

Reg. No. :

Question Paper Code : 72061

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/
Computer Science and Engineering/Electrical and Electronics
Engineering/Electronics and Communication Engineering/Electronics and
Instrumentation Engineering/Environmental Engineering/Geoinformatics
Engineering/Industrial Engineering/Industrial Engineering and
Management/Instrumentation and Control Engineering/Manufacturing
Engineering/Materials Science and Engineering/Mechanical Engineering/
Mechanical and Automation Engineering/Mechatronics Engineering/Medical
Electronics Engineering/Metallurgical Engineering/Petrochemical Engineering/
Production Engineering/Robotics and Automation Engineering/ Biotechnology/
Chemical Engineering/Chemical and Electrochemical Engineering/Fashion
Technology/Food Technology/Handloom & Textile Technology/Industrial
Biotechnology/Information Technology/Leather Technology/Petrochemical
Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic
Technology/Polymer Technology/Rubber and Plastics Technology/Textile
Chemistry/Textile Technology/Textile Technology (Fashion Technology)/
Textile Technology (Textile Chemistry))

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

- Two eigenvalues of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. What is the third eigenvalue? What is the product of the eigenvalues of A ?
- Find the constants a and b such that the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as its eigenvalues.

3. Test the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty$.
4. Examine the convergence of the sequence $u_n = 2n$.
5. Define Evolute and Involute.
6. Find the envelop of the family of lines $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$, θ being the parameter,
7. If $u = \sin^{-1} \left[\frac{x^3 - y^2}{x + y} \right]$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$.
8. Find $\frac{du}{dt}$, if $u = \frac{x}{y}$, where $x = e^t$, $y = \log t$.
9. Evaluate: $\int_0^{\pi} \int_0^{\sin \theta} r dr d\theta$.
10. Evaluate: $\int_1^3 \int_1^4 \int_1^4 xyz dx dy dz$.

PART B — (5 × 16 = 80 marks)

11. (a) Verify Cayley-Hamilton theorem find A^4 and A^{-1} when

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Or

- (b) Reduce the matrix $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$ to diagonal form. (16)

12. (a) (i) Test the convergence and absolute convergence of the series. (8)

$$\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{5}+1} + \dots$$

- (ii) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$. (8)

Or

- (b) (i) Test the series $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - n)$. (8)

- (ii) Test the convergence of the sum

$$\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \dots$$
 (8)

13. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, considering it as the envelope of its normals. (8)

- (ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by $a^2 + b^2 = c^2$, c being a constant. (8)

Or

- (b) (i) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos \frac{\theta}{2}$. (8)

- (ii) Find the circle of curvature at (3,4) on $xy = 12$. (8)

14. (a) (i) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (8)

- (ii) Find the minimum values of x^2yz^3 subject to the condition $2x + y + 3z = a$. (8)

Or

- (b) (i) Obtain the Taylor series of $x^3 + y^3 + xy^2$ at (1,2). (8)

- (ii) If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{-1}{2} \cot u$. (8)

15. (a) (i) Change the order of integration and hence evaluate it

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dy \, dx. \quad (8)$$

(ii) Evaluate : $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) \, dx \, dy \, dz.$ (8)

Or

(b) (i) Evaluate $\iint (x - y) \, dx \, dy$ over the region between the line $y = x$ and the parabola $y = x^2$. (8)

(ii) Find the value of $\iiint xyz \, dx \, dy \, dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$. (8)