

Properties of ROC

- Property-1 :** The ROC of X(s) consists of strips parallel to the $j\Omega$ - axis in the s-plane.
- Property-2 :** If x(t) is of finite duration and is absolutely integrable, then the ROC is the entire s- plane.
- Property-3 :** If x(t) is right sided, and if the line passing through $\text{Re}(s) = \sigma_0$ is in ROC, then all values of s for which $\text{Re}(s) > \sigma_0$ will also be in ROC.
- Property-4 :** If x(t) is left sided, and if the line passing through $\text{Re}(s) = \sigma_0$ is in ROC, then all values of s for which $\text{Re}(s) < \sigma_0$ will also be in ROC.
- Property-5 :** If x(t) is two sided, and if the line passing through $\text{Re}(s) = \sigma_0$ is in ROC, then the ROC will consists of a strip in the s-plane that includes the line passing through $\text{Re}(s) = \sigma_0$.
- Property-6 :** If X(s) is rational, (where X(s) is Laplace transform of x(t)), then its ROC is bounded by poles or extends to infinity.
- Property-7 :** If X(s) is rational, (where X(s) is Laplace transform of x(t)), then ROC does not include any poles of X(s).
- Property-8 :** If X(s) is rational, (where X(s) is Laplace transform of x(t)), and if x(t) is right sided, then ROC is the region in s-plane to the right of the rightmost pole.
- Property-9 :** If X(s) is rational, (where X(s) is Laplace transform of x(t)), and if x(t) is left sided, then ROC is the region in s-plane to the left of the leftmost pole.

TWO MARKS QUESTIONS AND ANSWERS

1. Define CT signal

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by x(t).
Eg: AC waveform, ECG etc.

2. Compare double sided and single sided spectrums.

The method of representing spectrums of positive as well as negative frequencies are called double sided spectrums.
The method of representing spectrums only in the positive frequencies is known as single sided spectrums.

3. Define Quadrature Fourier Series.

Consider x(t) be a periodic signal. The fourier series can be written for this signal as follows

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 n t + \sum_{n=1}^{\infty} b_n \sin \omega_0 n t$$

This is known as Quadrature Fourier Series.

4. Define polar Fourier Series.

$$x(t) = D_0 + \sum_{n=1}^{\infty} D_n \cos((2\pi n t) / T_0)$$

The above form of representing a signal is known as Polar Fourier series.

5. Define exponential fourier series.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t / T_0}$$

The method of representing a signal by the above form is known as exponential fourier series.

6. State Dirichlets conditions.

- (i). The function x(t) should be single valued within the interval T0
- (ii). The function x(t) should have atmost a finite number of discontinuities in the interval T0

- (iii). The function $x(t)$ should have finite number of maxima and minima in the interval T_0
- (iv). The function should have absolutely integrable.

7. State Parseval's power theorem.

Parseval's power theorem states that the total average power of a periodic signal $x(t)$ is equal to the sum of the average powers of its phasor components.

8. Define Fourier Transform.

Let $x(t)$ be the signal which is the function of time t . The Fourier transform of $x(t)$ is given by

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

9. State the conditions for the existence of Fourier series.

- (i). The function $x(t)$ should be single valued in any finite time interval T
- (ii). The function $x(t)$ should have at most finite number of discontinuities in any finite time interval T .
- (iii). The function $x(t)$ should have finite number of maxima and minima in any time interval T .
- (iv) The function $x(t)$ should be absolutely integrable.

10. Find the Fourier transform of function $x(t) = \delta(t)$

Ans: 1

11. State Rayleigh's energy theorem.

Rayleigh's energy theorem states that the energy of the signal may be written in frequency domain as superposition of energies due to individual spectral frequencies of the signal.

12. Define Laplace transform.

Laplace transform is the another mathematical tool used for analysis of continuous time signals and systems. It is defined as

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

13. Obtain the Laplace transform of ramp function.

Ans: $1/s^2$

14. What are the methods for evaluating inverse Laplace transform.

The two methods for evaluating inverse Laplace transform are

- (i). By Partial fraction expansion method.
- (ii). By convolution integral.

15. State initial value theorem.

If $x(t) \leftrightarrow X(s)$, then value of $x(t)$ is given as,

$$\lim_{s \rightarrow \infty} sX(s)$$

$$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$$

provided that the first derivative of $x(t)$ should be Laplace transformable.

16. State final value theorem.

If $x(t)$ and $X(s)$ are Laplace transform pairs, then the final value of $x(t)$ is given as,

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

17. State the convolution property of Fourier transform.

If $x_1(t)$ and $x_2(t)$ are Fourier transform pairs and $X_1(f)$ and $X_2(f)$ are Fourier transform pairs, then

$$\int_{-\infty}^{\infty} x_1(t)x_2(f-t) dt \text{ is Fourier transform pair with } X_1(f)X_2(f)$$

18. What is the relationship between Fourier transform and Laplace transform.

$$X(s) = X(j\omega) \text{ when } s = j\omega$$

This states that laplace transform is same as fourier transform when $s = j\omega$.

19. Find the fourier transform of sgn function.

$$\text{Ans: } 2/j\omega$$

20. Find out the laplace transform of $f(t) = e^{at}$

$$\text{Ans: } 1/(s-a)$$