Reg. No. :

Question Paper Code : 20746

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS – I

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Computer Science and Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics Engineering/Metallurgical Engineering/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Biotechnology/Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom & Textile Technology/Industrial Biotechnology/Information Technology/Leather Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Rubber and Plastics Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion Technology)/Textile Technology (Textile Chemistry))

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If one of the eigenvalues of the matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & -2 & -6 \\ 3 & 1 & -5 \end{bmatrix}$ is "2" and the matrix is singular, find the other two eigenvalues.

2. Test the nature of the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$.

3. Discuss the convergence of the series $\sum_{1}^{\infty}(-1)^{r+1}$.

4. Define conditionally convergent series.

5. Find the radius of curvature of $y = \log \sin x$ at $x = \frac{\pi}{2}$.

6. Define envelope of a family of curves.

7. If $u = f(x-y, y-z, z-x)$, Find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

8. If $u = \frac{2x-y}{2}$; $v = \frac{y}{2}$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

9. Evaluate $\int_{0}^{\frac{\pi}{2}}\int_{0}^{\frac{\pi}{2}} \sin(\phi+\theta)\, d\theta\, d\phi$.

10. Change the order of integration in $\int_{0}^{\infty}\int_{x}^{\infty} f(x, y)\, dx\, dy$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. (8)

(ii) Verify if the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ satisfies its own characteristic equation. If so, find A^{-2}. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$ to the canonical form by means of an orthogonal transformation. Hence find the rank, index, signature and nature of the quadratic form. (16)

12. (a) (i) Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{n^p}$, by comparison test. (8)

(ii) Examine the convergence of the series

$1-\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}-\frac{1}{7^2}+-\cdots\ldots$ by Leibnitz's test. (8)

Or

(b) (i) Discuss the convergence of the series

$\frac{1}{1+x}+\frac{1}{1+2x^2}+\frac{1}{1+3x^3}+\cdots\ldots\ldots\ldots$; for $x>0$. (10)

(ii) Test the convergence of $\sum_{n=2}^{\infty}\frac{1}{n\log n}$ by integral test. (6)

13. (a) (i) Find the equation of the circle of curvature at (c, c) on $xy=c^2$. (8)

(ii) Find the envelope of a system of concentric ellipses with their axes along the co-ordinate axes and of constant area. (8)

Or

(b) (i) Find the radius of curvature of the following :

$x=a(\theta+\sin\theta);\ y=a(1-\cos\theta)$. (6)

(ii) Find the Evolute of the tractrix $x=a\left(\cos\theta+\log\tan\left(\frac{\theta}{2}\right)\right)$; $y=a\sin\theta$, treating it as the envelope of its normals. (10)

14. (a) (i) Examine if the following functions are functionally dependent. If they are, find also the functional relationship;

$u=x+y+z;\ v=x^2+y^2+z^2;\ w=xy+yz+zx$. (4)

(ii) Examine for extreme values : $x^3+y^3-12x-3y+20$. (12)

Or

(b) (i) Expand $\sin(xy)$ as a Taylor's series in powers of $(x-1)$ and $\left(y-\frac{\pi}{4}\right)$. (8)

(ii) If $z=f(x, y)$, where $x=u\cos\alpha-v\sin\alpha$ and $y=\sin\alpha+v\cos\alpha$, show that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}$. (8)

15. (a) (i) Find the area between $y^2 = 4x$ and $2x - 3y + 4 = 0$. (6)

(ii) Change the order of integration in $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx\,dy$ and then evaluate it. (10)

Or

(b) (i) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + \frac{y}{2} + \frac{z}{3} = 1$. (6)

(ii) Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dx\,dy$ over the positive quadrant of the circle by $x^2 + y^2 = 1$ changing into polar coordinates. (10)
