Reg. No. :

Question Paper Code : 50772

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

First Semester

Civil Engineering

MA 6151 – MATHEMATICS – I

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Electrical and Electronics Engineering/ Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/ Manufacturing Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/ Mechatronics Engineering/Medical Electronics Engineering/Metallurgical Engineering/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Biotechnology/Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom & Textile Technology/Industrial Biotechnology/Information Technology/Leather Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Rubber and plastics Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion Technology)/Textile Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Find the sum and product of the eigenvalues of a 3 × 3 matrix A whose characteristic equation is $\lambda^3 - 7\lambda^2 + 36 = 0$.

2. If $\lambda (\neq 0)$ is an eigenvalue of a square matrix A, then show that λ^{-1} is an eigenvalue of A^{-1}.

3. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$, using integral test.

4. Show that an absolutely convergent series is convergent.

5. Define geometrically curvature of the curve and centre of curvature at a point.

6. Define the evolute and involute of the curves.

7. Find du/dt when $u = x^2 y$, $x = t^2$ and $y = e^t$.

8. If $x = u(1 + v)$ and $y = v(1 + u)$, find $\partial(x, y)/\partial(u, v)$.

9. Find the area bounded by the line $y = x$ and parabola $x^2 = y$.

10. Evaluate the triple integral $\int_1^3 \int_2^3 \int_1^2 x^2 yz \, dx \, dy \, dz$.

PART – B (5×16=80 Marks)

11. a) i) Show that $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ satisfies its own characteristic equation and hence find A^{-1}. (8)

ii) The eigenvectors of a 3 × 3 real symmetric matrix A corresponding to eigenvalues 1, 3 and 3 are $(1\ 0\ -1)^T$, $(1\ 0\ 1)^T$ and $(0\ 1\ 0)^T$ respectively. Find the matrix A by an orthogonal transformation. (8)

(OR)

b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into the canonical form by an orthogonal transformation and find the index, signature and nature of the quadratic form. (16)

12. a) i) Examine the character of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots\text{to } \infty$ where $0 < x < 1$. (8)

ii) Test for the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{(n^2+1)} - n\right)$, using comparison test. (8)

(OR)

b) i) Find the interval of convergence of the series

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots$ to ∞. (8)

ii) Test whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ is conditionally convergent or absolutely convergent. (8)

13. a) i) Find the radius of the curvature at (a, 0) on the curve $xy^2 = a^3 - x^3$. (8)

ii) Find the evolute of the parabola $x^2 = 4ay$. (8)

(OR)

b) i) Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at the point (3, 6). (10)

ii) Find the envelope of the family of straight lines given by $x \cos\alpha + y \sin\alpha = a \sec\alpha$, where α is the parameter. (6)

14. a) i) Examine the function $f(x, y) = x^3 y^2 (12 - x - y)$ for extreme values. (8)

ii) Expand $\sin(xy)$ in powers of $(x - 1)$ and $(y - (\pi/2))$ up to second degree terms by using Taylor's series. (8)

(OR)

b) i) If $z = f(x, y)$, where $x = e^u \cos v$ and $y = e^u \sin v$, then show that

$x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$. (8)

ii) The temperature T at any point (x, y, z) in a space is $T = 400\,xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (8)

15. a) i) Evaluate integral $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ by changing the order of integration. (8)

ii) Find, by using triple integrals, the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and x + y + z = a. (8)

(OR)

b) i) Evaluate $\iint r^3\,dr\,d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. (8)

ii) Evaluate $\iiint_V \frac{1}{x^2 + y^2 + z^2}\,dx\,dy\,dz$, where V is the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by changing to spherical polar coordinates. (8)
