Reg. No. :

Question Paper Code : 10852

M.E./M.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

First Semester

Communication Systems

MA 5154 — APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Common to M.E. Communication and Networking)

(Regulation 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the generalized eigenvector of rank 3 corresponding to the eigenvalue $\lambda = 7$ for the matrix $A = \begin{pmatrix} 7 & 1 & 2 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{pmatrix}$.

2. Define Toeplitz matrix.

3. Define feasible solution and basic feasible solution to a general L.P.P.

4. What is meant by degeneracy in transportation problem?

5. Write down Adams-Bash forth predictor-corrector transportation formulae.

6. When is a numerical method called as unconditionally unstable?

7. If A, B and C are any 3 events such that $P(A) = P(B) = P(C) = \frac{1}{4}$, $P(A \cap B) = P(B \cap C) = 0$; $P(C \cap A) = \frac{1}{8}$. Find the probability that at least 1 of the events A, B and C occurs.

8. The joint probability mass function of (X, Y) is given $p(x, y) = k(2x + 3y)$, $x = 0, 1, 2; y = 1, 2, 3$. Find the value of k.

9. Write down the Little's formulae for the average waiting time in the system and in the queue for an $(M/M/1):(K/FIFO)$ queuing model.

10. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. What is the probability that it will take him more than 10 minutes to complete his call?

PART B — (5 × 13 = 65 marks)

11. (a) Obtain the QR factorization of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. (13)

Or

(b) Construct singular value decomposition for the matrix $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$. (13)

12. (a) Use simplex method to maximize $z = 2x_1 - x_2 + x_3$

Subject to the constraints :

$3x_1 + x_2 + x_3 \le 60,$

$x_1 - x_2 + 2x_3 \le 10$

$x_1 + x_2 - x_3 \le 20$

$x_1, x_2, x_3 \ge 0$ (13)

Or

(b) A pharmaceutical company producing a simple product is selling it through five agencies located in different cities. All of a sudden, there is a demand for the product in another five cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distance is minimized. The distance between the surplus and deficit cities (in kilometer) is given in the following table:

		Deficit cities				
		a	b	c	d	e
	A	85	75	65	125	75
	B	90	78	66	132	78
	C	75	66	57	114	69
Surplus cities	D	80	72	60	120	72
	E	76	64	56	112	68

Determine the optimum assignment schedule. (13)

13. (a) Apply the fourth order Runge-Kutta method to find $y(0.2)$ given that $f(x,y)=\frac{y-x}{y+x}, y(0)=1$ by taking $h=0.1$. (13)

Or

(b) Solve the boundary value problem $y''=6x$ with $y(1)=2, y(2)=9$ by shooting method. (Assuming a suitable guess value for $y'(1)$). (13)

14. (a) There are 3 true coins and 1 false coin with 'head' on both sides. A coin is chosen at random and tossed 4 times. If 'head' occurs all the 4 times, what is the probability that the false coin has been chosen and used? (13)

Or

(b) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x,y)=xy^2+\frac{x^2}{8}$, $0 \le x \le 2, 0 \le y \le 1$. Compute $P(X>1), P\left(Y<\frac{1}{2}\right), P\left(X>1/Y<\frac{1}{2}\right), P\left(Y<\frac{1}{2}/X>1\right)$ and $P(X<Y)$. (13)

15. (a) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.

(i) What is the probability of all the typists will be busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter to spend for being typed in the system? (13)

Or

(b) A car servicing station has 2 bays where service can be offered simultaneously. Because of space imitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the boys is exponentially distributed with μ = 8 cars per day per bay. Find the average number of cars in the service station, the average number of cars waiting for service and average time a car spends in the system. (13)

PART C — (1 × 15 = 15 marks)

16. (a) If X and Y are independent random variables with probability density functions $f(x)=e^{-x}, x \ge 0$ and $f(y)=e^{-y}, y \ge 0$, find the density functions of $U=\frac{X}{X+Y}$ and $V=X+Y$. Are U and V independent.

Or

(b) A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time of 3 min per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time for the customers if each teller could handle both withdrawals and deposits?

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