MA8151 MATHEMATICS – I

13 Marks Question Bank

Part-B

Unit-I

1. Guess the value of the limit (if it exists) for the function $\lim_{x\to o}\frac{e^{5x}-1}{x}$ by evaluating the function at the given numbers x=$\pm$ 0.5, $\pm$ 0.1, $\pm$ 0.01, $\pm$ 0.001, $\pm$ 0.0001 (correct to six decimal places.
2. For the function f(x) = 2 + 2x^2 – x^4 , find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.
3. (i) Find the values of a and b that make f continuous on $(-\infty,\infty)$.

   f(x) = $\begin{cases} \frac{x^3-8}{x-2}, if\ x<2 \\ ax^2-bx+3, if\ 2x\ \le x\le 3 \\ 2x-a+b, if\ x\ge 3 \end{cases}$

   (ii)find the derivative of f(x) =$\cos^{-1}\left(\frac{b+acosx}{a+bcosx}\right)$

   (iii)find y$'$ for $\cos(xy) = 1+\sin y$

4. If f(x)= $\frac{1-x}{2+x}$ then, find the equation for f ’(x) using the concept of derivatives.
5. Find the derivative of f(x) = $\tanh^{-1}\left[\tan\frac{x}{2}\right]$.
6. For the function f(x) = 2x^3+3x^2-36x.

   (i) Find the intervals on which it is increasing and decreasing.

   (ii) Find the local maximum and minimum values of f.

   (iii) find the intervals of concavity and the inflection points.

7. For what value of the constant “c” is the function “f” continuous on (-∞,∞), $f(x) = \begin{cases} cx^2+2x; x<2 \\ x^3-cx,\ x\ge 2 \end{cases}$
8. Find the local maximum and minimum values of f(x) = $\sqrt{x} - \sqrt[4]{x}$ using both the first and second derivative tests.
9. Find y’ if x^4+y^4 = 16

10. Find the tangent line to the equation $x^3+y^3 = 6xy$ at the point (3,3) and at what point the tangent line horizontal in the first quadrant.
11. Find $\frac{dy}{dx}$ if $y = x^2 e^{2x}(x^2+1)^4$.
12. For what value of the constant b, is the function f continuous on $(-\infty,\infty)$ if $f(x) = \begin{cases} bx^2 + 2x, if \ \ x < 2 \\ x^3 - bx, if \ \ x \geq 2 \end{cases}$
13. If $f(x)=2x^3+3x^3-36x$, find the intervals on which it is increasing or decreasing, the local maximum and local minimum values of f(x).
14. Show that the function $f(x) = 1- \sqrt{1-x^2}$ is continuous in the interval [-1,1].
15. Calculate the absolute maximum and minimum of the function $f(x) = 3x^4-4x^3-12x^2+1$ in [-2,3].

Unit-II

1. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, find $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z}$
2. Find the maxima and minima of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$
3. Find the Taylor's series expansion of function of $f(x) = \sqrt{1+x+y^2}$ powers of (x,y) and y up to second degree terms.
4. Find the minimum distance from the point (1,2,0) to the cone $z^2 = x^2 + y^2$.
5. For the given function $z=\tan^{-1}\left(\frac{x}{y}\right) - (xy)$, verify whether the statement $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$.
6. A thin closed rectangular box is to have one edge equal to twice the other and constant volume 72 m^3. Find the least surface area of the box.
7. Obtain the Taylor's series expansion of $e^x \sin y$ in terms of powers of x and y upto third degree terms.
8. Find the maximum or minimum values of the function $f(x,y) = x^2+y^2+6x+12$.
9. If $u = (x^2+y^2+z^2)^{-1/2}$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
10. Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq.cm.
11. Obtain the Taylor's series expansion of $x^3+y^3+xy^3$ in terms of power of (x-1) and (y-2) up to third degree terms.
12. Find the maximum or minimum value of $f(x,y) = 3x^2-y^2+x^3$.
13. If $u= f(2x-3y, 3y-4z, 4z-2x)$, then find $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$.

14. Find the shortest and the longest distances from the point (1, 2, -1) to the sphere $x^2+y^2+z^2=24$.
15. Examine $f(x,y) = x^3+3xy^2-15x^2-15y^2+72x$ for extreme values.

Unit-III

1. Using integration by parts, evaluate $\int \frac{(lnx)^2}{x^2} dx$
2. Evaluate $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{9x^2-1}}$.
3. Establish a reduction formula for $I_n = \int sin^n x\, dx$. Hence, find $\int_0^{\frac{\pi}{2}} sin^n x\, dx$.
4. Evaluate $\int e^x\, sinx\, dx$ by using integration by parts.
5. Evaluate $\int_0^x sin^2 x\, cos^4 x\, dx$.
6. Evaluate $\int_0^3 (X^3 - 6X)dx$ by using Riemann sum with n sub intervals.
7. Evaluate $\int \sqrt{a^2 - x^2}$ dx by using substitution rule.
8. Evaluate $\int \frac{tanx}{secx+cosx} dx$.
9. evaluate $\int e^{ax} \cos bx\, dx$ using integration by parts.
10. Evaluate $\int \frac{x}{\sqrt{x^2+x+1}} dx$.
11. Evaluate $\int_0^{\frac{\pi}{2}} cos^5 x\, dx$.
12. Evaluate $\int_0^\infty e^{-ax} sinbxdx (a > 0)$ using integration by parts.
13. Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinxcosx}{cos^2x+3cosx+2} dx$.
14. Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}} dx$.
15. Evaluate $\int_0^{\frac{\pi}{2}} x\, tan^2 x\, dx$.

Unit-IV

1. Evaluate $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.
2. Express $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} dxdy$ in polar coordinates and then evaluate it.
3. Find the area bounded by the parabolas $y^2 = 4 - x\ and\ y^2 = x$.
4. Evaluate $\iint (xy) dx\, dy$ over the positive quadrant of the circle $x^2+y^2=a^2$.

5. Change the order of integration for the given integral $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} (x^2 + y^2)dy\,dx$ and evaluate it.
6. Find the area bounded by $y^2 = 4x$ and $x^2 = 4y$ by using double integrals.
7. Evaluate $\int_0^{2a} \int_0^x \int_y^x (x\,y\,z)dz\,dy\,dx$.
8. Evaluate by changing to polar coordinates $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.
9. Change the order of integration for the given integral $\int_0^a \int_0^{2\sqrt{ax}} (x^2)dy\,dx$ and evaluate it.
10. Evaluate $\iiint (xyz)dx\,dy\,dz$ over the first octant $x^2+y^2+z^2 = a^2$.
11. Using double integral, find the area bounded by $y = x$ and $y = x^2$.
12. Change the order of integration in $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\,dx$ and then evaluate it.
13. Evaluate by changing the polar coordinates $\int_0^\propto \int_y^\propto \frac{x^2}{\sqrt{x^2+y^2}} dy\,dx$
14. Evaluate $\iint xydxdy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$.
15. Find the value of $\iiint xyz\,dz\,dy\,dx$ through the positive spherical octant for which $x^2+y^2+z^2 \le a^2$.

Unit-V

1. Solve $(D^2+4D+5)\,y = e^x + x^3 + cos2x + 1$.
2. Solve $x^2 \frac{d^2y}{dx} - x\frac{dy}{dx} + y = \left(\frac{\ln x}{x}\right)^2$
3. Solve $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t,\;\; \frac{dx}{dt} - 2x + \frac{dy}{dt} = sin2t$.
4. Solve $y" - 4y' + 4y = (x+1)e^{2x}$ by the method of variation of parameters.
5. Solve the simultaneous differential equation Dx + y = sin 2t and -X +Dy =cos 2t.
6. Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4$.
7. Solve $\frac{d^2y}{dx^2}$+y = cosec x by using the method of variation of parameters.
8. Solve $(D^2+3D+2)\,y = 4e^{2x} + x$ by using the method of undetermined coefficients.
9. Solve $\frac{d^2y}{dx^2}$+y = cot x by using method of variation of parameters.
10. Solve $(D^2-2D)\,y = 5e^x \cos x$ by using method of undetermined coefficients.
11. Solve $[(x+1)^2D^2+(x+1)\,D+1]\,y = 4 \cos \log (x+1)$.
12. Solve by method of variation of parameters: $\frac{d^2y}{dx^2} + a^2y = \tan ax$.

13. Solve $(D^2+2D+1)\ y = e^x \sin 2x$ by using the method of undetermined coefficients.
14. Solve $\frac{dx}{dt} + \frac{dy}{dt} + 3x = sint, \frac{dx}{dt} = y - x = cost.$
15. Using method of undetermined coefficients solve $(D^2-D-2)\ y = 4x^2$.