Reg. No. : [ ]

Question Paper Code : 70761

M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

First Semester

Biometrics and Cyber Security

MA5160 – APPLIED PROBABILITY AND STATISTICS

(Common to M.E. Computer Science and Engineering/M.E. Computer Science and Engineering (With Specialization in Networks)/M.E. Industrial Engineering/ M.E. Manufacturing Engineering/M.E. Multimedia Technology/M.E. Software Engineering/M.Tech. Information Technology)

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Statistical table may be permitted

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. The probability that a student passes a Physics test is 2/3 and the probability that he passes both Physics and an English test is 14/45. The probability that he passes at least one test is 4/5. What is the probability that he passes the English test ?

2. If X has a Poisson distribution such that $P(X = 1) = P(X = 2)$, find $P(X = 4)$.

3. If the joint p.d.f. of (X, Y) is $f(x, y) = 6\, e^{-2x - 3y}$, $x \geq 0$, $y \geq 0$, find the marginal density of X.

4. The correlation coefficient between two random variables X and Y is $r = 0.6$. If $\sigma_x = 1.5$, $\sigma_y = 2$, $\overline{X} = 10$ and $\overline{Y} = 20$, find the regression of Y on X.

5. If T is an unbiased estimator for θ, show that T^2 is a biased estimator for θ^2.

6. State the normal equations to the parabola $y = a + bx + cx^2$.

7. What is the standard error of the sample proportion when the population proportion is known ?

8. State the main uses of Chi-square distribution.

9. State the additional properties of the Multivariate normal distribution.

10. Define the second principle component.

PART – B (5×13=65 Marks)

11. a) i) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bags are white ? (6)

ii) A random variable X has the following probability distribution

x :	– 2	– 1	0	1	2	3
p (x) :	0.1	k	0.2	2k	0.3	3k

Find k, $P(X < 2)$, $P(-2 < X < 2)$, the cdf of X. (7)

(OR)

b) i) Find the moment generating function of the random variable with the probability law $P(X = x) = q^{x-1}p$, x = 1, 2, ... Find the mean and variance. (6)

ii) Suppose the duration X in minutes of long distance calls from your home, follows exponential law with p.d.f. $f(x) = \begin{cases} \frac{1}{5}e^{\frac{-x}{5}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

Find $P(X > 5)$, $P(3 \le X \le 6)$, the mean and variance of X. (7)

12. a) The joint p.d.f of the random variables X and Y is given by

$f(x, y) = \begin{cases} 8xy, & 0 < x < 1, 0 < y < x \\ 0, & \text{otherwise} \end{cases}$

Find $f_X(x)$, $f_Y(y)$, $f(y/x)$, $f(x/y)$ and covariance of (X, Y). (13)

(OR)

b) If $(x, y) = \frac{6-x-y}{8}$, $0 \le x \le 2$, $2 \le y \le 4$ find the correlation coefficient between X and Y. (13)

13. a) i) Let $x_1, x_2, ..., x_n$ be a random sample of a Poisson random variable with unknow parameter λ, determine the maximum likelihood estimator of λ. (6)

ii) Fit a straight line to the following data by the principle of least squares :

x	1	2	3	4	5
y	16	19	23	26	30

(7)

(OR)

b) i) Let $x_1, x_2, ..., x_n$ be a random sample of size n from the normal population $N(\mu, \sigma^2)$. Obtain the estimators of μ and σ^2 by the method moments. (6)

ii) Find the most likely price in Mumbai corresponding to the price of Rs. 70 at Kolkata from the following : **(7)**

	Kolkata	Mumbai
Average price	65	67
Standard deviation	2.5	3.5

Correlation coefficient between the prices of commodities in the two cities is 0.8.

14. a) i) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence, 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5 % level. **(6)**

ii) The following data, give the number of aircraft accidents that occurred during the various days of a week :

Day :	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
No. of accidents :	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. **(7)**

(OR)

b) i) The means of two large samples of 1000 and 2000 members are 67.5 and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches ? **(6)**

ii) The following data represents the biological values of protein from cow's milk and buffalo's milk at a certain level.

Cow's milk :	1.82	2.02	1.88	1.61	1.81	1.54
Buffalo's milk :	2.00	1.83	1.86	2.03	2.19	1.88

Examine if the average values of protein in the two samples significantly differ. **(7)**

15. a) For $\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$, find ρ_{13}, the correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$. (13)

(OR)

b) The random variables X_1 and X_2 have the covariance and correlations matrices $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$, $\rho = \begin{pmatrix} 1 & .4 \\ .4 & 1 \end{pmatrix}$, find the principal components. (13)

PART – C (1×15=15 Marks)

16. a) The p.d.f. of a random variable X is given by $f(x) = kx(2-x)$, $0 \le x \le 2$. Find k, mean, variance, r^{th} moment and the moment generating function of X. (15)

(OR)

b) If the joint p.d.f. of (X, Y) is given by $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$.

Find $P\left(X > \frac{1}{2}\right)$, $P(Y < 1)$, $P\left(Y < \frac{1}{2} \Big/ X < \frac{1}{2}\right)$ and cov (X, Y). (15)