Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 77188

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

2. Find α such that $\vec{F} = (3x-2y+z)\vec{i} + (4x+\alpha y-z)\vec{j} + (x-y+2z)\vec{k}$ is solenoidal.

3. Solve : $(D^3 + D^2 + 4D + 4)y = 0$.

4. Transform the equation $(2x+3)^2 y'' - 2(2x+3)y' + 2y = 6x$ in to a linear differential equation with constant coefficients.

5. State the sufficiency condition for the existence of Laplace transform.

6. Evaluate $\int_0^\infty te^{-2t}\sin t \, dt$ using Laplace transform.

7. Show that $|z|^2$ is not analytic at any point.

8. Find the invariant points of the transformation $w = \frac{z-1}{z+1}$.

9. State Cauchy's integral theorem.

10. Identify the type of singularity of function $\sin\left(\frac{1}{1-z}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. Hence find the value of $\nabla^2\left(\frac{1}{r}\right)$. (8)

(ii) Using Green's theorem, evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (16)

12. (a) (i) Solve : $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$. (8)

(ii) Solve the simultaneous differential equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t, \quad \frac{dx}{dt} + y - x = \cos t. \quad (8)$$

Or

(b) (i) Solve : $(x^2D^2 - xD + 1)y = \log x + \pi$. (8)

(ii) Solve, by the method of variation of parameters, $y'' - 2y' + y = e^x \log x$. (8)

13. (a) (i) Find the Laplace transform of the triangular wave function $f(t)$ defined by

$$f(t) = \begin{cases} t & \text{in } 0 < t \le c \\ 2c - t & \text{in } c < t < 2c \end{cases} \text{ and } f(t+2c) = f(t) \text{ for all } t. \quad (8)$$

(ii) Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$. (8)

Or

(b) (i) Solve the differential equation $y'' - 3y' + 2y = 4t + e^{3t}$, where $y(0) = 1$ and $y'(0) = -1$ using Laplace transforms. (10)

(ii) Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (6)

14. (a) (i) Determine the analytic function $w = u + iv$ if $u = e^{2x}(x\cos 2y - y\sin 2y)$. (8)

(ii) Show that a harmonic function 'u' satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ and hence prove that $\log|f'(z)|$ is harmonic, where $f(z)$ is a regular function. (8)

Or

(b) (i) Find the image in the w-plane of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively. (8)

15. (a) (i) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is $|z| = 3$. (8)

(ii) Find the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ valid in the region $|z| < 2$ and $2 < |z| < 3$. (8)

Or

(b) Evaluate $\int_0^\infty \frac{dx}{\left(x^2 + a^2\right)^2}$, $(a > 0)$ using contour integration. (16)
