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M.E. Structures

Question Paper Code : 40722

M.E./M.Tech. DEGREE EXAMINATION, JANUARY 2019.

First Semester

Aeronautical Engineering

MA 5151 – ADVANCED MATHEMATICAL METHODS

(Common to M.E. Aerospace Technology/M.E. Soil Mechanics and Foundation Engineering/M.E. Structural Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $f''(t)$.

2. Show that $\int_0^t J_0(u)J_0(t-u)du = \sin t$.

3. If the Fourier transform of $f(x)$ is $F(\alpha)$, the find the Fourier transform of $f(x)\cos ax$.

4. State the Parseval's identity for Fourier transforms.

5. State the fundamental lemma of Calculus of Variations.

6. Find the tranversality condition for the functional of the form $v[y(x)] = \int_{x_0}^{x_1} A(x, y)\sqrt{1+y'^2}\,dx$ with the right boundary moving along $y_1 = \varphi(x_1)$.

7. Show that the transformation $z = F(w) + iG(w)$ maps the curve C in the z-plane given by $x = F(t)$, $y = G(t)$ onto the real axis of the w–plane.

8. Find the stagnation points of the flow represented by the complex potential $\Omega(z) = z^2$.

9. If f is a function of 'n' variables x^i, then write the differential of f using summation convention.

10. State the Quotient law of tensors.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Laplace transform of $erf\left(t^{1/2}\right)$. (8)

(ii) Using complex inversion formula, find the inverse laplace transform of $\frac{1}{(s+1)(s-2)^2}$. (8)

Or

(b) Using Laplace Transform method, solve the following problem :

$u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$

$u(0, t) = u(1, t) = 0$, $t > 0$

$u(x, 0) = \sin \pi x$, $u_t(x, 0) = -\sin \pi x$, $0 < x < 1$. (16)

12. (a) Using Fourier Transform method, solve the following problem :

$ku_{xx} = u_t$, $0 \le x < \infty$, $t \ge 0$

$u(x, 0) = 0$, $0 < x < \infty$

$u(0, t) = u_0$, $t \ge 0$ and $u, u_x \to 0$ as $x \to \infty$. (16)

Or

(b) Using Fourier Transforms, solve the Laplace equation in the half-plane $y > 0$ described by :

$u_{xx} + u_{yy} = 0$, $-\infty < x < \infty$, $y > 0$ subject to $u(x, 0) = f(x)$ for $-\infty < x < \infty$;

u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. (16)

13. (a) (i) Find the extremals of the functional

$$v[y(x), z(x)] = \int_{x_0}^{x_1} \left(2yz - 2y^2 + y'^2 - z'^2\right) dx.$$ (8)

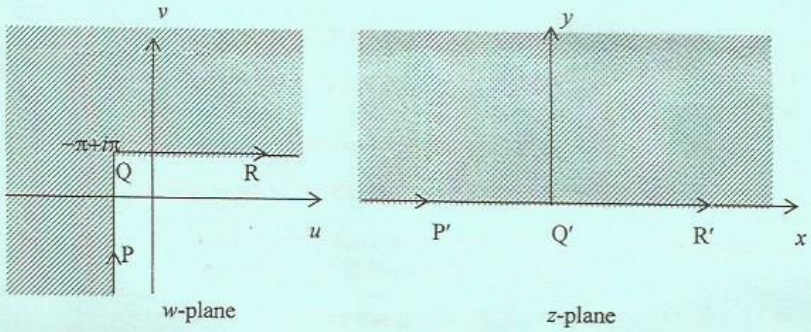
(ii) Find the curve connecting the points A and B which is traversed by a particle sliding from A to B in the shortest time. (8)

Or

(b) (i) Using Ritz's method, solve the boundary value problem $y'' - y + x = 0,\ 0 \le x \le 1,\ y(0) = y(1) = 0$. (8)

(ii) Find the plane curve of fixed perimeter and maximum area. (8)

14. (a) (i) Find a transformation which maps a polygon in the w–plane on to the unit circle in the z-plane. (8)



(ii) Find a function harmonic in the upper half of the z-plane, which takes the prescribed values on the x-axis given by $G(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$. (8)

Or

(b) Find the complex potential due to a source at $z = -a$ and a sink at $z = a$ of equal strengths k. Determine the equipotential lines and streamlines and represent graphically. Also find the speed of the fluid at any point. (16)

15. (a) Find the components of the metric tensor and the conjugate tensor in cylindrical coordinates. (16)

Or

(b) Prove that the covariant derivative of g^{ij} is zero. (16)