

MA8151 MATHEMATICS – I**13 Mark Important Questions****Part-B****Unit-I**

1. Guess the value of the limit (if it exists) for the function $\lim_{x \rightarrow 0} \frac{e^{5x}-1}{x}$ by evaluating the function at the given numbers $x = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$ (correct to six decimal places).
2. For the function $f(x) = 2 + 2x^2 - x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.
3. (i) Find the values of a and b that make f continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} \frac{x^3-8}{x-2}, & \text{if } x < 2 \\ ax^2 - bx + 3, & \text{if } 2x \leq x \leq 3 \\ 2x - a + b, & \text{if } x \geq 3 \end{cases}$$

(ii) find the derivative of $f(x) = \cos^{-1} \left(\frac{b+a\cos x}{a+b\cos x} \right)$

(iii) find y' for $\cos(xy) = 1 + \sin y$

4. If $f(x) = \frac{1-x}{2+x}$ then, find the equation for $f'(x)$ using the concept of derivatives.
5. Find the derivative of $f(x) = \tanh^{-1} \left[\tan \frac{x}{2} \right]$.

Unit-II

1. If $u = f \left(\frac{y-x}{xy}, \frac{z-x}{xz} \right)$, find $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$.
2. Find the maxima and minima of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.
3. Find the Taylor's series expansion of function of $f(x) = \sqrt{1+x+y^2}$ powers of (x, y) and y up to second degree terms.
4. Find the minimum distance from the point $(1, 2, 0)$ to the cone $z^2 = x^2 + y^2$.
5. For the given function $z = \tan^{-1} \left(\frac{x}{y} \right) - (xy)$, verify whether the statement $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

Unit-III

- Using integration by parts, evaluate $\int \frac{(\ln x)^2}{x^2} dx$
- Evaluate $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5 \sqrt{9x^2 - 1}}$.
- Establish a reduction formula for $I_n = \int \sin^n x dx$. Hence, find $\int_0^{\frac{\pi}{2}} \sin^n x dx$.
- Evaluate $\int e^x \sin x dx$ by using integration by parts.
- Evaluate $\int_0^x \sin^2 x \cos^4 x dx$.

Unit-IV

- Evaluate $\iint xy(x+y) dx dy$ over the area between $y = x^2$ and $y = x$.
- Express $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} dx dy$ in polar coordinates and then evaluate it.
- Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$.
- Evaluate $\iint (xy) dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.
- Change the order of integration for the given integral $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} (x^2 + y^2) dy dx$ and evaluate it.

Unit-V

- Solve $(D^2 + 4D + 5)y = e^x + x^3 + \cos 2x + 1$.
- Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \left(\frac{\ln x}{x}\right)^2$
- Solve $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t$, $\frac{dx}{dt} - 2x + \frac{dy}{dt} = \sin 2t$.
- Solve $y'' - 4y' + 4y = (x + 1)e^{2x}$ by the method of variation of parameters.
- Solve the simultaneous differential equation $Dx + y = \sin 2t$ and $-X + Dy = \cos 2t$.