Reg. No. :

Question Paper Code : 10849

M.E./M.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

First Semester

Aeronautical Engineering

MA 5151 – ADVANCED MATHEMATICAL METHODS

(Common to M.E. Aerospace Technology/M.E. Soil Mechanics and Foundation Engineering/M.E. Structural Engineering)

(Regulation 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find $L\{t^2 e^{2t}\}$.

2. Find the inverse Laplace transform of the function $\frac{2p+1}{p\,(p+1)}$.

3. If $F(s)$ is the Fourier transform of $f(x)$, then show that the Fourier transform of $f(ax)$ is $\frac{1}{a} F\left(\frac{s}{a}\right)$.

4. State Parseval's identity for Fourier transform.

5. When will the curves $y = y(x)$ and $y = y_1(x)$ be close in the sense of first order proximity?

6. For what value of 'a', the variational problem $v[y(x)] = \int_0^1 (y^2 + x^2 y')dx; y(0) = 0$, $y(1) = a$ has no extremal.

7. State any two properties of conformal mapping.

8. Find all the points at which the mapping $z(z^4 - 5)$ is not conformal.

9. Define inner product of two tensors.

10. If A_i is a covariant tensor, then prove that $\frac{\partial A_i}{\partial x^j}$ does not form a tensor.

PART B — (5 × 13 = 65 marks)

11\. (a) (i) Use the convolution theorem to find $L^{-1}\left[\frac{p^2}{(p^2+4)^2}\right]$. (6)

(ii) Prove that $L\{J_0(t)\} = \frac{1}{\sqrt{1+p^2}}$ and hence deduce that $L\{J_0(at)\} = \frac{1}{\sqrt{p^2+a^2}}$. (7)

Or

(b) A string is stretched between two fixed points $(0,0)$ and $(c,0)$. If it is displaced into the curve $y = b\sin\left(\frac{\pi x}{a}\right)$ and released from rest in that position at time $t = 0$, find its displacement at any time $t > 0$ and at any point $0 < x < c$. [Use Laplace transform to solve]. (13)

12\. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\left(\frac{x}{2}\right)dx$. (13)

Or

(b) If the initial temperature of an infinite bar is given by $u(x,0) = \begin{cases} 1 & \text{for } -c < x < c \\ 0 & \text{otherwise} \end{cases}$, determine the temperature at any point x and at any time $t(> 0)$. (13)

13\. (a) The problem of the brachistochrone: find the curve connecting given points A and B which is traversed by a particle sliding from A to B in the shortest time (friction and resistance of the medium are ignored) (A and B do not lie on a vertical line). (13)

Or

(b) By Ritz method, find a solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x, y)$ inside a rectangle D, $0 \le x \le a$, $0 \le y \le b$, that vanishes on the boundary of D. (13)

14. (a) (i) Show that the bilinear transformation $w = \frac{1}{z}$ maps the totality of circles and straight lines in the z-plane onto the totality of circles and straight lines in the w-plane. (6)

(ii) Find the bilinear transformation that maps $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. (7)

Or

(b) (i) Find the transformation which maps the semi infinite strip of width π bounded by the lines $v = 0, v = \pi$ and $u = 0$ into the upper half of the z-plane. [Use Schwarz-Christoffel transformation]. (6)

(ii) Find the streamlines, potential lines and complex velocity of the motion of a fluid with complex potential $\Omega(z) = i\,k \ln z$, $k > 0$. (7)

15. (a) If the metric is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1 dx^2 + 4dx^2.dx^3$, evaluate g and g^{ij}. (13)

Or

(b) Prove that (i) $[ij, m] = g_{km}\begin{bmatrix} k \\ i\,j \end{bmatrix}$ (ii) $[ik, j] + [jk, i] = \frac{\partial g_{ij}}{\partial x^k}$

(iii) $\frac{\partial g_{ij}}{\partial x^k} = -g^{il}\begin{bmatrix} i \\ l\,k \end{bmatrix} - g^{im}\begin{bmatrix} j \\ m\,k \end{bmatrix}$. (13)

PART C — (1 × 15 = 15 marks)

16. (a) Define unit step function and find its Laplace transform. Write the following function using unit step functions and find its Laplace transform (15)

$$f(t) = \begin{cases} 2 & if \quad 0 < t < 1 \\ \frac{1}{2}t^2 & if \quad 1 < t < \frac{\pi}{2} \\ \cos t & if \quad t > \frac{\pi}{2} \end{cases}$$

Or

(b) Obtain the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ and show that $f(x)$ is self reciprocal under Fourier transform. (15)

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