponential signal,	$x(t) = A\, e^{j\Omega_0 t}\, u(t)$	Causal signals
Exponential signal,	$x(t) = A\, e^{bt}$; for all t	Noncausal signals
Complex exponential signal,	$x(t) = A\, e^{j\Omega_0 t}$; for all t	Noncausal signals

Note : On multiplying a noncausal signal by u(t), it becomes causal.

1.8 SYSTEMS AND ITS CLASSIFICATION

Classification of Continuous Time Systems

The continuous time systems are classified based on their characteristics. Some of the classifications of continuous time systems are,

1. Static and dynamic systems
2. Time invariant and time variant systems
3. Linear and nonlinear systems
4. Causal and noncausal systems
5. Stable and unstable systems
6. Feedback and nonfeedback systems

1.8.1 Static and Dynamic Systems

A continuous time system is called ***static*** or ***memoryless*** if its output at any instant of time t depends at most on the input signal at the same time but not on the past or future input. In any other case, the system is said to be ***dynamic*** or to have memory.

Example :

$y(t) = a\, x(t)$
$y(t) = t\, x(t) + 6\, x^3(t)$
} Static systems

$y(t) = t\, x(t) + 3\, x(t^2)$
$y(t) = x(t) + 3\, x(t-2)$
} Dynamic systems

1.8.2 Time Invariant and Time Variant Systems

A system is said to be ***time invariant*** if its input-output characteristics does not change with time.

Definition : A relaxed system $\mathcal{H}$ is ***time invariant*** or ***shift invariant*** if and only if

$$x(t) \xrightarrow{\mathcal{H}} y(t) \text{ implies that, } x(t-m) \xrightarrow{\mathcal{H}} y(t-m)$$

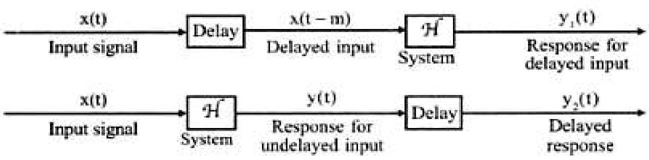
for every input signal x(t) and every time shift m.

i.e., in time invariant systems, if $\mathbf{y(t) = \mathcal{H}\{x(t)\}}$ then $\mathbf{y(t-m) = \mathcal{H}\{x(t-m)\}}$.

Alternative Definition for Time Invariance

A system $\mathcal{H}$ is ***time invariant*** if the response to a shifted (or delayed) version of the input is identical to a shifted (or delayed) version of the response based on the unshifted (or undelayed) input.

The diagrammatic explanation of the above definition of time invariance is shown in fig 2.41.



If $y_1(t) = y_2(t)$ then the system is time invariant

Diagrammatic explanation of time invariance.

Procedure to test for time invariance

1. Delay the input signal by m units of time and determine the response of the system for this delayed input signal. Let this response be $y_1(t)$.
2. Delay the response of the system for unshifted input by m units of time. Let this delayed response be $y_2(t)$.
3. Check whether $y_1(t) = y_2(t)$. If they are equal then the system is time invariant. Otherwise the system is time variant.

Example

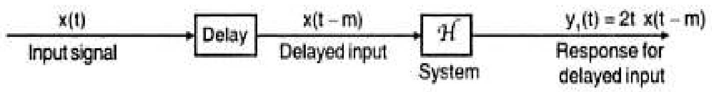
State whether the following systems are time invariant or not.

a) $y(t) = 2t\,x(t)$ b) $y(t) = x(t)\sin 20\pi t$ c) $y(t) = 3x(t^2)$ d) $y(t) = x(-t)$

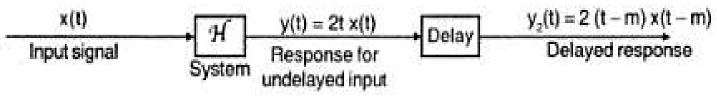
Solution

a) Given that, $y(t) = 2t\,x(t)$

Test 1 : Response for delayed input



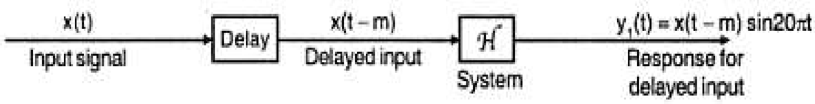
Test 2 : Delayed response



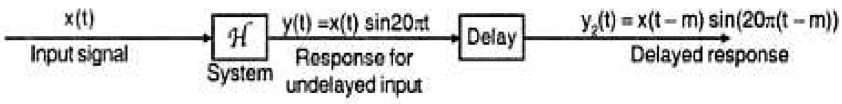
Conclusion : Here, $y_1(t) \neq y_2(t)$, therefore the system is time variant.

b) Given that, $y(t) = x(t)\sin 20\pi t$

Test 1 : Response for delayed input



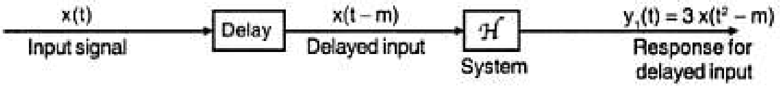
Test 2 : Delayed response



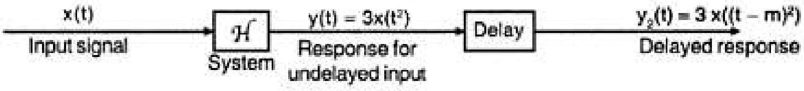
Conclusion : Here, $y_1(t) \neq y_2(t)$, therefore the system is time variant.

c) Given that, $y(t) = 3x(t^2)$

Test 1 : Response for delayed input



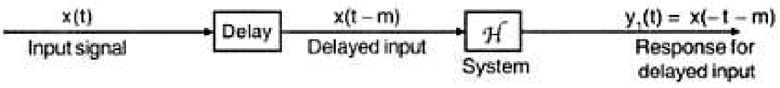
Test 2 : Delayed response



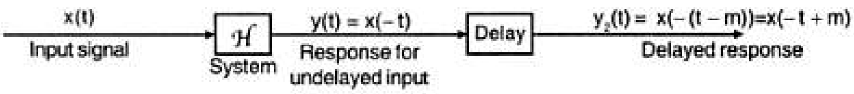
Conclusion : Here, $y_1(t) \neq y_2(t)$, therefore the system is time variant.

d) Given that, $y(t) = x(-t)$

Test 1 : Response for delayed input



Test 2 : Delayed response



Conclusion : Here, $y_1(t) \neq y_2(t)$, therefore the system is time variant.

Example

State whether the following systems are time invariant or not.

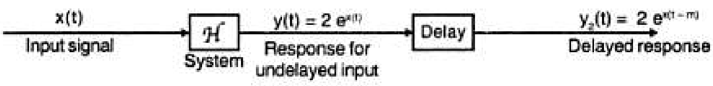
a) $y(t) = 2\,e^{x(t)}$ b) $y(t) = x(t) + C$ c) $y(t) = 3x^2(t)$ d) $y(t) = x(t) + \frac{dx(t)}{dt}$ e) $y(t) = x(t) + \int x(t)\,dt$

Solution

a) Given that, $y(t) = 2\,e^{x(t)}$

Test 1 : Response for delayed input

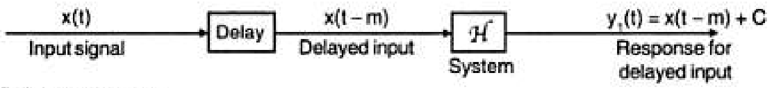
Test 2 : Delayed response



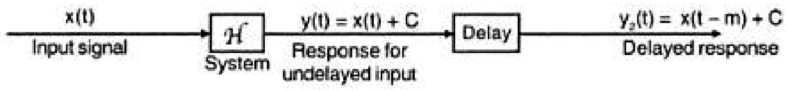
Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

b) Given that, $y(t) = x(t) + C$

Test 1 : Response for delayed input



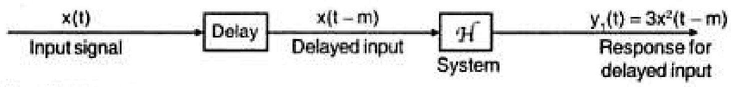
Test 2 : Delayed response



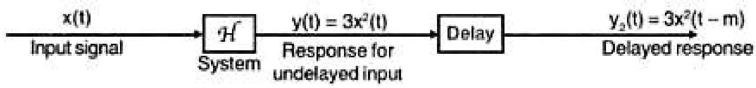
Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

c) Given that, $y(t) = 3x^2(t)$

Test 1 : Response for delayed input



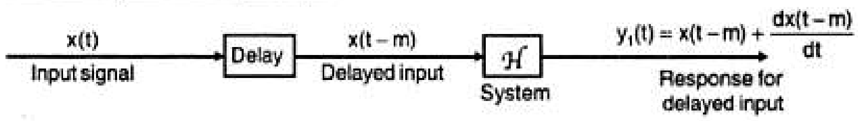
Test 2 : Delayed response



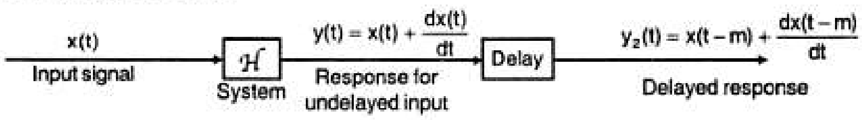
Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

d) Given that, $y(t) = x(t) + \frac{dx(t)}{dt}$

Test 1 : Response for delayed input



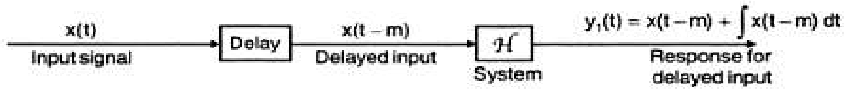
Test 2 : Delayed response



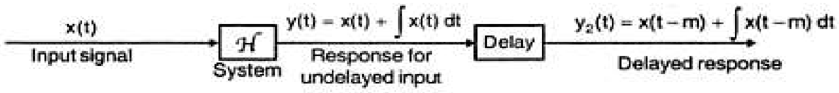
Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

e) Given that, $y(t) = x(t) + \int x(t)\, dt$

Test 1 : Response for delayed input



Test 2 : Delayed response



Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

1.8.3 Linear and Nonlinear Systems

A ***linear system*** is the one that satisfies the superposition principle.

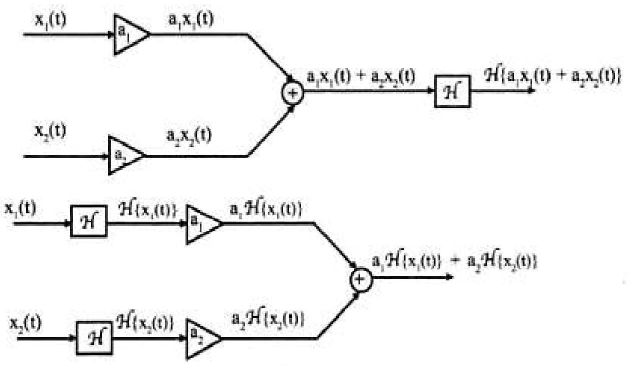
The ***principle of superposition*** requires that the response of a system to a weighted sum of the signals is equal to the corresponding weighted sum of the responses to each of the individual input signals.

Definition : A relaxed system $\mathcal{H}$ is ***linear*** if

$$\mathcal{H}\{a_1\, x_1(t) + a_2\, x_2(t)\} = a_1\, \mathcal{H}\{x_1(t)\} + a_2\, \mathcal{H}\{x_2(t)\}$$

for any arbitrary input signal $x_1(t)$ and $x_2(t)$ and for any arbitrary constants a_1 and a_2.

If a relaxed system does not satisfy the superposition principle as given by the above definition, the system is ***nonlinear***. The diagrammatic explanation of linearity is shown in fig. 2.42.



The system, $\mathcal{H}$ is linear if and only if, $\mathcal{H}\{a_1\, x_1(t) + a_2\, x_2(t)\} = a_1\, \mathcal{H}\{x_1(t)\} + a_2\, \mathcal{H}\{x_2(t)\}$

Fig 2.42 : Diagrammatic explanation of linearity.

Procedure to test for linearity

1. Let $x_1(t)$ and $x_2(t)$ be two inputs to the system $\mathcal{H}$, and $y_1(t)$ and $y_2(t)$ be the corresponding responses.
2. Consider a signal, $x_3(t) = a_1\, x_1(t) + a_2\, x_2(t)$ which is a weighed sum of $x_1(t)$ and $x_2(t)$.
3. Let $y_3(t)$ be the response for $x_3(t)$.
4. Check whether $y_3(t) = a_1\, y_1(t) + a_2\, y_2(t)$. If equal then the system is linear, otherwise it is nonlinear.

Example

Test the following systems for linearity.

a) $y(t) = t\,x(t)$, b) $y(t) = x(t^2)$, c) $y(t) = x^2(t)$, d) $y(t) = A\,x(t) + B$, e) $y(t) = e^{x(t)}$.

Solution

a) Given that, $y(t) = t\,x(t)$

Let $\mathcal{H}$ be the system operating on $x(t)$ to produce, $y(t) = \mathcal{H}\{x(t)\} = t\,x(t)$.

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = t\,x_1(t) \qquad \text{....(1)}$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = t\,x_2(t) \qquad \text{.....(2)}$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. [A linear combination of inputs $x_1(t)$ and $x_2(t)$]

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\} = \mathcal{H}\{a_1 x_1(t) + a_2 x_2(t)\}$$

$$= t(a_1 x_1(t) + a_2 x_2(t)) = a_1 t\,x_1(t) + a_2 t\,x_2(t)$$

$$= a_1 y_1(t) + a_2 y_2(t)) \qquad \text{[Using equations (1) and (2)]}$$

Since, $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, the given system is linear.

b) Given that, $y(t) = x(t^2)$

Let $\mathcal{H}$ be the system operating on $x(t)$ to produce, $y(t) = \mathcal{H}\{x(t)\} = x(t^2)$.

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = x_1(t^2) \qquad \text{....(1)}$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = x_2(t^2) \qquad \text{.....(2)}$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. [A linear combination of inputs $x_1(t)$ and $x_2(t)$]

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\} = \mathcal{H}\{a_1 x_1(t) + a_2 x_2(t)\}$$

$$= (a_1 x_1(t^2) + a_2 x_2(t^2))$$

$$= a_1 y_1(t) + a_2 y_2(t)) \qquad \text{[Using equations (1) and (2)]}$$

Since, $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, the given system is linear.

c) Given that, $y(t) = x^2(t)$

Let $\mathcal{H}$ be the system operating on $x(t)$ to produce, $y(t) = \mathcal{H}\{x(t)\} = x^2(t)$.

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = x_1^2(t) \qquad \text{.....(1)}$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = x_2^2(t) \qquad \text{.....(2)}$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. [A linear combination of inputs $x_1(t)$ and $x_2(t)$]

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\} = \mathcal{H}\{a_1 x_1(t) + a_2 x_2(t)\} = (a_1 x_1(t) + a_2 x_2(t))^2$$

$$= a_1^2\, x_1^2(t) + a_2^2\, x_2^2(t) + 2\, a_1\, a_2\, x_1(t)\, x_1(t)$$

$$= a_1^2\, y_1(t) + a_2^2\, y_2(t) + 2\, a_1\, a_2\, x_1(t)\, x_1(t) \qquad \text{[Using equations (1) and (2)]}$$

Here, $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$. Hence the given system is nonlinear.

d) Given that, $y(t) = A\,x(t) + B$.

Let $\mathcal{H}$ be the system operating on x(t) to produce, $y(t) = \mathcal{H}\{x(t)\} = A\,x(t) + B$.

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = A\,x_1(t) + B \qquad .....(1)$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = A\,x_2(t) + B \qquad .....(2)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. | A linear combination of inputs $x_1(t)$ and $x_2(t)$

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\} = \mathcal{H}\{a_1 x_1(t) + a_2 x_2(t)\}$$

$$= A\,(a_1 x_1(t) + a_2 x_2(t)) + B = A\,a_1 x_1(t) + A\,a_2 x_2(t) + B$$

$$= a_1\,A\,x_1(t) + a_2\,A\,x_2(t) + B$$

$$= a_1\,(y_1(t) - B) + a_2\,(y_2(t) - B) + B$$ | Using equations (1) and (2)

Here, $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$. Hence the given system is nonlinear.

e) Given that, $y(t) = e^{x(t)}$

Let $\mathcal{H}$ be the system operating on x(t) to produce, $y(t) = \mathcal{H}\{x(t)\} = e^{x(t)}$.

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = e^{x_1(t)} \qquad .....(1)$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = e^{x_2(t)} \qquad .....(2)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. | A linear combination of inputs $x_1(t)$ and $x_2(t)$

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\} = \mathcal{H}\{a_1 x_1(t) + a_2 x_2(t)\}$$

$$= e^{(a_1 x_1(t) + a_2 x_2(t))} = e^{a_1 x_1(t)}\ e^{a_2 x_2(t)}$$

$$= \left(e^{x_1(t)}\right)^{a_1}\ \left(e^{x_2(t)}\right)^{a_2} = (y_1(t))^{a_1}\ (y_2(t))^{a_2}$$ | Using equations (1) and (2)

Here, $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$. Hence the given system is nonlinear.

Example

Test the following systems for linearity.

a) $y(t) = 4\,x(t) + 2\,\dfrac{dx(t)}{dt}$ b) $\dfrac{d^2y(t)}{dt^2} + 2\,\dfrac{dy(t)}{dt} + 3\,y(t) = x(t)$

Solution

a) Given that, $y(t) = 4\,x(t) + 2\,\dfrac{dx(t)}{dt}$

Let $\mathcal{H}$ be the system operating on x(t) to produce, $y(t) = \mathcal{H}\{x(t)\} = 4\,x(t) + 2\,\dfrac{dx(t)}{dt}$

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

$$\therefore\ y_1(t) = \mathcal{H}\{x_1(t)\} = 4\,x_1(t) + 2\,\frac{dx_1(t)}{dt} \qquad .....(1)$$

$$y_2(t) = \mathcal{H}\{x_2(t)\} = 4\,x_2(t) + 2\,\frac{dx_2(t)}{dt} \qquad .....(2)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$. | A linear combination of inputs $x_1(t)$ and $x_2(t)$

Let $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

$$\therefore\ y_3(t) = \mathcal{H}\{x_3(t)\}$$

$$= 4\,x_3(t) + 2\,\frac{dx_3(t)}{dt}$$

$$= 4(a_1 x_1(t) + a_2 x_2(t)) + 2\,\frac{d}{dt}(a_1 x_1(t) + a_2 x_2(t))$$

$$= 4a_1 x_1(t) + 4a_2 x_2(t) + 2a_1\,\frac{dx_1(t)}{dt} + 2a_2\,\frac{dx_2(t)}{dt}$$

$$= a_1\left(4\,x_1(t) + 2\,\frac{dx_1(t)}{dt}\right) + a_2\left(4\,x_2(t) + 2\,\frac{dx_2(t)}{dt}\right)$$ | Using equations (1) and (2)

$$= a_1\,y_1(t) + a_2\,y_2(t)$$

Since, $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, the given system is linear.

b) Given that, $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = x(t)$

Let $\mathcal{H}$ be the system operating on x(t) to produce, y(t).

Consider two signals $x_1(t)$ and $x_2(t)$.

Let $y_1(t)$ and $y_2(t)$ be the response of the system $\mathcal{H}$ for inputs $x_1(t)$ and $x_2(t)$ respectively.

When the input is $x_1(t)$, the response is $y_1(t)$. Hence the system equation for the input $x_1(t)$ can be written as,

$$\frac{d^2y_1(t)}{dt^2} + 2\frac{dy_1(t)}{dt} + 3y_1(t) = x_1(t) \quad .....(1)$$

When the input is $x_2(t)$, the response is $y_2(t)$. Hence the system equation for the input $x_2(t)$ can be written as,

$$\frac{d^2y_2(t)}{dt^2} + 2\frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t) \quad .....(2)$$

Let, $x_3(t) = a_1x_1(t) + a_2x_2(t)$.

A linear combination of inputs $x_1(t)$ and $x_2(t)$

Let, $y_3(t)$ be the response of the system $\mathcal{H}$ for input $x_3(t)$.

When the input is $x_3(t)$, the response is $y_3(t)$. Hence the system equation for the input $x_3(t)$ is given by,

$$\frac{d^2y_3(t)}{dt^2} + 2\frac{dy_3(t)}{dt} + 3y_3(t) = x_3(t) \quad .....(3)$$

Let us multiply equation (1) by a_1.

$$\therefore a_1\frac{d^2y_1(t)}{dt^2} + 2a_1\frac{dy_1(t)}{dt} + 3a_1y_1(t) = a_1x_1(t) \quad .....(4)$$

Let us multiply equation (2) by a_2.

$$\therefore a_2\frac{d^2y_2(t)}{dt^2} + 2a_2\frac{dy_2(t)}{dt} + 3a_2y_2(t) = a_2x_2(t) \quad .....(5)$$

On adding equation (4) and (5) we get,

$$a_1\frac{d^2y_1(t)}{dt^2} + 2a_1\frac{dy_1(t)}{dt} + 3a_1y_1(t) + a_2\frac{d^2y_2(t)}{dt^2} + 2a_2\frac{dy_2(t)}{dt} + 3a_2y_2(t) = a_1x_1(t) + a_2x_2(t)$$

$$\frac{d^2}{dt^2}[a_1y_1(t) + a_2y_2(t)] + 2\frac{d}{dt}[a_1y_1(t) + a_2y_2(t)] + 3[a_1y_1(t) + a_2y_2(t)] = a_1x_1(t) + a_2x_2(t) \quad .....(6)$$

On comparing equations (3) and (6) we can say that,

if, $x_3(t) = a_1x_1(t) + a_2x_2(t)$, then $y_3(t) = a_1y_1(t) + a_2y_2(t)$

Hence the sytem is linear.

1.8.4 Causal and Noncausal Systems

Definition : A system is said to be ***causal*** if the output of the system at any time t depends only on the present input, past inputs and past outputs but does not depend on the future inputs and outputs.

If the system output at any time t depends on future inputs or outputs then the system is called a ***noncausal*** system.

The causality refers to a system that is realizable in real time. It can be shown that an LTI system is causal if and only if the impulse response is zero for $t < 0$, (i.e., $h(t) = 0$ for $t < 0$).

Example

Test the casuality of the following systems.

a) $y(t) = x(t) - x(t-1)$ b) $y(t) = x(t) + 2x(3-t)$ c) $y(t) = t\,x(t)$

d) $y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda$ e) $y(t) = x(t) + \int_0^{3t} x(\lambda)\,d\lambda$ f) $y(t) = 2x(t) + \frac{dx(t)}{dt}$

Solution

a) Given that, $y(t) = x(t) - x(t-1)$

When $t = 0$, $y(0) = x(0) - x(-1)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on the present input $x(0)$ and past input $x(-1)$.

When $t = 1$, $y(1) = x(1) - x(0)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on the present input $x(1)$ and past input $x(0)$.

From the above analysis we can say that for any value of t, the system output depends on present and past inputs. Hence the system is causal.

b) Given that, $y(t) = x(t) + 2x(3-t)$

When $t = -1$, $y(-1) = x(-1) + 2x(4)$ $\Rightarrow$ The response at $t = -1$, i.e., $y(-1)$ depends on the present input $x(-1)$ and future input $x(4)$.

When $t = 0$, $y(0) = x(0) + 2x(3)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on the present input $x(0)$ and future input $x(3)$.

When $t = 1$, $y(1) = x(1) + 2x(2)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on the present input $x(1)$ and future input $x(2)$.

When $t = 2$, $y(2) = x(2) + 2x(1)$ $\Rightarrow$ The response at $t = 2$, i.e., $y(2)$ depends on the present input $x(2)$ and past input $x(1)$.

From the above analysis we can say that for $t < 2$, the system output depends on present and future inputs. Hence the system is noncausal.

c) Given that, $y(t) = t\,x(t)$

When $t = 0$, $y(0) = 0 \times x(0)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on the present input $x(0)$.

When $t = 1$, $y(1) = 1 \times x(1)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on the present input $x(1)$.

When $t = 2$, $y(2) = 2 \times x(2)$ $\Rightarrow$ The response at $t = 2$, i.e., $y(2)$ depends on the present input $x(2)$.

From the above analysis we can say that the response for any value of t depends on the present input. Hence the system is causal.

d) Given that, $y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda$

$$y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda = x(t) + [z(\lambda)]_0^t = x(t) + z(t) - z(0), \qquad \text{where, } z(\lambda) = \int x(\lambda)\,d\lambda$$

When $t = 0$, $y(0) = x(0) + z(0) - z(0)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on present input.

When $t = 1$, $y(1) = x(1) + z(1) - z(0)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on present and past input.

When $t = 2$, $y(2) = x(2) + z(1) - z(0)$ $\Rightarrow$ The response at $t = 2$, i.e., $y(2)$ depends on present and past input.

From the above analysis we can say that the response for any value of t depends on the present and past input. Hence the system is causal.

e) Given that, $y(t) = x(t) + \int_0^{3t} x(\lambda)\, d\lambda$

$$y(t) = x(t) + \int_0^{3t} x(\lambda)\, d\lambda = x(t) + [z(\lambda)]_0^{3t} = x(t) + z(3t) - z(0), \qquad \text{where } z(\lambda) = \int x(\lambda)\, d\lambda$$

When $t = 0$, $y(0) = x(0) + z(0) - z(0)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on present input.

When $t = 1$, $y(1) = x(1) + z(3) - z(0)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on present, past and future inputs.

When $t = 2$, $y(2) = x(2) + z(6) - z(0)$ $\Rightarrow$ The response at $t = 2$, i.e., $y(2)$ depends on present, past and future inputs.

From the above analysis we can say that the response for $t > 0$ depends on the present, past and future inputs. Hence the system is noncausal.

f) Given that, $y(t) = 2x(t) + \frac{dx(t)}{dt}$

$$y(t) = 2x(t) + \frac{dx(t)}{dt}$$

$$= 2x(t) + \underset{\Delta t \to 0}{\text{Lt}} \frac{x(t) - x(t - \Delta t)}{\Delta t} \quad \text{(Using definition of differentiation, refer section 2.4.6)}$$

In the above equation, for any value of t, the $x(t)$ is present input and $x(t-\Delta t)$ is the past input.

Therefore we can say that the response for any value of t depends on present and past input. Hence the system is causal.

Example

Test the causality of the following systems.

a) $y(t) = x(t) + 3\,x(t + 4)$ b) $y(t) = x(t^2)$

c) $y(t) = x(2t)$ d) $y(t) = x(-t)$

Solution

a) Given that, $y(t) = x(t) + 3\,x(t + 4)$

When $t = 0$, $y(0) = x(0) + 3\,x(4)$ $\Rightarrow$ The response at $t = 0$, i.e., $y(0)$ depends on the present input $x(0)$ and future input $x(4)$.

When $t = 1$, $y(1) = x(1) + 3\,x(5)$ $\Rightarrow$ The response at $t = 1$, i.e., $y(1)$ depends on the present input $x(1)$ and future input $x(5)$.

From the above analysis we can say that the response for any value of t depends on present and future inputs. Hence the system is noncausal.

b) Given that, $y(t) = x(t^2)$

When $t = -1$; $y(-1) = x(1)$ $\Rightarrow$ The response at $t = -1$, depends on the future input $x(1)$.

When $t = 0$; $y(0) = x(0)$ $\Rightarrow$ The response at $t = 0$, depends on the present input $x(0)$.

When $t = 1$; $y(1) = x(1)$ $\Rightarrow$ The response at $t = 1$, depends on the present input $x(1)$.

When $t = 2$; $y(2) = x(4)$ $\Rightarrow$ The response at $t = 2$, depends on the future input $x(4)$.

From the above analysis we can say that the response for any value of t (except $t = 0$ & $t = 1$) depends on future input. Hence the system is noncausal.

c) Given that, $y(t) = x(2t)$

When $t = -1$; $y(-1) = x(-2)$ $\Rightarrow$ The response at $t = -1$, depends on the past input $x(-2)$.

When $t = 0$; $y(0) = x(0)$ $\Rightarrow$ The response at $t = 0$, depends on the present input $x(0)$.

When $t = 1$; $y(1) = x(2)$ $\Rightarrow$ The response at $t = 1$, depends on the future input $x(2)$.

From the above analysis we can say that the response of the system for $t > 0$, depends on future input. Hence the system is noncausal.

d) Given that, $y(t) = x(-t)$

When $t = -2$; $y(-2) = x(2)$ ⇒ The response at $t = -2$, depends on the future input $x(2)$.

When $t = -1$; $y(-1) = x(1)$ ⇒ The response at $t = -1$, depends on the future input $x(1)$.

When $t = 0$; $y(0) = x(0)$ ⇒ The response at $t = 0$, depends on the present input $x(0)$.

When $t = 1$; $y(1) = x(-1)$ ⇒ The response at $t = 1$, depends on the past input $x(-1)$.

From the above analysis we can say that the response of the system for $t < 0$ depends on future input. Hence the system is noncausal.

1.8.5 Stable and Unstable Systems

Definition : An arbitrary relaxed system is said to be ***BIBO stable*** (Bounded Input-Bounded Output stable) if and only if every bounded input produces a bounded output.

Let x(t) be the input of continuous time system and y(t) be the response or output for x(t).

The term ***bounded input*** refers to finite value of the input signal x(t) for any value of t. Hence if input x(t) is bounded then there exists a constant M_x such that $|x(t)| \le M_x$ and $M_x < \infty$, for all t.

> Examples of bounded input signal are step signal, decaying exponential signal and impulse signal.
>
> Examples of unbounded input signal are ramp signal and increasing exponential signal.

The term ***bounded output*** refers to finite and predictable output for any value of t. Hence if output y(t) is bounded then there exists a constant M_y such that $|y(t)| \le M_y$ and $M_y < \infty$, for all t.

In general, the test for stability of the system is performed by applying specific input. On applying a bounded input to a system if the output is bounded then the system is said to be BIBO stable.

Condition for Stability of an LTI System

For an LTI (Linear Time Invariant) system, the condition for BIBO stability can be transformed to a condition on impulse response, h(t). For BIBO stability of an LTI continuous time system, the integral of impulse response should be finite.

$$\therefore \int_{-\infty}^{+\infty} |h(t)|\, dt < \infty \text{, for stability of an LTI system.}$$

Example

Test the stability of the following systems.

a) $y(t) = \cos(x(t))$ b) $y(t) = x(-t-2)$ c) $y(t) = t\,x(t)$

Solution

a) Given that, $y(t) = \cos(x(t))$

The given system is a nonlinear system, and so the test for stability should be performed for specific inputs.

The value of $\cos\theta$ lies between -1 to $+1$ for any value of θ. Therefore the output y(t) is bounded for any value of input x(t). Hence the given system is stable.

---

b) Given that, $y(t) = x(-t-2)$

The given system is a time variant system, and so the test for stability should be performed for specific inputs.

The operations performed by the system on the input signal are folding and shifting. A bounded input signal will remain bounded even after folding and shifting. Therefore in the given system, the output will be bounded as long as input is bounded. Hence the given system is BIBO stable.

c) Given that, y(t) = t x(t)

The given system is a time variant system, and so the test for stability should be performed for specific inputs.

Case i : Let x(t) tends to ∞ or constant, as t tends to infinity. In this case, y(t) = t x(t) will be infinity as t tends to inifnity and so the system is unstable.

Case ii : Let x(t) tends to 0, as t tends to infinity. In this case y(t) = t x(t) will be zero as t tends to infinity and so the system is stable.

1.9 INVERSE SYSTEM

Inverse System and Deconvolution

Inverse System

The ***inverse system*** is used to recover the input from the response of a system. For a given system, the inverse system exists, if distinct inputs to a system leads to distinct outputs. The inverse systems exist for all LTI systems. Consider an amplifier with gain "A". When we pass the output of the amplifier through an attenuator with gain "1/A" then the output of the attenuator will be same as that of input of the amplifier. Therefore the attenuator with gain "1/A" is an inverse system for the amplifier with gain "A".

The inverse system is denoted by $\mathcal{H}^{-1}$. If x(t) is the input and y(t) is the output of a system, then y(t) is the input and x(t) is the output of its inverse system.

Fig 2.46a : System. *Fig 2.46b : Inverse system.*

Fig 2.46 : A system and its inverse system.

Let h(t) be the impulse response of a system and h'(t) be the impulse response of inverse system. Let us connect the system and its inverse in cascade as shown in fig 2.47.

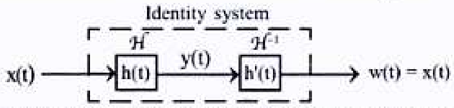


Fig 2.47 : Cascade connection of a system and its inverse.

Now it can be proved that,

$$h(t) * h'(t) = \delta(t)$$

Therefore the cascade of a system and its inverse is the identity system.

Deconvolution

In an LTI system the response y(t) is given by convolution of input x(t) and impulse response h(t).

i.e., $y(t) = x(t) * h(t)$

The process of recovering the input signal from the response of a system is called ***deconvolution***. Alternatively, the process of recovering x(t) from y(t), where y(t) = x(t)*h(t), is called ***deconvolution***.

1.10 PROPERTIES OF IMPULSE SIGNAL

Properties of Impulse Signal

Property-1: $\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$

Proof:

Consider a narrow pulse signal, $P_\Delta(t)$ of width $\Delta\lambda$ and height $1/\Delta\lambda$ as shown in fig 2.30.

Now the pulse signal is defined as,

$$P_\Delta(t) = \frac{1}{\Delta\lambda} \quad ; 0 \le t \le \Delta\lambda$$
$$= 0 \quad ; t > \Delta\lambda$$

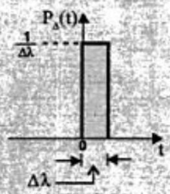


Fig 2.30.

Now the impulse signal can be represented as,

$$\delta(t) = \underset{\Delta\lambda \to 0}{Lt} P_\Delta(t)$$

On integrating the above equation we get,

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = \int_{-\infty}^{+\infty} \underset{\Delta\lambda \to 0}{Lt} P_\Delta(t)\,dt = \underset{\Delta\lambda \to 0}{Lt} \int_{-\infty}^{+\infty} P_\Delta(t)\,dt = \underset{\Delta\lambda \to 0}{Lt} \int_0^{\Delta\lambda} \frac{1}{\Delta\lambda}dt = \underset{\Delta\lambda \to 0}{Lt} \frac{1}{\Delta\lambda}\int_0^{\Delta\lambda} dt$$
$$= \underset{\Delta\lambda \to 0}{Lt} \frac{1}{\Delta\lambda}[t]_0^{\Delta\lambda} = \underset{\Delta\lambda \to 0}{Lt} \frac{1}{\Delta\lambda}[\Delta\lambda - 0] = \underset{\Delta\lambda \to 0}{Lt} 1 = 1$$

Property-2: $\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt = x(0)$

Proof:

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{+\infty} x(0)\,\delta(t)\,dt$$
$$= x(0)\int_{-\infty}^{+\infty} \delta(t)\,dt = x(0) \times 1 = x(0)$$

Since $\delta(t)$ is nonzero only at t=0, x(t) is replaced by x(0).

Since x(0) is constant, it is taken outside integration.

Using property-1

Property-3: $\int_{-\infty}^{+\infty} x(t)\,\delta(t - t_0)\,dt = x(t_0)$

Proof:

$$\int_{-\infty}^{+\infty} x(t)\,\delta(t - t_0)\,dt = \int_{-\infty}^{+\infty} x(t_0)\,\delta(t - t_0)\,dt$$
$$= x(t_0)\int_{-\infty}^{+\infty} \delta(t - t_0)\,dt = x(t_0) \times 1 = x(t_0)$$

Since $\delta(t_0)$ is nonzero only at $t=t_0$, x(t) is replaced by $x(t_0)$.

Since $x(t_0)$ is constant, it is taken outside integration.

Using property-1

Property-4: $\int_{-\infty}^{+\infty} x(\lambda)\,\delta(t - \lambda)\,d\lambda = x(t)$

Proof:

Consider the property-3 of impulse signal. $\int_{-\infty}^{+\infty} x(t)\,\delta(t - t_0)\,dt = x(t_0)$

On substituting $t = \lambda$ in the above equation we get, $\int_{-\infty}^{+\infty} x(\lambda)\,\delta(\lambda - t_0)\,d\lambda = x(t_0)$

On substituting $t_0 = t$ in the above equation we get, $\int_{-\infty}^{+\infty} x(\lambda)\,\delta(\lambda - t)\,d\lambda = x(t)$

Since impulse signal is even, $\delta(\lambda - t) = \delta(t - \lambda)$, Therefore the above equation is written as shown below.

$$\int_{-\infty}^{+\infty} x(\lambda)\,\delta(t - \lambda)\,d\lambda = x(t)$$

SUMMARY

1. An analog signal is a continuous function of an independent variable.
2. When the independent variable of an analog signal is time 't' then the analog signal is called continuous time (CT) signal.
3. In a continuous time signal the magnitude and the independent variable are continuous.
4. The sinusoidal and complex exponential signals are always periodic.
5. The sum of two periodic signals is also periodic if the ratio of their fundamental periods is a rational number.
6. A signal which is neither even nor odd can be expressed as a sum of even and odd signals.
7. Periodic signals are power signals and nonperiodic signals are energy signals.
8. The power of an energy signal is zero and the energy of a power signal is infinite.
9. The causal signals are defined only for $t \geq 0$.
10. The noncausal signals are defined for either $t \leq 0$ or all t.
11. Ideally, an impulse signal is a signal with infinite magnitude and zero duration.
12. Practically, an impulse signal is a signal with large magnitude and short duration.
13. The homogenous solution is the response of a system when there is no input, whereas the particular solution is the response for specific input.
14. The free or natural response is the response of the system due to initial stored energy, whereas the forced response is the response due to a particular input when there is no initial energy.
15. The response (or total response) of an LTI system is the sum of natural and forced response.
16. The response of a static system depends on present input whereas, the response of a dynamic system depends on present, past and future inputs.
17. The dynamic systems require memory whereas the static systems do not require memory.
18. In time invariant systems the input-output characteristics do not change with time.
19. In time invariant systems, if a delay is introduced either at input or at output, the response remains same.
20. Linear systems will satisfy the principle of superposition.
21. In linear systems, the response for weighted sum of inputs is equal to similar weighted sum of individual responses.
22. In causal systems, the response depends only on past and present values of inputs/outputs.
23. In noncausal systems, the response depends on future inputs/outputs.
24. In stable systems, for any bounded input, the output will be bounded.
25. The present output of a feedback system depends on past outputs.
26. The response of an LTI system is given by convolution of input and impulse response.
27. The unit step response of an LTI system is given by the integral of its impulse response.
28. The convolution operation satisfies the commutative, associative and distributive properties.
29. When LTI systems are connected in cascade, the overall impulse response is given by the convolution of individual impulse responses.
30. When LTI systems are connected in parallel, the overall impulse response is given by the sum of its individual impulse responses.
31. The inverse system is used to recover the input from the response of a system.
32. The cascade of a system and its inverse is the identity system.
33. The deconvolution is the process of recovering input from the response of a system.