Reg. No. :

Question Paper Code : 70757

M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019
First Semester
CAD/CAM
MA 5156 – APPLIED MATHEMATICS FOR ENGINEERS
(Common to M.E. Computer Aided Design/M.E. Computer Integrated Manufacturing/M.E. Engineering Design/M.E. Mechatronics Engineering/ M.E. Product Design and Development)
(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Find the generalized Eigenvector of rank 3 corresponding to the Eigenvalue $\lambda = 6$ for the matrix $A = \begin{pmatrix} 6 & 1 & 2 \\ 0 & 6 & 1 \\ 0 & 0 & 6 \end{pmatrix}$.

2. What do you mean by canonical basis ?

3. Define the variation of a functional.

4. Prove that the shortest distance between two points in a plane is a straight line.

5. A problem is given to 3 students A, B, C whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the probability that it is solved ?

6. A continuous random variable has the probability density function given by $f(x) = K(x-1)^3$, $1 \le x \le 3$, find K.

7. Find the inverse Laplace transform of $\frac{1}{(s+a)^n}$.

8. If $L[f(t); s] = F(s)$, then show that $L\left[t^n f(t); s\right] = (-1)^n \frac{d^n}{ds^n} F(s)$.

9. If $F(\alpha)$ is the Fourier transform of f(x), then find the Fourier transform of f(x) cos ax.

10. State the convolution theorem for Fourier transform.

PART – B (5×13=65 Marks)

11. a) i) Find the Cholesky decomposition for the matrix $A = \begin{pmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{pmatrix}$. (7)

ii) Find the least square solution of AX = B, where $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \end{pmatrix}$. (6)

(OR)

b) Find QR factorization for the matrix $A = \begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$. (13)

12. a) i) Solve the variational problem $v[y(x)] = \int_{-l}^{l}\left(\frac{1}{2}\mu y''^2 + \rho y\right)dx$ that satisfies the boundary conditions $y(-l) = 0, y'(-l) = 0, y(l) = 0, y'(l) = 0$. (7)

ii) Show that the curve which extremizes the functional $\int_0^{\pi/4}(y''^2 - y^2 + x^2)\,dx$ under the conditions $y(0) = 0, y'(0) = 1, y\left(\pi/4\right) = y'\left(\pi/4\right) = 1/\sqrt{2}$ is y = sinx. (6)

(OR)

b) i) Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume. (7)

ii) Find the extremum of the functional $v[y(x)] = \int_0^1 (y'^2 + y^2)\,dx, y(0) = 0,$ $y(1) = 1$ using Rayleigh-Ritz method. (6)

13. a) i) Find the mean, variance and the moment generating function of a Poisson random variable. (7)

ii) If X is uniformly distributed in the interval [0, 10], find $P(X < 4)$, $P(X > 7)$ and $P(1 < X < 6)$. (6)

(OR)

b) i) State and prove the memoryless property of an exponential distribution. (7)

ii) A trainee Soldier shoots a target in an independent fashion. If the probability that the target is hit is 0.8, what is the probability that it takes him less than 5 shots ? (6)

14. a) i) Find the inverse Laplace transform of $\frac{1}{(s+1)(s-2)^2}$ by complex inversion formula. (7)

ii) Apply the convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2};t\right]$. (6)

(OR)

b) An infinitely long string having one end at $x = 0$ is initially at rest on the x-axis. The end $x = 0$ undergoes a periodic transverse displacement described by $A_0 \sin \omega t$, $t > 0$. Using Laplace transform, find the displacement of any point on the string at any time t. (13)

15. a) Obtain the solution of free vibrations of a semi-infinite string governed by $u_{tt} = c^2 u_{xx}$, $0 < x < \infty$, $t > 0$ with initial conditions; $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$. (13)

(OR)

b) Using the Fourier cosine transform, find the temperature u(x, t) in a semi-infinite rod $0 \leq x < \infty$, determined by the PDE $ku_{xx} = u_t$, $0 < x < \infty$, $t > 0$ subject to the IC : $u(x, 0) = 0$, $0 \leq x < \infty$, and the BC : $U_x(0, t) = -u_0$(a constant) when $x = 0$ and $t > 0$; u, $\partial u/\partial x$ both tend to zero as $x \to \infty$. (13)

PART – C (1×15=15 Marks)

16. a) Find singular value decomposition for the matrix $A = \begin{pmatrix} 1 & -1 \\ -1 & 2 \\ 2 & -2 \end{pmatrix}$. (15)

(OR)

b) A very long homogeneous rod, one end of which is exposed to a time-varying heat reservoir. If the initial temperature distribution is 0°C along the rod, then solve $u_{xx} = a^{-2}u_t$, $0 < x < \infty$, $t > 0$; B.C. : $u(0, t) = f(t)$, $u(x, t) \to 0$ as $x \to \infty$; I.C. : $U(x, 0) = 0$, $0 < x < \infty$ by applying Laplace transform. (15)

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