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**Question Paper Code : 41303**

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

Common to Mechanical Engineering (Sandwich) Aeronautical Engineering/  
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Civil  
Engineering/Civil Engineering and Computer Based Construction/Computer  
Science and Engineering/Computer and Communication Engineering/Electrical  
and Electronics Engineering/Electronics and Communication Engineering/  
Electronics and Instrumentation Engineering/Environmental Engineering/  
Geoinformatics Engineering/Industrial Engineering/Industrial Engineering  
and Management/Instrumentation and Control Engineering/Manufacturing  
Engineering/Material Science and Engineering/Mechanical and Automation  
Engineering/Mechatronics Engineering/Medical Electronics/Metallurgical  
Engineering/Petrochemical Engineering/Production Engineering/Robotics and  
Automation Engineering/B.E./B.Tech. (Common to all Branches except Marine  
Engg.)/Bio Technology/Chemical Engineering/Chemical and Electrochemical  
Engineering/Fashion Technology/Food Technology/Handloom and Textile  
Technology/Industrial Bio Technology/Information Technology/Leather  
Technology/ Petrochemical Technology/ Petroleum Engineering/Pharmaceutical  
Technology/Plastic Technology/Polymer Technology/Rubber and Plastics  
Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion  
Technology)/Textile Technology (Textile Chemistry)  
(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the eigen values of the matrix  $A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$  and hence find the eigen values of  $A^{-1}$ .
2. Discuss the nature of the quadratic form  $2x^2 + 2xy + 3y^2$ .

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3. State the necessary condition for the convergence of series of positive terms.
4. Define absolutely convergent and conditionally convergent of a series.
5. Find the curvature of the curve  $2x^2 + 2y^2 + 5x - 2y + 1 = 0$ .
6. List two important properties of the evolute.
7. If  $x = r^2 - \theta^2$ ,  $y = 2r\theta$  find  $\frac{\partial r}{\partial x}$ .
8. When is a function said to be stationary at a point  $(x, y)$  ?
9. Evaluate  $\int_{-1}^2 \int_x^{x+2} dy dx$ .
10. Evaluate  $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^4 r^3 \sin \theta dr d\theta d\phi$ .

PART - B

(5×16=80 Marks)

11. a) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$  and hence find  $A^{-1}$ . (10 + 6)

(OR)

- b) Reduce the following quadratic form to a canonical form by orthogonal reduction and find the rank, index signature and the nature of the quadratic form :  
 $(-x^2 + y^2 + 4yz + 4zx)$ . (8+2+2+2+2)
12. a) i) Use integral test to check the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$  (8)  
 ii) Test for the convergence of the series  $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$  by D'Alembert's Ratio test. (8)
- (OR)
- b) i) Discuss the convergence of the series  $\frac{5}{2} - \frac{7}{4} + \frac{9}{6} - \frac{11}{8} + \dots$  by Leibnitz's rule. (8)  
 ii) Test  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$  for convergence and absolute convergence. (8)



13. a) i) Find the circle of the curvature at (0, 0) on  $x + y = x^2 + y^2 + x^3$ . (8)

ii) Find the evolute of the four cusped hypocycloid  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  (8)

(OR)

b) i) Find the envelope of  $\frac{x}{a} + \frac{y}{b} = 1$  subject to  $a^n + b^n = c^n$  given  $c$  is a known constant. (8)

ii) Considering the evolute of a curve as the envelope of the normals, find the

evolute of  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (8)

14. a) i) If  $f_1 = u - x - y - z = 0$ ,  $f_2 = uv - y - z = 0$ ,  $f_3 = uvw - z = 0$  then prove that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v \quad (8)$$

ii) Find the Taylors series expansion for  $f(x, y) = x^2 + y^2 + 2xy$  at (1, 1) upto second degree terms. (8)

(OR)

b) i) Find the maxima and minima of  $xy(a - x - y)$ . (8)

ii) The temperature  $u(x, y, z)$  at any point in space is  $u = 400xyz^2$ . Find the highest temperature on surface of the sphere  $x^2 + y^2 + z^2 = 1$ . (8)

15. a) i) Change the order of integration in  $I = \int_{-a}^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2) dy dx$ . (4)

ii) Evaluate  $\iint_A (x^2 + y^2) dx dy$  where  $A$  is the area bounded by the curves

$$x^2 = y, x = 1, x = 2 \text{ and the } x \text{ axis.} \quad (12)$$

(OR)

b) i) Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  and hence evaluate  $\int_0^\infty e^{-x^2} dx$ . (6+2)

ii) Find the volume of the tetrahedron bounded by the coordinate planes and  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ . (8)